A TREATISE

ON

LAND SURVEYING

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ON

LAND SURVEYING

BY JOHN AINSLIE

A NEW AND ENLARGED EDITION

EMBRACING

RAILWAY, MILITARY, MARINE, & GEODETICAL SURVEYING

BY WILLIAM GALBRAITH

M.A. F.R.A.S.

ILLUSTRATED BY THIRTY-TWO ENGRAVINGS ON STEEL, BY W. & A. K. JOHNSTON, AND ONE HUNDRED AND SIXTY-SEVEN ON WOOD, BY BRANSTON

WILLIAM BLACKWOOD AND SONS
EDINBURGH AND LONDON
MDCCCXLIX

PREFACE.

GREAT improvements have been effected of late years in the art of Surveying, alike in the construction of instruments, in the methods of observation, and in the processes of reduction; and it is hoped there will be found in this edition a corresponding advance in accuracy and completeness.

The present Treatise comprehends the whole art of Land-Surveying, and contains examples, progressing from the measurement and planning of a private estate to the delineation of a whole country. Descriptions are given of the various instruments employed by surveyors, including the chain, cross-staff, plane-table, circumferentor, prismatic compass, sextant, and theodolite; together with the best methods of using them in the measurement of boundaries of every species of figure, and in computing the areas of every variety of surface. From the great experience acquired by the late Mr Ainslie in the discharge of his professional duties, this portion of the work will be found peculiarly instructive; and all improvements posterior to his time, in this department, have been now introduced by the Editor. A section is also devoted to the important branch of Military and Marine Surveying, and the Projection of Maps and Charts; while another details the simplest methods, in colouring and shading, by which plans can be made complete and effective.

The subject of Railway Surveying is treated with a care corresponding to its importance; and Tables are given for executing the curves, for estimating the effects of gradients, and for comparing the relative merits of railways—together with instructive examples from lines already constructed.

The entire section treating of the higher departments of Trigonometrical Surveying and Levelling has been added by the Editor; as also the following one, containing a description of the requisite instruments and the manner of using them, with illustrative plates and diagrams. In the Trigonometrical Surveys of their respective territories, the governments of Europe have exhibited somewhat of a national rivalry; and the results have been worthy of this emulation. But the interesting and profound works in which these processes are detailed, are not only so expensive as to be inaccessible to students in general, but many of them are also so recondite as to be unintelligible to the ordinary classes of professional men. To remove such obstacles is the object of the present Treatise, in which the results deducible from these works, and from the Editor's own investigations, are condensed in the form of formulæ, rules, and tables.

Appended is a series of Tables, chiefly useful to the practical man, and which therefore have been rendered full and complete, and clearly explained by numerous examples. A volume of Plates accompanies the work, in which will be found all the varieties of maps and plans used in the various branches of Surveying.

EDINBURGH, *9th May* 1849.

CONTENTS.

LIST OF PLATES.

*** The Plates are bound in a separate volume for convenience of reference.

ERRATA.

Page 179, line 20, for "hypotenuse," read "square of the hypotenuse."
238, line 14, for "position," read "proposition."
255, line 9, for "diagram," read "diaphragm."
259, line 7 from bottom, for "square yards," read "solid yards."
320, line 8 from bottom, for "A F=387.55," read "386.55."
381, line 12 from bottom, for "$45° \times \frac{1}{2} l$," read "$45° + \frac{1}{2} l$."
384, formula 2, for "0.08 a''," read "0.08 σ''."
432, line 9 from bottom, for "cosecant" read "secant."
438, line 6 from bottom, supply "8," under "4."
442, for "Beinnorsh," read "Beinnoosh."

P.S.—In the note at the bottom of page 526, it ought also to have been stated, that the *last term* of Mr Bailey's formula, XLIII., from which Mr Simms' table was computed, is erroneous—in place of $\frac{1}{4}$, the true coefficient is $\frac{1}{8}$, as correctly given in formula (1), page 527.—W. G.

TREATISE

ON

PRACTICAL LAND-SURVEYING.

SECTION FIRST.

ART. I.—DEFINITIONS AND PROBLEMS.

In this Treatise, which is strictly practical, the mathematical demonstrations are omitted. The Surveyor may, however, be assured that the principles upon which the problems are founded are susceptible of strict demonstration. A facility in performing these problems is of the greatest use in practice. But, before proceeding to the problems, it may be proper to lay down the following definitions:—

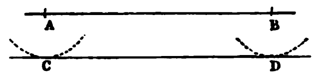
LAND-SURVEYING has for its object the determination of the extent of area contained in horizontal surfaces; for no greater number of poles could be planted perpendicularly upon the surface of a hill, than what can find room to stand upon the plane of its base. Of course, no greater number of plants or trees, all of which grow upright, could find room upon the hill's surface than what there is room for on its base.

Surfaces consist of length and breadth only, and do not, like solids, infer their constitution from the three dimensions of length, breadth, and thickness.

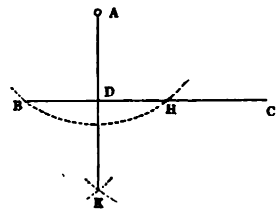
Lines, whether straight lines or curves, are the mere boundaries of surfaces, and, as such, are to be considered as having only length without breadth.

A *Point* is the termination of a line, or the intersection of two lines, and, as such, has neither length nor breadth.

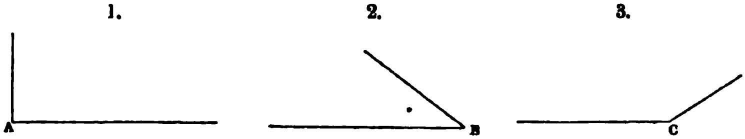
Parallel lines are lines placed equidistant from each other, and which, however far extended, can never meet; as the lines A B and C D.



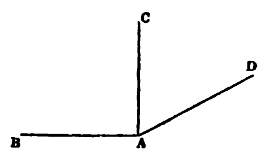
Angles are formed by the meeting of lines drawn in different directions. When a line, as A D, falls upon the line B C, so that the two angles on the opposite sides of the line A D, at the point D, are equal, then these two angles are each of them right angles, and the line A D is called a perpendicular to B C.



1. A represents a right angle; 2. B is an acute angle, which is less than a right angle; 3. C is an obtuse angle, which is greater than a right angle. The space which the two lines forming



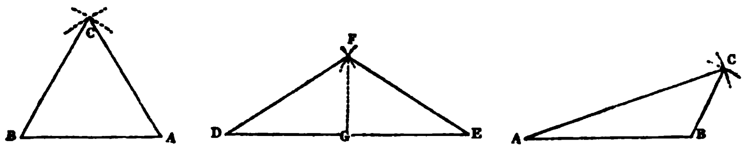
the angle diverge from the point where they meet, characterises the nature of the angle, as consisting of a certain number of degrees; which will be explained when the circle is treated of. It may be proper here to observe, that when only two lines, in different directions, meet at a point, and of course form only one angle at the point of junction, the angle is marked and designated by a single letter. But if three or more lines meet a point, and form two or more angles, three letters are required to mark and designate these different angles, and, in naming them, the letter at the point of junction is placed in the middle. Thus, the angle formed by the junction of the lines C A and B A, is designed the angle B A C or C A B; that formed by the lines C A and D A is named the angle D A C or C A D, and that by A B and A D the angle B A D.



Figures are the portions of space completely enclosed and bounded by lines, either right or curved; those bounded by the former being designated *Rectilinear* figures, those bounded by the latter *Curvilinear.*

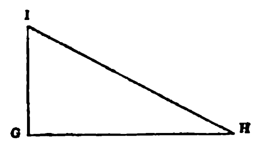
I. *Rectilinear Figures* comprehend *Triangles*, or spaces bounded by three right lines. Of these there are three kinds, as characterised by their bounding lines: 1. The *Equilateral triangle*, of which all the three sides are equally represented by the triangle

1. 2. 3.



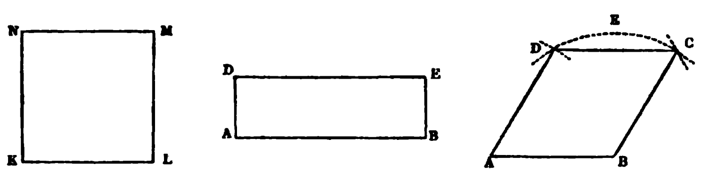
B C A; 2. The *Isosceles triangle*, of which two sides are equal; 3. The *Scalene triangle*, where the three sides are unequal. Triangles are also characterised by their angles.

All the three angles of a triangle are equal to two right angles; so that, if one angle is a right angle, (or greater than a right angle,) none of the other two can be so great as a right angle. If one of the angles is a right angle, the triangle is a *right angled triangle;* if one angle is obtuse, it is an *obtuse angled triangle;* if all the three angles are acute, it is an *acute angled triangle.*



II. *Quadrilateral figures* are spaces bounded by four straight lines. These comprehend the *Square*, 1, in which all the four sides are equal, the opposite sides parallel, and all the angles are right angles; the *Rectangular Parallelogram*, or oblong, 2, of which the

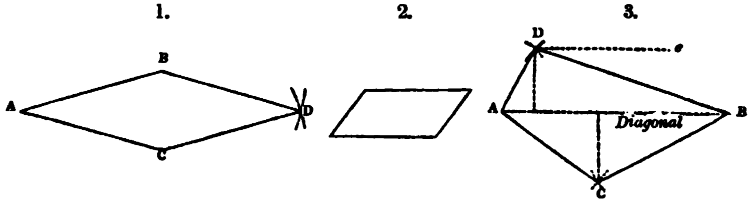
1. 2. 3.



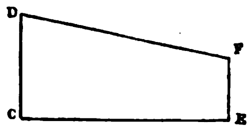
two opposite sides are equal and parallel, all the angles are right angles, but all the four sides are not equal; the *Rhombus*, 3, of which all the sides are equal and parallel, but none of the angles are right angles; the perfect or regular *Rhombus*, 3, has two of its angles of 120 degrees, and two of 60.

The *Rhomboid*, 1, has the opposite sides equal and parallel, but the sides are not all equal, and none of the angles are right angles.

1. 2. 3.

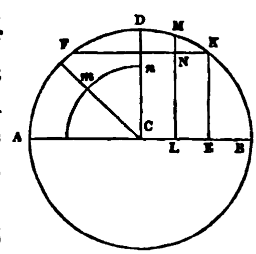


The *Trapezium*, 3, is a figure in which the four sides are unequal, none of them parallel, and none of the angles right angles. The *Trapezoid* is that in which two of the sides are parallel but not equal. In every quadrilateral figure the four angles are equal to four right angles. All quadrilateral figures, which are neither squares, parallelograms, rhombuses, nor rhomboids, are called *Trapeziums*. As every four-sided figure contains four angles, they are also named *quadrangular*. If the figure is bounded by more than four sides, it is called a *polygon* or a *multilateral figure*.



Curvilinear figures. These comprehend the *Circle*, the *Ellipse*, or *oval*, &c. Several of the principles of the art of surveying are referable to the properties of the circle; and the practical surveyor may sometimes be called upon to trace out an oval upon pleasure-grounds.

The *Circle* may be considered as a figure traced by a point moving round a fixed point called the Centre, and keeping always at the same distance from the centre, till it arrives at the place from whence it set out. It may be traced upon a slate or paper with a pair of compasses, by fixing one foot in the centre, and making the other revolve round it, extending the compasses to the width required in the circle, whilst a writing, steel, or slate pen, fixed to the moving foot, traces the line of its course, called the circumference. In tracing a large circle upon the ground, a convenient way is to fix a pin in the centre, to put the end of a cord with an eye over the pin, and, at the distance required, to move round with the other end of the cord, marking the line it makes by pins at short distances. A straight line, drawn in any direction from the centre of the circle to its circumference, is called the *Radius* of the circle; and from the mode in which a circle is formed, it is self-evident that all the radii of a circle are equal. Any straight line drawn across the circle, passing through its centre, and terminated by the circumference, is called the *diameter of the circle*, and divides it into two equal parts; each of these parts is named a *semicircle*; the part of a circle cut off by any right line drawn across it, which does not pass through the centre, is called the *segment of a circle*. A F D B is the circumference of the circle; C is its centre; the line A B is its diameter; the lines drawn from the centre to the circumference A C, F C, D C, and B C, are radii of the circle; the line D C,

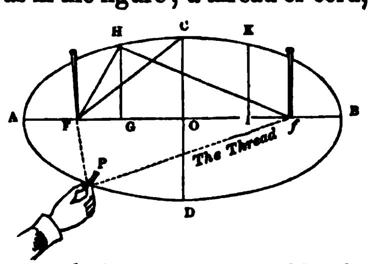


perpendicular to A B, if prolonged, would divide the circle into four equal parts; the portions of the circle A C D, or B C D, are named *Quadrants* of the circle.

In the circle A E × E B = E K², and A L × L B = L M², whence any number of points in the circumference of the circle may be computed when the diameter is given. This will enable a surveyor to lay out a circus in a town, or to stake off the curves of railways. Compute E K, in which K forms the extremity of the straight line F K, then compute L M, whence L M − E K = L M − L N = M N. In this way, as many distances from the straight line E K as may be thought necessary may be computed.

The circle is used to measure the divergence of the two lines forming an angle: for this purpose the circumference of the circle is conceived to be divided into 360 equal parts, called *degrees;* the degree is divided into 60 parts called minutes, and the minute into 60 parts called seconds; the semicircle contains 180 of these degrees; the quadrant A C D contains 90 degrees; the divergence of the two lines forming an angle, is ascertained by the number of degrees of the circumference of a circle, which these two lines would intercept, were we to form the circle from the point, as its centre, where these lines meet; and the angle receives its name from the degrees so intercepted: thus the lines A C and C D intercept the fourth part of the circumference, or 90 degrees; and the angle A C D is called an angle of 90 degrees, or a right angle; the line F C intercepts with the line D C one half of these, or 45 degrees, therefore the angle F C D is called an angle of 45 degrees, and F C B = 45° + 90° = 135°. It is the same thing whether the circle, by the intercepted portions of whose circumference circles are measured, be a greater or lesser circle; for *m n* is just the eighth part of the circumference of the lesser circle, as F D is of the larger one, drawn from the same common centre C.

The *Ellipse* is regularly formed, somewhat similar to a circle of two centres. Two pins are planted, as in the figure; a thread or cord, with its ends fastened together, is thrown over the pins; a black lead pencil, or other marker, is then held upright in the hand, within the double of the cord, and is carried round the pins at the full stretch of the cord, from the two planted pins or centres, and marks



the progress round the centres of the angle formed in the cord by the marker. Ellipses, it is evident, may thus be formed with any proportion of their length to their breadth; the nearer the distance of the two centre pins, with the same length of cord, the more nearly will the ellipse approach to the form of a circle; and the same thing will take place, in proportion to the length of cord, with the same distance of the centre pins.

The longer diameter A B is called the major axis, the shorter C D is called the minor axis, cutting each other at right angles in O the centre of the ellipse. The points F, f, are called the foci, the line F C is called the mean distance from F, and is equal to A O half the major axis. Also F O is called the eccentricity, and $AG \times GB : AI \times IB :: GH^2 : IK^2$. Hence if A B and D C be known, any number of points, H, K, &c., may be computed so as to form the front of an elliptical circus in large towns.

Since from the foci F, f, two straight lines drawn to any point P in the circumference are, together, equal to the major axis A B, it follows that an ellipse may be constructed by points through which, by hand merely, or a bent spring passing over them, the curve may be traced. Divide the major axis into any number of equal parts, as 10, 100, &c., or to be easy in execution, into numbers capable of constant bisection, as 8, 16, 32, 64, 128, &c. Then take the extent A G in the compasses, with F as a centre describe one arc. With G B the other portion of the major axis and centre f, describe another arc, intersecting the former in H, giving one point in the curve. In like manner, any number of points may be found by intersections on both sides of the curve at the same time.

We now proceed to describe the mode of performing a few Geometrical Problems, in the exercise of which the practical surveyor should endeavour to acquire a readiness.

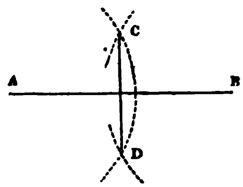
ART. II.—GEOMETRICAL PROBLEMS.

PROB. I.

To draw a line parallel to a given line A B.—With a pair of compasses take the distance you want to make one line distant from another; then set one foot of the compasses in A, (fig. to definition of parallel lines, page 2,) and describe an arc with the other foot at C; remove the compasses with the same extent, and put one foot of the compasses at B, and with the other foot describe an arc at D; draw the line C D, which will be parallel to A B.

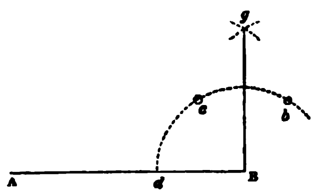
Prob. II.

To divide a line into two equal parts.—Let A B be the line to be divided. Stretch the compasses to any extent, exceeding half the length of the line A B; fix one foot in A, and sweep the arc C D; then, with the compasses at the same extent, fix one foot at B, and cut the former arc in the points C and D; draw the line C D through the points of intersection, which will divide the line A B into two equal parts. This is the best mode of raising a perpendicular upon the middle of a line, if you have room below at D.



Prob. III.

From the end of a line, as A B, *to raise a perpendicular at* B.—With any extent in a pair of compasses set one foot in B, and describe an arc *d c b*; with the same extent put one foot of the compasses in *d*, and turn the compasses twice upon the arc, marking the points *c* and *b*; and from these points describe the arcs intersecting each other at *g*; then draw the line *g* B, which will be perpendicular to the line A B, at the end of the line at B.

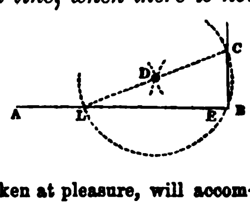


Prob. IV.

To let fall a perpendicular from a point at A *upon the given line* B C.—With any extent greater than the distance from A to D, (fig. page 2,) put one foot of the compasses in A, and describe the arc H B; then put one foot of the compasses in the intersection at H, and describe an arc at K; with the same extent, on B as a centre, intersect the arc at K; then draw the straight line A D from the point A to the point of intersection at K, and the line A D will be perpendicular to the line B C. In practice, the line may terminate at D.

Prob. V.

To raise a perpendicular from the end of a line, when there is not room on one side to extend the arc d e b, as in last figure.—With any extent you think proper between your compasses, put one foot at the end of the line at B, and describe an arc passing through D;* with the same ex-

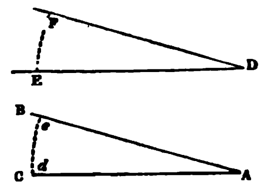


* Any point D, evidently within the angle A B C, taken at pleasure, will accomplish the same purpose when the other foot of the compasses is extended to B.

tent, put one foot of the compasses in any part of the line A B, suppose at L, and intersect the former arc in D; with the same extent in D as a centre sweep the arc L E C; draw a line from L through the centre D to the arc at C; lastly, draw the line C B, which will be the perpendicular to the line A B at B. Various other methods might be shown how to raise and let fall perpendiculars; but what has been pointed out is thought sufficient.

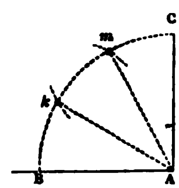
PROB. VI.

At a given point, to make an angle equal to a given angle.—Let B A C be the given angle; set one foot of your compasses in the point A, and with any extent sweep an arc *e d*, intersecting the two lines B A and C A, which form the given angle; then with the same extent fix one foot in the point D, where the required angle is to be formed, and sweep the arc E F; take the arc *e d* in your compasses, and apply it to the arc E F, marking those points; then draw straight lines from the points E and F to the angular point D, and you will have the angle E D F equal to the angle B A C.



PROB. VII.

To divide a right angle in three equal parts.—From the angular point A describe an arc B C, and with the same extent set one foot of your compasses upon the arc at B, and make a mark upon the arc at *m*; then with the same extent set one foot of the compasses on the arc at C, and make another mark with the other foot at *k*; draw straight lines from the points *m* and *k* to the angular point A, which will divide the right angle into three angles of 30 degrees each.



PROB. VIII.

To make an equilateral triangle whose base is A B.—Take the length of A B (fig. 1 to def., page 3,) with a pair of compasses, and with that extent set one foot of your compasses in A, and describe an arc at C; then, with the same extent, set one foot of the compasses in B, and with the other foot intersect the former arc in C; draw lines from the point C to the points A and B, then A B C will be an equilateral triangle.

PROB. IX.

To construct an isosceles triangle upon the line D E.—With the extent of the line D F (fig. 2 to def., page 3,) set one foot of the compasses in D, and describe an arc passing through F; then, with the same extent set one foot of the compasses in E, and intersect the former arc at F; draw the lines F D and F E, and it is formed.

PROB. X.

To construct a scalene triangle whose sides are all unequal, as A B, C B, *and* C A.—Take the length of A B, (fig. 3 to def., page 3,) and lay that distance off upon a line drawn at pleasure; then, with your compasses, take the length of the line A C, and with one foot in A describe an arc through C; then take the length of the line B C, and with one foot in B intersect the former arc at C; draw lines from the point of intersection at C to A and B, and it is made.

PROB. XI.

To form a right angled triangle on the line G H.—At the point G (fig. to def., page 3,) raise the given perpendicular G I, as described in page 7, and draw the line H I, and it is finished.

PROB. XII.

To form a square whose sides shall be equal to K L.—Raise a perpendicular from K to N, (fig. 1 to def., page 3,) and another from L to M, both of an equal length with K L; join M and N, and it is done. Or you may draw K N perpendicular to L K, from the point K; then take the length of L K, and lay off that distance to N; and, with the same extent in your compasses, put one foot in N, and describe an arc through M; and, with the same extent, put one foot of your compasses in L, and with the other foot intersect the arc in M; and, from the point of intersection at M, draw the lines M L and M N, which form the square L K N M.

PROB. XIII.

To form a rectangular parallelogram whose sides shall be equal to the given lines A B *and* A D.—Lay down a line equal to the length A B, (fig. 2 to def., page 3;) at the point A raise a perpendicular of the length of A D; then, with your compasses extended to the length of A B, put one foot in D, and describe an arc at E; then, with the extent of A D, fix one foot in B, and intersect the arc at E; from the point of intersection draw the lines E D and E B, and it is done.

PROB. XIV.

To construct a rhombus upon a given line A B.—Take the length in your compasses, and with one foot in B (fig. 3 to def., page 3) describe the arc D E C; with the same extent on A as a centre describe an arc cutting the former; and on D as a centre describe an arc cutting D E C in C; draw the lines A D, B C, and C D, which form a rhombus. This is called the regular rhombus. The opposite angles of a rhombus may have any magnitude except right angles.

PROB. XV.

To form a rhombus of any given angle, suppose B A C.—First, by PROB. VI., make an angle equal to B A C, (fig. 1 to def., page 3;) take any length you please between your compasses, suppose A B; with the same extent make a mark on the line from A to C; and with the same distance on B as a centre describe an arc through D; then with the same extent, and on C as a centre, cut that arc in D; lastly, draw the lines B D and C D, and it is formed.

A rhomboid has its opposite sides equal and parallel, but not perpendicular to one another.

PROB. XVI.

To construct a trapezoid upon the line C E, *whose parallel lines shall be the lines* C D *and* E E.—With a pair of compasses lay off the distance of C E (fig. to def., page 4;) then at one end of the line C raise a perpendicular of the length of C D; at the other end of the line at E raise another perpendicular of the length of the given line E F; join those lines, and it is done.

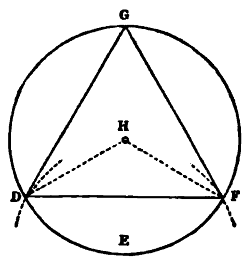
PROB. XVII.

To construct a trapezium equal to a given trapezium, A D B C.—From any angle of the given trapezium draw a line to its opposite angle, which is called a diagonal, suppose the line A B (fig. 3 to def., page 3;) lay down a line of the same length as A B, and extend your compasses to the length of the line A B of the given figure; fix one foot in the end at B, and with the other foot sweep an arc the length of B D; then take the length from the given figure A D, and put one foot of the compasses in A, and intersect the arc at D, draw in the lines A D and D B; then take the length of the line from the given figure A C, and put one foot of your compasses in A, and describe an arc at C; then, with your compasses extended to the length of B C, and in like manner from the point B, intersect the arc in C; draw the lines A C and B C, and the trapezium is formed equal to the one given.

Note.—The dotted line D *e* in the figure has no connexion with the above construction, nor has the dotted lines from D and C to the diagonal. Their use will be explained in another place, relative to the computation of areas.

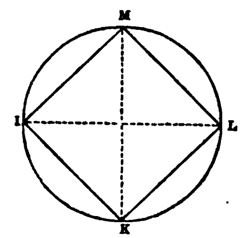
Prob. XVIII.

To construct an equilateral triangle within a circle. Let D E F G *be the circle, of which* H *is the centre.*—With the same radius by which the circle is drawn set one foot of the compasses on any part of the circumference you please, suppose on E, and with the other foot intersect the circumference in D and F, and draw the line F D; then take the distance of the line F D, and set one foot in F, and with the other intersect the arc in G; draw the lines G F and G D, and it is formed.



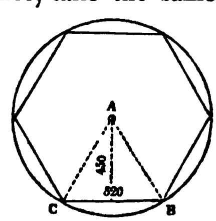
Prob. XIX.

To construct a square within a circle.—Divide the circle into quadrants; first, by drawing a line through the centre of the figure, suppose from I to L, and another line perpendicular through the centre from K to M; then draw lines from I to K, K to L, L to M, and M to I, and the square is formed.



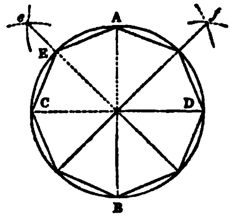
Prob. XX.

To construct a regular hexagon in a circle.—The radius of a circle being transferable six times on its circumference, take the same radius the circle is drawn by, put one foot of the compasses on any part of the circumference, and divide the circumference into six equal parts; lastly, draw lines from every point to the one next it till you have gone all round, and the hexagon is finished.



Prob. XXI.

To construct an octagon, or eight-sided figure.—Describe a circle, and divide it into four equal parts by two diameters perpendicular to one another (as in the figure); then set one foot of the compasses in A, and with any distance you please sweep an arc at *e*, and another at *f*; then set one foot of the compasses in C, and with the other intersect the arc in *e*; and with the same extent on D as a centre, intersect the former arc in *f*; then draw the dotted lines from the points *e* and *f* exactly through the centre to the other side of the circumference, which will divide the circle in eight equal parts; lastly, draw straight lines from every point where the above straight lines touch the circumference to the next point touched, and the octagon is formed. When one of the sides, such as A E, must be of a given length, it will be readily constructed by the sector, as described in cases of mathematical instruments.

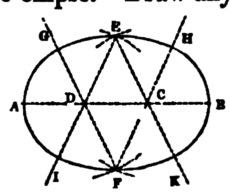


Prob. XXII.

To describe an ellipse.—The *ellipse* is formed by a curve drawn about two centres, as represented in the figure, page 5. Two pins are fastenedat the points F, *f* (which points are termed the foci of the ellipse); a thread or cord is to be doubled, and the ends fastened; the thread is then thrown over the pins with its double extended to C or D; and with a pen or pencil, by keeping the thread equally tight about the pins (taking care to hold the pen or pencil upright) the figure may be easily described. It is evident the nearer the two pins approach to one another, the nearer does the figure approach to a circle.

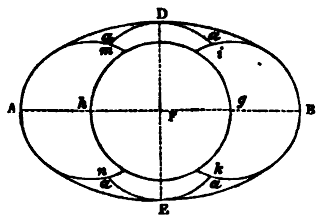
Prob. XXIII.

By the following method a figure may be formed with arcs of circles only, and which will nearly resemble the ellipse. Draw any line, as A B, and upon it describe two isosceles triangles, D E C and D F C; produce their sides to H, G, I, and K; then on the vertex of each triangle E and F, with the distance E F, describe the arcs G H and I K; lastly, on C and D as centres, with the distance C H or D G, describe the arcs H K and G I, and it is done. This figure is generally preferred for a grass plot, and sometimes in the construction of arches.

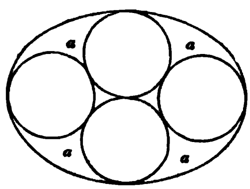


Prob. XXIV.

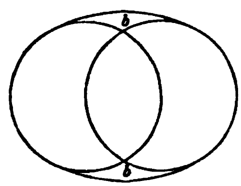
Easy modes of forming oval figures by compasses.—Draw a line the length of the oval, as A B, and let fall a perpendicular through the centre F from D to E; with one foot of the compasses in E describe a circle, the diameter of which shall be half the length of the line A B; then put one foot of the compasses where the arc intersects the line A B at *g*, and with the same radius as the circle already made describe the arc *i* B *k*; remove the compasses to *h*, and put one foot in *h*, and sweep the arc *m* A *n*; then take a stretch of half the breadth you wish to give to the oval, and put one foot of the compasses in the centre F, and with the other foot describe an arc at D and another at E; and then with your hand cut off the four corners *a a a a*, and the figure is formed.



A figure somewhat resembling the ellipse, made by drawing four circles, and with the hand cutting off the four corners *a a a a*.

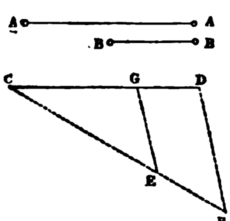


This represents also a figure which is formed by two circles. These are approximations only to the ellipse, part of it marked *b b* being drawn with the hand.



Prob. XXV.

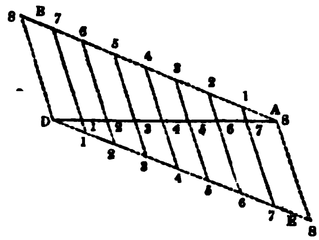
To divide a line into two parts, which shall be in the same ratio to each other as two given lines.—Let A A be one line and B B the other; let C D be the given line, to be divided into two parts, bearing the same proportion to each other as A A does to B B.



First, from C draw a line at pleasure, as C F; then with a pair of compasses take the length of the line A A, and lay that distance off upon the line C F, which will reach to a point at E; likewise take off the length of the line B B, and set that distance off from E, which will reach to the point F; then draw a line from F to D; lastly, draw a line parallel E G with F D, from the point at E, this last line will cut the line C D at G, and will make the line C G in the same ratio to G D that A A is to B B.

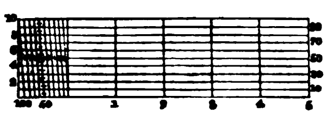
Prob. XXVI.

To divide a line into any number of equal parts, suppose eight.—Lay down a line of any length at pleasure, suppose A B, from one end of the given line A D, making any small angle you please above that line, and another parallel to it below, as D E; lay any convenient distance off eight times upon the upper line, and with the same extent eight times upon the under line, and draw lines from the corresponding alternate point to point, namely, 1 to 7, 2 to 6, 3 to 5, &c., from the upper line to the under one, these will divide the middle line A D into eight equal parts. This mode is very useful in dividing scales into equal parts.



Prob. XXVII.

To construct a diagonal scale, suppose one-fourth of an inch to each primary division.—Draw a line any length you wish to make your scale, suppose an inch and a half; raise a perpendicular at both ends of an equal breadth, which divide into ten horizontal parallel spaces, by eleven parallel lines at equal distances, the whole length of the scale; then divide the length into six equal divisions, and draw vertical lines parallel with the perpendiculars, which will be one fourth of an inch each division; then divide the left hand division into ten equal parts, both at the top and bottom; draw a diagonal line from 0 to the first of the small divisions at the top of the scale; then draw another line from the first division at the bottom to the second division at the top of the scale, and go on in this way till the whole of the ten lines are drawn; then insert the figures as represented on the diagram; the first figure is sometimes called *ten*, the second *twenty*, the third *thirty*, &c. In the first case, each of the small divisions is one: the first figure is sometimes called 100, the second 200, the third 300, the fourth 400, the fifth 500. In that case, each small division is called *ten*, two is



termed *twenty*, and the vertical figures *units*, and so on in a decimal ratio.

Diagonal scales may be made of various dimensions, in the same way as above, half-inch or inch scales.

To take off a distance from a diagonal scale, suppose 446 links, when the figure *one* upon the scale is termed 100. Place one foot of the compasses in the fourth vertical line, at the sixth line up, and extend the other foot along the parallel line to the fourth diagonal, which will be the distance required.

SECTION SECOND.

DESCRIPTION AND METHOD OF SURVEYING WITH VARIOUS INSTRUMENTS.

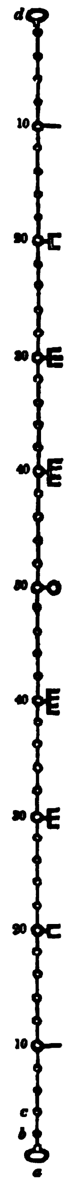
ART. I.—OF THE CHAIN.

THE land-surveyor's chain, commonly called Gunter's chain, is divided into one hundred equal parts, denominated links, reckoned from each end towards the middle by means of brass marks at every ten links. The best chains have welded iron or brass handles attached to each end. The end of the marks which is opposite that fixed to the chain, is divided into as many parts, or points, as the number of tens of links from each end of the chain. In reckoning the odd links, care must be taken to observe on which side of the fifty, which is indicated by a round piece of brass or ring without divisions, the last pin was put down, otherwise an error may be easily committed, by calling the marks 10, 20, 30, or 40, instead of 90, 80, 70, or 60; because, when the pin is past the circular mark indicating 50, then 40 next the fore or leading end of the chain will be 60, 30 will be 70, 20 will be 80, and 10 will be 90. The chain being thus divided, as shown in the figure, it is immaterial which end or handle the leader or foremost chain-bearer takes hold of. Indeed, in the course of the survey it may be advantageously changed as occasion might require.

In the figure, the handles are indicated by *a* and *d*, while *a b*, *b c*, &c., show the links, though, to avoid confusion, these occupy the real length of two links in the chain, while the number of points in the brass marks appended to it show the number of tens from each end.

When it is necessary to unfold the chain, take both handles in the left hand and throw it from you with the right, taking care to keep hold of the handles, then stretch it out to its full extent.

The most easy and expeditious method of folding up the chain,



is to begin at the circular mark, or fifty, and fold it up double, which, when done, should, for the convenience of carriage, be enclosed in a belt, with a buckle to make it fast when sufficiently tight.

The pins should also be carefully tied up to prevent losing any of them.

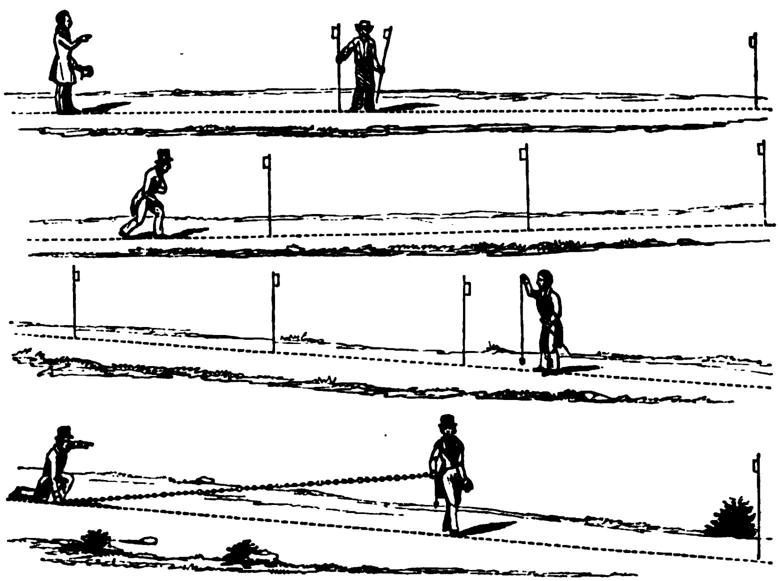
The offset staff may be advantageously used for setting off straight lines, or diagonals, when a sufficient number of assistants are not at hand; for, by looking first to the one pole and then to the other, when even invisible from one another by an intervening height, if both poles are seen through one of the grooves of the head of the cross staff, the poles and staff are all in the same straight line; otherwise, not. If they are not in the same straight line, the position of the staff must be altered till they are so; then remove the staff, and put a signal in its place. This will be readily understood from the four adjacent figures, exhibiting the whole process.

The measurer should also be provided with ten pins, made of wood or strong iron wire, about 12 or 15 inches long, having a piece of red cloth tied to the head of each pin, so that they may be easily found. It is requisite to be provided with three or four poles, called station staffs, about 8 or 10 feet in length, with a red or white flag tied to the top of each, to be readily seen when placed at a distance; which, if shod with iron, will be easily stuck in the ground. The measurer ought also to have an offset staff* ten links long, and divided and numbered from one to ten, for taking offsets into the bends and angles in the fences; or a tape, which is better, divided into links in place of feet, such as carpenters, masons, and painters use for measuring their work. These tapes can be bought at most of the hardware shops.

Being provided with the above articles, place one of the poles where you intend to measure to, and leave the other at the mark you begin at. Let the foremost assistant take one end of the chain and the ten pins; and the other assistant, when the chain is stretched out, must direct him in a line with the pole they are to measure to. If he is not exactly on the line at first, the hindmost assistant must cause him to move to the right or left till he is exactly on the

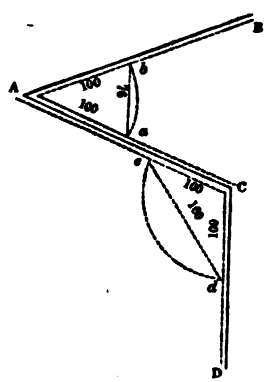
* The cross staff should be divided into links, to serve this purpose when applicable. Its length in that case should be 8 links exactly, so as to be of moderate or convenient dimensions.

line with the station staff, where he is ordered to stick down one of the pins at the end of the chain. The foremost chain-man goes forward and the hindmost one follows, and stands with his hand above the first pin, and moves the foremost assistant by signal to the right or left, till he is exactly on the line; when he is ordered to stick in his second pin. The hindmost chain-man lifts the first pin at the same time the other sticks his second pin. The one chain-man goes forward, and the other follows to the second pin; the foremost man then sticks in his third pin by the direction of the hindmost man. It will be proper to observe, that each of the chain-men should be very careful in keeping the line very correct, which they can both know exactly; the foremost assistant will always see the back pole and the hindmost assistant in a line, and the hindmost assistant never allows the foremost one to stick in his pin till he sees that he is exactly on the line to the pole they are measuring to; and the one assistant should always



lift the pin with the *same hand* that he has the chain in, and the other should stick down his pin with the *same hand* that he draws the chain with. The chain-men can now direct each other; but if they should deviate in the least from the line, they can put themselves exactly upon it again, by moving a little to the right or left till they are exactly upon it again, which they can easily ascertain when the hindmost assistant sees the foremost assistant in a line with the pole, and the foremost assistant observes the hindmost assistant in a line with the pole left at the place where the measuring began. If great care is not taken to keep the line, particularly where offsets are taken, none of them will be right, but either too short or too long, according to the distance they are to the right or left of the line. The foremost man always sticks in his pins, and the hindmost lifts them up, till they are all spent, which should be *counted* to see if none are lost. This is what land-surveyors call a change, or 1000 links. The hindmost man gives the whole of the pins to the foremost chain-man, and proceeds measuring as before. They may either change at ten pins or eleven: if they change at eleven, the hindmost man sticks in one of the pins, and gives the foremost man only nine. They now continue measuring till the pins are all spent a second time. This is called two changes, or 2000 links. The pins ought to be regularly counted at *each change*, so as immediately to detect any error in the measured distances without going over the *whole* measurement a second time. But we shall suppose the foremost chain-man comes to the pole before it was necessary to change pins the second time: in that case, the hindmost assistant's pins are counted, which we shall suppose 8 chains and 25 links. You insert for the length of that line in the field-book 1825. A land-surveyor never thinks of setting down in his field-book, or eye-draught, chains or links at the end of his figures, as every one that measures with the chain generally inserts their distances in links.

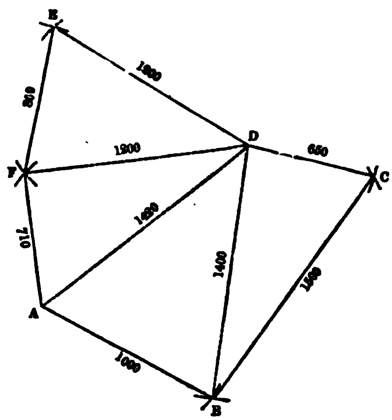
Some surveyors, in some of the counties in England, survey with a chain 10 yards in length; in Scotland, some measure with a chain 10 ells in length, each ell being 37 inches; and in Ireland with a chain of 2 perches, or 42 feet, in length. These measures are, since the year 1826, generally discontinued, on the passing of an act of parliament relative to weights and measures.



This figure shows the method of taking an angle with the chain, when you have no other instrument to take angles with in the field; it saves the trouble of measuring the diagonal. Many surveyors prefer taking an angle with it, on account of its simplicity, to any other instrument. Suppose the angle B A C is wanted from the corner of the hedge A; measure one chain or 100 links from A on the line A C, and order the chain-man to leave a pin at *a*; then measure another chain's length from A to *b*: when that is done go up to the mark at *b*, and measure to *a*, which is 76 links, which note down on your field-book.

To protract the angle B A C, draw a line at pleasure, representing the dotted line A B; then take the length of one chain from a larger scale than the one you intend to plot the field by, and with one foot of the compasses in A sweep an arc, as *a b*; then take off 76 links from the same large scale the arc was described by, from *a* to *b*; then put one foot of the compasses in *b*, with the extent of 76 links, and make a mark upon the arc at *a*; draw the line A C through the point *a*, which will give the angle B A C. Suppose you want to lay off another angle from C towards D, lay that angle off as above directed, and take off 100 links from a large scale, suppose four times larger than you intend to plot the field by—the larger the scale the better; then draw an arch from the point, and make a prick upon the dotted line C A at *e*; then take off 160 links from the same scale the arch *e d* was described by; then put one foot of the compasses in *e*, and intersect the arch in *d*; lastly, draw a line from the angle C through the point *d*, and the angle A C D will be formed. Observe, when the fences are measured, that you measure the distance from the hedge to where you stand, on each side of the fences; and if a pole is placed at the same distance from the hedge, it will be exactly parallel. In *Fig*. 4. *Areas*, an angle was taken with the chain into the middle of the hedge, in the trapezium at A to *a* and *b*, and the distance across from *a* to *b* was 147; but it is the same thing, and answers the purpose better, to measure parallel with a fence, suppose from 5 to 10 links. The reason of having represented the dotted lines a little from the fence is, that obstructions are frequently met with when the measure is taken close to the fence. All other angles that are taken with the chain are taken in the same way in the field; which insert either upon an eye-sketch or in a field-book.

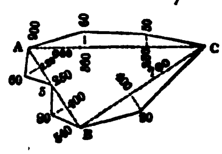
A B C D E F is a field of six sides, surveyed with the chain, the fences of which are all straight. The best method of measuring this field is to divide it into triangles on the spot, which is represented with dotted lines on the figure, and represent the lines that were measured to divide it into four triangles. Begin the measurement at any angle you please, suppose A, and measure to B, that is 1000 links, which insert in an eye-sketch; then measure the fence from B to C, 1500, and from C to D, 650, from D to E,



1200, from E to F, 800, and from F to A, 710, to where you began; all of which distances set down carefully in an eye-sketch or a field-book; then go to A, and measure across the field to D, which is 1420, then measure from D to F, which is 1200, then return to D, and measure to B, which is 1400, finishing the survey; and insert all the distances in an eye-sketch or a field-book, whichever you choose to keep.

To plot and delineate a plan of this fig. Draw any one of the lines you choose to begin at by random with a black-lead pencil, to represent either of the fences or dotted lines across the inclosure. Suppose you begin at A, take a thousand links from a scale of equal parts, and lay off that distance upon the black lead line, which will reach to B, then take the distance 1420 from the same scale, and put one foot of the compasses in A, and describe an arc at D; then take the distance of 1400 from the scale, and put one foot of the compasses in B, and intersect the arc in D; then take the distance from D to C, 650, and set one foot of the compasses in D, and sweep an arc at C; then take the distance from B to C, which is 1500, from the same scale, and put one foot of the compasses in B; intersect the arc in C; then take 1200 from the same scale, which is the length across the field from D to F, and put one foot of the compasses in D, and sweep an arc at F: then take the distance from A to F, which is 710, from the same scale, and put one foot of the compasses in A, and intersect the arc in F; then take the length from F to E, which is 800, from the scale, and with that extent put one foot of the compasses in F, and sweep an arc at E; then take the distance from D to E, which is 1200, from the same scale, and put one foot of the compasses in D, and cut the arc at E; then draw in the fences A B, B C, C D, D E, E F, and F A, which gives the exact shape of the enclosure, if the lengths are all right measured in the field, and the distances taken exactly from the scale of equal parts with the compasses. Let a field consist of ever so many sides, they must all be divided into triangles and trapeziums, either in the field or on a plan, before the area can be obtained.

There are numbers of enclosures that have their sides very crooked and irregular, as the fig. Let this enclosure be divided into a triangle, as A B C, whose side A B is 540, the line B C 760, and the side C A 900. Great care must be taken in measuring each line, and taking offsets into all the bends and angles, which are represented by dotted lines. On the figure where they are taken, not only the length of each offset



must be marked on an eye-sketch, but the *distance* from a given point where each offset is taken at. Suppose you begin the measurement at C, at 250, you take an offset to the bend of 50, and at 560 you take another offset of 60, and the length of the line C A is 900. Begin again at A, and measure to B; at 120 an offset is taken of 60, at 250 another is taken of 5 to the fence; at 400 another offset is taken of 80 to an angle in the fence, and the whole distance of the line A B is 540. In measuring the line B C, an offset is taken at 400 of 90, to an angle in the hedge, and the whole length of the line B C is 760. The above distances being all carefully marked in the field upon a field-book or eye-sketch, it is now to be delineated and laid down by scale and compasses. Draw a line at pleasure to represent the longest side, A C, with a black lead pencil, and take off the distance 900 from a scale of equal parts, which is the distance from A to C, and make a mark at C and another at A; then take the distance, which is 540, from the same scale, and put one foot of the compasses in A, and describe an arc at B; then take the distance, 760, from B to C; put one foot of the compasses in C, and with the other foot intersect the arc at B; then draw the line A B and B C with a black lead pencil, which will form the triangle A B C.

The different offsets are now to be laid off where each of them were taken. Most surveyors, for quickness, use a feather-edged scale (see page 33); others use a scale of equal parts and a pair of compasses. First, on the line A B, make a mark at 120, another at 250, and another at 400; at 120 lay off the offset 60, at 250 lay off 5, at 400 lay off the offset 80; then draw in the fence to the offset 60, from thence to the offset 5, from thence to the offset 80, and from thence to B, which will give you the boundary line from A to B; then either use the feather-edged scale or a pair of compasses, and prick off 400 upon the line B C, and opposite it prick off the offset 90 to the corner of the fence; then draw in the boundary from B to the offset 90, from thence to C; lastly, lay off 250 and the offset 50; also 560 and the offset 60; then draw in the fence from C to the offset 50, from thence to the offset 60, from thence to A, which closes the enclosure.

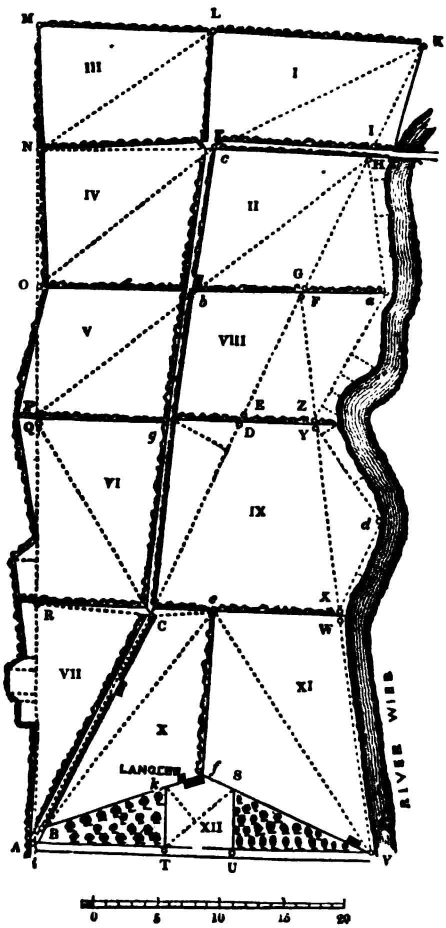
After inking it in, rub out the black lead lines, and you have the exact shape of the ground, if you have measured the lines correct. Almost all surveyors use the feather-edged scales, of about 12 inches long, while with another of 2 inches, set at right angles to the other, the offsets are pricked off—each scale of 12 inches having its corresponding two-inch offset scale.

FIELD-BOOK OF LANGLEE.

The river is about 200 *links wide.*

	Offsets.	Dist.	Offsets.			Offsets.	Dist.	Offsets.	
Closes at F, .		4460		On long line	House, . .		100		Long
		3450		Z			30		Broad
Crosses, .	..	3420	..	Hedge					
		3400		Y	From . .	S	480		To U
Crosses, .	..	2000	..	Hedge	Diagonal from	T	630		To S
		1900		X	Corner of wood	330	280		
Crosses, .		1830		Hedge	K . .		T		
		1800	60	W river					
		1700	10	River	From V .		1510		To *f*
Crosses, .	..	15	..	Hedge	From V .		1170		Corner of wood
		V	15	To river	Diagonal from *e*		2200		To V
Angle of wood,	15	2620	15	V	From B .		1200		To *f*
Corner of wood,	15	1535	15	U	From B .		1000		To corner wood
Corner of wood,	15	1000	15	T	From *e* .		1300		To *f*
		A		Road	From *e* .		550		To C
					Diagonal from B		2150		To *e*
Closes at A,		6450		End of line					
Enters on road,		6380			Closes . .		860		At Y
		5525	0	Touches fence			780	120	River
		5520	200	Corner of fence			300	100	River
		5330	200		From . .		*d*		
		5310	270	Fence			870		*d*
		5100	270	Fence			600	60	River
		5030	130	Boundary fence			340	50	River
		4600	180	R crosses fence	From . .		X		To *d*
		4190	200	Boundary					
Touches, .		4100	0	Boundary			1170		*a*
		3120		Q			600	290	River
Crosses hedge,		3100	150	Boundary			410	180	River
		3080		P			300	100	River
Crosses, .	..	2430	..	Hedge boundary			140	110	River
Boundary, .	80	2070	..	O	From . .		Z		To *a*
Boundary, .	20	1000	..	N					
		M			From C .		860		To R
					Diagonal from Q		1730		To C in road
		3000		M end of line					
		1600		L	From P .		1030		To *g*
Turns to left,		K			Diagonal from *b*		1590		To P
		7090		K end of line	From O .		1240		To *b*
		6160	110	I to river	Diagonal from C		1820		To O
Crosses, .		6090		Road					
Crosses, .		6050		Hedge	From N .		1440		To C
		6030		H	Diagonal from L		1700		To N
		4850		G					
Crosses, .	..	4820	..	Hedge	From C .		900		To L
		4800		F	Diagonal from K		1800		To C
		3720		E					
Crosses, .		3680		Hedge	From *c* .		1160		To *b*
	..	3630	..	D	From *b* .		830		To G
Corner of hedge,	430	3500			From H .		1170		To C
Leaves road,		2150		Crosses hedge	Diagonal from H		1700		To *b*
Breadth of,	20	2020	20	Road					
		2000		C	Closes at H .		1080	130	River
B, . .		130	20	Corner of wood			900	70	River
Breadth of, .	20	A	20	Road			540	80	River
							280	100	River
					From . .		*a*		To H
THE SURVEY BEGINS HERE.					From G .		670		To *a*
					CONTINUATION.				

The preceding is the field-book of the farm of *Langlee*, and this figure the corresponding plan; which plan is made out to give an idea how to measure and plot the farm. The survey was begun at A, and poles placed in A, B, C, and D, exactly in a line; the distances were then measured from A to B, C and D, and pits dug in the ground where each pole was placed. It will be proper to observe that a pit must be dug with a spade at each station, where a pole stood. A conspicuous pin will generally answer the purpose sufficiently well, each pin having its proper number marked upon it, or upon a piece of paper stuck into it, especially if not required to remain long. The poles were then removed from A and B, one of which was placed in E and the other in F, exactly in a line with the poles C and D, and the same line measured forward to E and F. The poles were then removed from C and D; the one was placed in G and the other in H, exactly in a line with F and E, and the distance measured forward to H. The poles were then removed from E and F; the one was placed in I and the other in K, and the line measured forward to I and K, which finishes the first long line. By the above method of measuring, a line may be continued for miles by letting two poles stand, and advancing the other two poles in the same line. Poles were put up in L and M, and the distance measured from K to L and M. Poles



were then placed up at M, N, O, P, all in a direct line, and the distance measured forward to N, O, and P. The poles were then removed from M and N, the one was placed in Q and the other in R, in a line with P and O. The pole was then removed from Q, and placed in A, in a line with R Q, and the distance measured forward to A, which makes a closing. Poles were then placed in A, T, U and V, and the distances measured forward to T, U, and V, which finishes the line along the road. Poles were then placed in W, X, Y and Z, and the line continued and measured from V to F, where it joins the first long line from A to K.

Before proceeding further, I shall now point out the method used in plotting the survey of what is done, and afterwards describe the way how to finish the enclosures.

First draw a line at pleasure, to represent the long line A, B, C, D, E, F, G, H, I, K; and with a pair of compasses prick off the distance 7090, which is the distance from A to K, from a large scale, suppose 2 chains in an inch, with a pair of large compasses;* then take off from the same scale 3000, which is the length of the line K, L, M; put one foot of the compasses in K, and describe an arc at M; then take off the distance 6450, which is the distance from M to A, and put one foot of the compasses in A, and intersect the arc at M; then, with a sharp-pointed black lead pencil, draw the line K, L, M, and the line M, N, O, P, Q, R, A, which will form the triangle A K M; then take off from the same scale 4800, which is the distance from A to F, and make the mark o, signifying station, at F, upon the line A K; then take off from the same scale 2620, which is the distance from A to V; put one foot of the compasses in A, and sweep an arc at V; then take 4460, which is the distance from V to F; put one foot of the compasses in F, and intersect the arc in V; then draw the lines A V and V F, which will form the triangle A V F.

All the intermediate distances and offsets, and where the hedges were crossed, having previously been inserted in the field-book, the next thing to be done is to prick off the distances from A to B, A to C, A to D, A to E, A to F, A to G, A to H, and from A to I; make the mark o (station) where each pole was placed, and insert the letter of reference at each mark; do the same upon the other lines; then begin and lay off the offsets taken to the different

* Beam compasses are the most convenient for this purpose, especially if they have scales graduated on the beam to suit the plan. It would be well if the brass and steel work were made to fit different beams, having various scales graduated on them.

angles and bends, and prick them all off from the same scale, and also where the hedges were crossed in measuring the different lines.

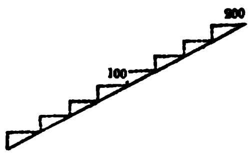
In measuring the line A K, the road was departed from at 2150, and an offset on the left at 3500 of 430 to *g;* which distance lay off, and draw in the road 40 wide from A to *g;* then lay off the distance 3680, where the hedge was crossed between D and E, and draw in that fence from *g;* then prick off the distance 4820, where the hedge was crossed between F and G; then lay off the distance 6050, where the hedge was crossed at the road, and also 40 for the breadth of the road, and an offset to the river of 110, to the end of the bridge; draw in the fence from the end of the bridge to K, and lay off the distance from K to L, which is 1600; draw in the fence from K to M: then lay off the offset 20 from N to the fence, and make a mark that a hedge goes off, and draw in the fence from M to the mark; then lay off the offset 80 from O; draw in the fence from N to the offset 80. In measuring the line from M to A, the boundary was crossed at 2430, which mark, and also where the hedge was crossed at 3100; and lay off the offset of 150 on the right to the boundary; then draw in the fence from the offset of 80 to where the boundary was crossed; from thence to the offset of 150, taken between P and Q; from thence to where the boundary was touched at 4100; then prick off from the scale 4190, and an offset of 200 to the corner of the boundary; lay off an offset of 180 at R, another of 120 at 5030, another of 270 at 5100, another of 270 at 5310, another of 200 at 5330, and another offset of 200 at 5520, and mark the corner of the boundary; then draw in the crooked boundary from the offset at R, to where the boundary was touched at the sharp angle, from thence to the offset of 20 at A; then draw in the road from A to V, 30 wide. In measuring the line V F, cross the hedge at 15, at the corner of the wood, and prick off an offset of 15 to the river; at 1700 prick off an offset of 10 to the river; at W prick off an offset of.60 to the river; then draw in the river from the road, *about two chains wide*, to the offset of 60 at W; and prick off 1850, where the hedge was crossed, between W and X; and prick off 3420, at crossing another hedge, between Y and Z.

I now come to show how to finish the measurement of the enclosures. Go to the mark at G, and measure to *a*, from thenee to H, and take the offsets to the river, and from H measure a diagonal to *b*, and from H to *c*, and from *c* to *b*, and from *b* to G, which finishes enclosure II.; then measure a diagonal from K to *c* and from *c* to L, which finishes enclosure I.; then measure the diagonal from L to N and from N to *c*, which finishes enclosure III.; then measure the diagonal from *c* to O, then a line along the hedge from O to *b*, which finishes the measurement of enclosure IV.; then measure the diagonal from *b* to P, and measure from P to *g*, which finishes enclosure V; then measure the diagonal from Q to C, and measure from C to R, which finishes enclosures VI. and VII. Begin again at Z and measure, and take four offsets to the river, which finishes enclosure VIII.; then measure from X to *d*, and take three offsets to the river, and measure from *d* to Y, and take two offsets to the river, which finishes enclosure IX.; then go to B, and measure a diagonal to *e*, and from *e* measure to C, also from *e* to *f* and from *f* to B, which finishes enclosure X.; then measure the diagonal from *e* to V, and from V, by the side of the wood, to *f*, which finishes enclosure XI.; then measure a diagonal across the yard from T to *s*, (in measuring that line an offset was taken to the corner of the wood;) then measure from S to U, and take the length and breadth of the house at *f*, which finishes the house, yard, and wood.

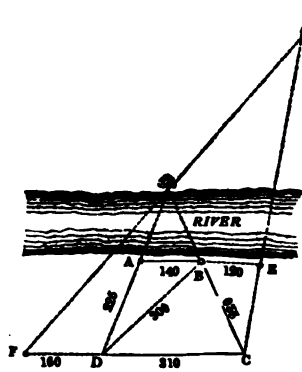
The triangle A K M, and the triangle A V F, being already plotted, I have now to show how to plot the different enclosures. Take the distance 670, which is the length from G to *a*, from the scale, and put one foot of the compasses in G, and describe an arc at *a*; then take 1080, which is the length from *a* to H, from the scale, and put one foot of the compasses in H, and intersect the arc in *a*; then draw in the fence from the mark near G to *a*; then lay off the offset 10 at *a* to the river, also an offset of 100 at 280, likewise an offset of 80 at 540, and the offset of 70 at 900 to the river, and the offset of 130 at H to the river; then draw in the river *about two chains wide* from the first offset to the others; then take 1700, which is the length of the diagonal from H to *b*, from the scale, and put one foot of the compasses in H, and sweep an arc with the other foot at *b*; then take 1170, which is the length from H to *c*, and put one foot of the compasses in H, and describe an arc at *c*; then take 1160 from the scale, and put one foot in *b*, and intersect the arc in *c*; then draw in the road, 40 wide, from *b* to *c*, also the road from *c* to the bridge, and the fence from *b* to the mark between F and G, and the fence from the corner of the road at *c* to L; and if the diagonal answers to 1800 between K and *c*, it is right, which finishes enclosures I. and II.; then take off from the scale 1700, which is the length of the diagonal from L to N; and if the distance from N to *c* answers to 1440, it is right; then draw in the fence from the offset at N to the corner of the road near *c*, which finishes the plotting of enclosure III.; then take 1820 from the scale, which is the length of the diagonal from *c* to O; if that distance answers, draw in the fence from the offset at O to the road opposite *b*, which finishes the plotting of enclosure IV.; then take from the scale 1590, which is the length of the diagonal *b* P; if it answers, draw in the road from *b* 40 wide to *g*, and also the fence from the mark between P and Q to *g*, which finishes enclosure V.; then take from the scale 1730, which is the length of the diagonal Q C; if it answers, draw in the fence from R to C, which finishes the plotting of enclosures VI. and VII.; then take 1170, which is the distance from Z to *a*; at 140 lay off an offset of 110 to the river; at 300 lay off an offset of 100 to the river; at 410 lay off another of 180 to the river; and at 600 lay off another of 290; then draw in the river from one offset to the other, and the river about two chains wide, which finishes the plotting of enclosure VIII.; begin again at X, and take off from the scale 870, which is the distance from X to *d*, and put one foot of the compasses in X, and sweep an arc at *d;* then take 860 from the scale, and put one foot of the compasses in Y, and intersect the arc in *d;* then lay off an offset of 60 at X, also one of 50 at 340, and another of 60 at 600, and draw in the river from offset to offset, from X to *d*, and from *d* lay off the offsets taken to Y, which finishes enclosure IX.; then take 2150 from the scale, which is the length of the diagonal from B to *e*, and put one foot of the compasses in B, and describe an arc at *e;* then take 550 from the scale, and put one foot of the compasses in C, and intersect the arc at *e;* then take 1300 from the scale, and put one foot of the compasses in *e*, and sweep an arc at *f*; next, take 1000 from the scale, and put one foot of the compasses in B, and intersect the arc in *f;* then take off from the scale 2200, which is the length of the diagonal from *e* to V; if it answers, draw in the fence from C to *e*, from thence to the mark between W and X to the river, and the fences from *e* to *f*, from *f* to V, and from *f* to B, which finishes the enclosures X. and XI.; then lay off from the scale 630, which is the length of the diagonal across the yard from T to S, and put one foot of the compasses in T, and describe an arc to the corner of the wood at S; then take from the scale 480, and put one foot of the compasses in U, and bisect the arc in S, (in measuring from T to S, an offset was taken on the left of 330 at 280 to the corner of the wood, which lay off, and also 100 by 30, the length and breadth of the house;) lastly, draw in the house, and also the fences—which finishes the wood and the yard, and also the plotting of the whole—which should be carefully inked in, and the black lead lines rubbed out with a piece of bread or Indian rubber. Area = 172 acres, 3 roods, 36 poles.

Different surveyors have objected to measuring with the chain, and assert that there are many things that cannot be done with it, such as measuring the horizontal distance of a steep bank, the distance across a wide river, or measuring a plantation that cannot be entered on account of brushwood, brambles, &c. A land-measurer that is well acquainted in measuring with the chain can overcome all those difficulties, notwithstanding its being more tedious than with a theodolite. I shall now suppose a steep bank is to be plotted upon a horizontal plane upon paper, which must be done correctly, otherwise it will not join to other parts of a plan.

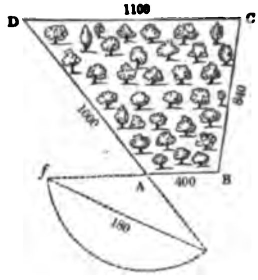
This figure represents a steep bank. The method used is by taking a short length. If the bank is not very steep, take half a chain's length; or, if very steep, take 25 links, or less. The foremost chain-man ascends the bank, and the hindmost chain-man takes 25 links, or a quarter of the chain, and orders the foremost man to stick in a pin, while the hindmost man holds the chain as nearly level as he can guess. Some surveyors, who wish to be still more correct, have a plumb, that they allow to hang over a mark, which makes them certain that the hand is exactly over the mark when the chain is held up to the level with the pin stuck in the bank by the foremost man. The plumb is made with a piece of lead, as a musket-ball, with a small cord fastened to it, about seven feet in length, and is held in the same hand that the chain is held with, which the hindmost man carries with him, and observes that the lead always hangs over the pins which the foremost man sticks in the face of the bank. By the figure, it will be observed that there were eight different pins stuck in the bank, which is two chains horizontal measure, whereas the measurement of the slope of the hill is two chains and 40 links. This shows the necessity of plotting plans by horizontal measure. If that measure is not allowed, it gives too little measure for the adjoining fields.



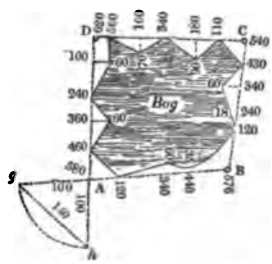
This figure is a river, of which it is requisite to have the exact width, and which cannot be measured across on account of its width and depth; and we will suppose the surveyor to have no other instrument but the chain. I shall suppose one of the stations is at A, nearly opposite a tree, T, close by the river side. The method of obtaining the breadth of it is to put a pole in at the station A and another at B, nearly opposite the tree; then go any length you please, and put in a pole at D, exactly in a line with the pole in A and the tree; then put in a pole at C at any distance you please, provided it is in a line with B and the tree on the opposite side of the river. When the poles are all placed, measure the distances from one pole to another, and insert each distance on an eye-draught, and measure a diagonal from A to C, or from B to D. First lay off the distance from A to B 140 links; then take the distance 225 from A to D from the same scale, and put one foot of the compasses in A, and describe an arc at D; then take the length of the diagonal, from B to D, which is 300, from the same scale, and put one foot of the compasses in B, and intersect the arc in D, which fixes that point; then take the distance from B to C, which is 220, and put one foot of the compasses in B, and describe an arc at C; then take the distance from D to C, which is 310, and put one foot of the compasses in D, and intersect the arc in C; then lay a ruler upon the marks C and B, and draw a line across the river from the point C through the point B; then draw another line from the point D through the point A, across the river; and where the one line intersects the other, is the distance across the river to the tree; or if there be any other distance you want to know, suppose the church, which stands a considerable way off the river, continue the line C D 160 to F, in the line of the tree and church; then draw from F a line through the tree to the church; produce the line A B to E, in a line with C and the church, and measure to E 120 links; lay a ruler upon C and E, and draw a line towards the church; and the intersection of the other line is the distance from the tree to the church. If you apply a pair of compasses to the tree and the intersection at the church, and lay the extent upon the same scale the rest of the work was plotted by, then you will have the number of chains and links the church is from the tree. Thus it is evident, that inaccessible distances may be ascertained with a chain only.



A B C D represents a coppice of wood, which we will suppose very much run over with brushwood and brambles, so that it cannot be measured through. In that case, take an angle with the chain, by measuring 100 links from A to *e*, keeping yourself in a line with the fence A D, and put in a mark at *e*; then measure 100 links from A to *f*, in a line with the fence A B, and measure the distance from *f* to *e*, which is 180; then measure on the outside of the wood from A to B, which is 400, and from B to C, which is 840, and from C to D, which is 1100, and from D to A, which is 1000; all those distances being carefully inserted either on an eye-draught or a field-book. To plot the wood, draw a line at pleasure, to represent the line *e* A D; then put one foot of the compasses in A, and sweep an arc from *e* to *f*, after having taken off 100 links from a large scale; then take off 180 links from the same scale, put one foot of the compasses in *e*, and with the other cut the line at *f*; then lay a ruler upon the point of intersection at *f* and the angle of the wood at A, and draw the fence A B, and with the distance 400 lay off from A to B, by any scale you want to plot the wood by, which you may make much smaller than the scale used for laying down the angle; then take the distance from A to D, which is 1000 links, which lay off upon the line from A to D by the same scale you used from A to B; then take the distance of the line B C, which is 840, and put one foot of the compasses in B, and describe an arc at C; then take the distance from D to C, which is 1100; next, put one foot of the compasses in D, and intersect the arc at C, which gives the exact shape of the wood.



This is an irregular bog or marsh, which is very wet. First put up poles at A, B, C, and D. It is necessary that an angle be taken with the chain, it being impossible to measure a diagonal across the marsh. First measure 100 links to *h*, in a line with the poles A and D; then measure out 100 links in a line with the pole in A and the pole in B, and put in a mark at *g*; measure from *g* to *h*, which is 140 links; then take the distance from *g* to *h* from the scale, and with one foot of the compasses in *g* intersect the line at *h*; then lay a ruler upon the point in A and the point *g*, and draw the line A B; then lay a ruler upon the point *h* and the point A, and draw the line A D, which forms the angle D A B.



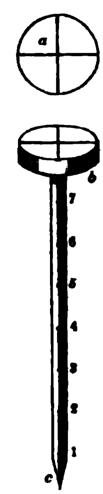
The distance from A to B is 576, from B to C 540, C to D 620, and the distance from D to A is 580. Those distances being all laid off from the field-book upon a plan, you have then to lay down the offsets, which you also take from the field-book, in the same way as described in page 22. After all the offsets are pricked off, draw the outline of the bog from offset to offset, all round till it closes, and you will have a plan of it.

The passage of any obstruction in the course of tracing a long line may be accomplished by going off at a right angle a distance sufficient to pass the obstruction; at right angles to this, or parallel to the original, a distance requisite to pass it; then, returning at right angles the same distance as that first measured: the chain will then be on the original line. An equilateral or isosceles triangle formed in a similar manner will do the same thing.

Many more examples might be given for taking angles with the chain, and for measuring very irregular pieces of land; but what has been already said on the subject shows, that pieces of land, however irregular, may be measured with the chain, a few poles, and ten pins, without the aid of any other instrument.

ART. II.—OF THE CROSS STAFF.

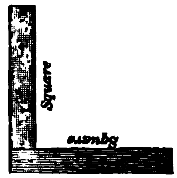
This figure is the representation of a cross staff—a very simple but useful instrument, and can be easily procured. Some surveyors, particularly in the inland counties of England, have the top very finely mounted in brass; others, that are very partial to this small instrument, have them mounted with plain sights, resembling those on a common theodolite, and prefer it to the best instruments that are made, and use no other. A cross staff, made of ash, with a neatly turned head, answers the same purpose as well as those that use them mounted in brass, and is more convenient to be carried. The circle marked *a*, with two lines crossing one another at right angles, should be three or four inches in diameter, and sawn across with a fine saw about half an inch deep, and about the sixteenth part of an inch wide; *b* is a socket for fixing the staff to the cross; *c* is the point of the staff, shod with iron for sticking it in the ground, which must be done at every observation that is taken with it. It will be found very useful in taking the perpendiculars to offsets, and for keeping the chain-men in a line between



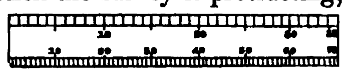
two objects or poles in measuring from one station to another. The cross staff always gives a right angle, and points out exactly that part of the chain when you are opposite any bend or angle to a fence or boundary you want to take an offset to; the angle which the fences make with one another can be ascertained with great precision; and any field, however intricate, can be measured with it and the chain. Although in some fields it is more tedious and laborious to measure with, and draw a plan from, the dimensions taken in the field, than either the plain table or theodolite; yet any piece of ground can be surveyed with the cross staff and the chain with great accuracy.

The staff should be divided into links (about eight only) for the purpose of measuring small offsets. Four links make an easy pace, sufficiently accurate for many ordinary purposes.

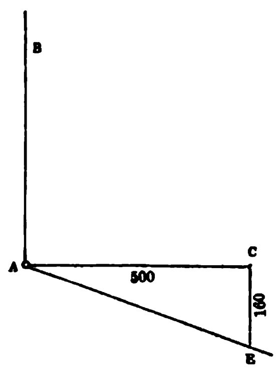
This figure represents a rectangular or carpenter's square, which is made use of when you protract the survey taken with a cross staff, as by laying one edge of it upon any line, the other edge gives a right angle or perpendicular, which the cross staff gives in the field; it is commonly made about 8 or 9 inches in length, but if longer so much the better. The common protractor will answer this purpose in most cases.



This figure is a feather-edged scale, made use of for pricking off distances that are taken in the field when the survey is protracting, which is much quicker than taking off the distances with compasses from a scale of equal parts. Its use is very general by all surveyors in great practice.



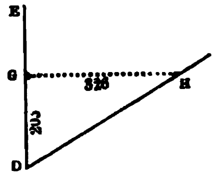
Suppose you want to take the angle B A E in the field with the cross staff, stick it in at the corner of the fence, and look through one of the slits upon the head of the staff parallel with the fence A B; then look through the opposite slit, having previously sent one of your assistants forward with a pole a few chain lengths, suppose 500 links; sign to him to move to the right or left hand till you see him; there cause him to place up a pole at C; measure to that pole; then remove the cross staff, and stick in a pole where it stood at A, and



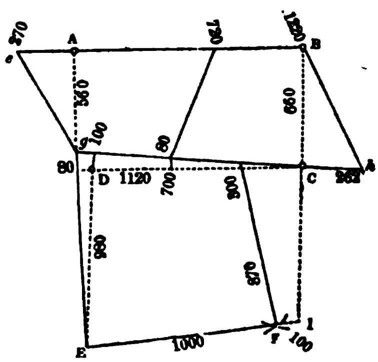
set up the staff at C, where your assistant's pole was placed; look through one of the slits to the pole left at A, and order your assistant to go to the fence A E; and cause him move to the right or left till you see him through the other slit; then measure from C to the fence at E, which is 160 links.

To protract that angle upon paper, draw the line A B at pleasure, and apply the square (see page 33) to the line A B; and draw the line A C, which is perpendicular with A B; with the feather-edged scale, or a pair of compasses, prick off 500, the length from A to C; then lay one edge of the square upon the line A C, and the other edge at C will give the perpendicular to E; then lay off the distance from C to E, which is 160, and make a mark at E; then lay a ruler, or the edge of the square, upon the angle A and the mark at E, and draw the fence, which gives the angle B A E.

Suppose it is required to take the angle E D H with the cross staff, which is less than a right angle, stick in the cross staff at any convenient distance upon the fence E D, suppose at G, and measure from G to D, which is 205 links; look through the sight or slit parallel with the fence D E; then look through the other slit, and cause one of your assistants to go with a pole towards the fence D H, and cause him to move either to the right or left till you see the pole which he fixes at H; measure to H, which is 326; then, upon your paper, draw a line at pleasure, to represent the line D E; then prick off 205 from D to G, and lay the square upon the line D E and the angle of the square at G, and the other edge will be the perpendicular to H; lastly, draw in the fence from D through the mark at H, which gives the angle E D H.



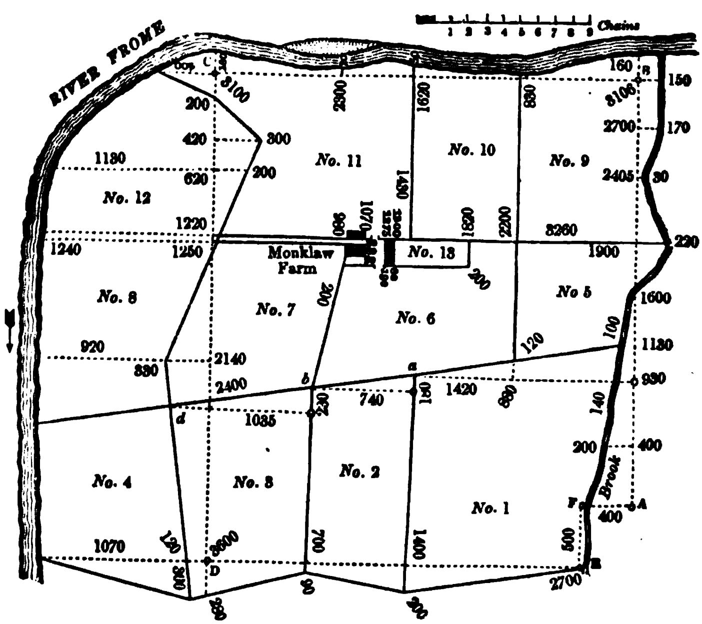
This figure is three enclosures measured with the chain and cross staff, and a plan made out of those enclosures. The cross staff was placed at A, and a distance of 270 links measured into the corner of the fence at *e*; another line was measured from A to *g* 560, the other corner, which is perpendicular with the line A B; in measuring the line A B, crossed a hedge at 720, and continued



the same line to B, which is 1220. The cross staff was set up at B, and a back sight taken to A, and a fore sight taken to C, by looking through one slit to A and the other slit to C, which is perpendicular with the line A B, and the distance measured to C, which is 660. The staff was set up at C, and a back sight taken to B, and a fore sight to D, which is perpendicular with the line B C; an offset was taken at C of 262 to the corner of a hedge at *h*. In measuring the line C D, crossed a hedge at 300, (where a mark was left;) at 700 an offset was taken of 80 to the corner of a straight hedge, which was crossed on the line A B, and the distance to D is 1120, and an offset of 100 was taken to the fence on the right, and another to the fence opposite of 80, which set down on an eye-draught, as well as all the other offsets and distances that are taken.

Set up the cross staff in D, and take a back sight to C, and a fore sight to E, by looking through the slits, which are perpendicular with the line C D; measure to E, which is 980; then measure to F, which is 1000; from thence to the mark left where you crossed the hedge at 300 on the line C D, which is 870; at F an offset was taken of 100, which is in a line with the line B C. All the above distances being inserted on an eye-sketch, I now have to show the method of making out a plan from the eye-draught that was kept when the survey was taken.

Draw a line at pleasure to represent the line A B; choose any scale you think proper, suppose two chains to an inch; lay the interior angle of the square upon the point A; and by laying one edge of it upon the line A B, the other edge will give the perpendicular to *g*; then with a pair of compasses take off 560 from the scale, and lay that distance off from the point A to *g*; then lay off 270 from A to the corner of the hedge at *e*; draw in the fence from *e* to *g*; then lay off the distance of 720, and make a mark where the line A B crossed a hedge; then lay off 1220, which is the whole distance from A to B; then lay the edge of the square upon the line A B, and the interior angle of it upon the point B, and the other edge will give the perpendicular to C, which draw in with a black lead pencil; then lay off the distance upon the line B C, which is 660, and also the offset of 262 at *h* to the corner of the enclosure; then draw in the fence from *e* close by A to B; from thence to *h*, and lay the edge of the square upon the line B C, and with the other edge draw the perpendicular to D; then lay off the distance by the scale and compasses from C to where the line crossed the hedge at 300, which mark with the point of the compasses, or a black lead pencil; then lay off 700, where an offset was taken to the corner of a hedge of 80, which offset mark; then from the same scale lay off the distance from C to D, which is 1120; and also the offset of 80, and another of 100 to the fence; draw in the fence from the corner at *h* to *g*; you also draw in the fence from the offset at 80 to the mark made at 720 on the line A B, which finishes two of the enclosures; then lay the edge of the square upon the line D C, and the other edge gives the perpendicular to E; lay off the distance from D to E, which is 980; then draw the fence from *g* to E, and take off with the compasses 1000, which is the distance from E to F, and on E describe an arc at F; then take the distance from the scale from F to the mark left at 300 on the line C D, which is 870; by placing one foot of the compasses in that mark the other will bisect the arc in F; then draw the fence from E to F, and from F to the mark at 300 on the line C D, which finishes the outline of the three enclosures. A short line of 100 was measured from F to *i*, merely to try how the line corresponded with the line B C; and it was found to be right. The fences ought to be drawn in with ink, and all the pencil lines rubbed out with bread or Indian rubber.



This figure represents the farm of Monklaw, which consists of twelve enclosures, measured with the chain and the cross staff. I began at A on the line A B, and a perpendicular taken to F of 400, an offset of 200 at 400 on the line A B to the brook; at 930 a perpendicular was measured to *a* to the corner of a hedge; in measuring to *a* crossed the brook or rivulet; at 140 an offset was taken of 120, opposite 880, to a corner of an enclosure; and the whole distance to *a* is 1420 to the angle of the enclosure No. I. (all of which distances I insert in my eye-draught.) Returned to 930, and continued measuring the line A B; at 1130 an offset was taken of 100 to the brook, where a hedge goes off, and crossed the brook at 1600; at 1900 crossed a straight fence, where an offset was taken of 220 to the brook, which is the boundary of the farm; at 2405 an offset to the brook of 30, at 2700 one of 170 to the brook; and the whole distance from A to B is 3106, where another offset was taken to the brook of 150. The cross staff was placed at B, and a back sight taken to A and a fore sight to C, by looking through one slit to A, and through the other to C, which gives the perpendicular from the line A B to C; at B an offset of 160 to the river; at 830, close by the river Frome, where a fence is crossed upon the line B C, and crosses another hedge at 1620, and an offset of 50 to the river at 2300, another offset of 20 to the river, and the whole distance to C is 3100 (inserted in my field-book or eye-sketch.) Again an offset was taken of 160 to the river at C, and another of 400 to the corner of the river and a fence; the cross staff was placed at C, and a back sight taken to B, and a fore sight or perpendicular to D (which is always known when your assistant is seen through the other slit,) where he fixes his pole, by the observer's directions, either to the right or left, till he is exactly perpendicular, when he is desired to stick in his pole. In measuring the line C D a fence is crossed at 200, at 420 an offset of 300 is taken to an angle in the fence, at 620 a perpendicular is taken to the river, which is 1130, and an offset to the fence of 200; at 1220 set up the cross staff, and take a perpendicular along the road to the river, which is 1240; at 1250 crossed the fence, at 2140 the cross staff is set up, and a perpendicular taken to the river; at 330 crosses a fence, and it is 920 more to the river; at 2400 crosses a fence, and the whole length of the line C D is 3600, which insert, and all other distances that are taken in measuring the different lines; set up the cross staff in D, take a back sight to C, and a perpendicular to E, also a perpendicular to the river Frome; crosses a fence at 120, and it is 1070 more to the river, where the fence leaves it; at D is an offset to the boundary of 280. In measuring the line D E, crosses a fence at 700, where there is an offset of 90;

crosses another fence at 1400, and an offset 200 taken to the boundary; the whole distance from D to E is 2700: fix the cross staff in E, and take a back sight to D, and a perpendicular to F, and measure the distance to F, which is 500, which enter in the eye-draught, and write Closes at F. Set every distance carefully down in the field-book or eye-sketch: you need not mind whether the eye-sketch is very like the ground, only make it in such a way as to give yourself a just idea of what you are doing, and be careful to make the figures legible, and to mark the offsets distinctly and where they were taken at.

I shall now point out the method of making out a plan from the dimensions taken in the field, and inserted in the field-book; and afterwards the manner in which the enclosures are to be finished that are not already completed.

The mode of laying off the perpendiculars having been already particularly mentioned in the last three figures, a repetition in what immediately follows would be superfluous.

First, a perpendicular was taken from A to F, and the distance 500 measured to F; 500 must be taken from a scale of equal parts, and laid off from A to F; a perpendicular was taken upon the line A B at 400, an offset of 200 to the brook, another at 930 to the corner of a hedge, where the brook was crossed at 140, and an offset taken at 880 of 120 to the corner of a hedge, the whole distance to *a* is 1420; lay all those distances off, also 1130, and the offset of 100 to the brook; then sketch in the brook with a black lead pencil from F to the offset of 200, from thence to where the brook was crossed at 140, and to the offset of 100 at the corner of the fence; then draw in the fence from that corner to *a*, and make a mark; then lay off 1600 at crossing the brook, and 1900 where the line crossed a straight hedge, and an offset of 220 to the brook; lay off those distances, and draw in the brook to where it was crossed at 1600, from thence to the offset at 220; then lay off 2405, and the offset 30, also the offset of 170 at 2700, and the whole distance from A to B 3106, and the offset of 150 to the brook; then sketch in the brook from 220 to 30, from thence to 170, and from thence to the offset taken at B to the brook of 150; then lay off upon your plan a perpendicular line with the square from B, and lay off the offset to the river of 160, and make a mark where the line crosses the hedge at 830, and draw in the river from 160 to 830, which river is about 150 links wide; lay off 1620, and make a mark where you crossed the fence and the offset of 50, also lay off 2300 and the offset 20; then the whole length of the line

B C, which is 3100, and the offsets 160 and 400; next, draw in the river from 830 to the offset 50, from thence to the offset 20, thence to 160, and from thence to 400; observe the sand bank on the other side of the river opposite the offset of 20, that was taken at 2300; from C lay off a perpendicular to D, on the line C D lay off from the same scale 200, where you cross a fence at 420, lay off the offset 300 to the fence; at 620 a perpendicular was taken to the river of 1130, and an offset to the fence of 200; draw in the fence from the water to where the fence was crossed at 200, from thence to the angle of the fence at 300; then to the offset of 200 taken at 620; you may also sketch in the river from the corner of the hedge to the mark left at the river at the end of the perpendicular, which measured 1130; lay off 1220, and a perpendicular to the river, and the distance 1240 along a road which is 20 links wide, and draw in the river from the mark at the end of the perpendicular at 1130 to the mark at 1240; lay off another perpendicular from 2140 to the river, and mark 330 at crossing the hedge, and lay off 920 more to the river; then lay off from the scale 2400 at crossing a hedge, and lay off the whole length of the line from C to D, which is 3600, and the offset 280 to the boundary; lay off another perpendicular from D to the river, which is exactly at where the boundary joins the river; and from the scale take first off 120 where it crosses the hedge, and 1070 more to the edge of the river; you may now draw in the river from the bottom of the road to the mark at 920, from thence to where the boundary joins the river Frome, which is about 150 links wide; also draw in the fence from 1220 to the offset 330, from thence to 120, and continue that line forward to the boundary, which is 300 beyond where the hedge was crossed; at 120 lay a perpendicular off with the square from D to E, lay off from D 700, where the fence was crossed, and also an offset of 90 to the boundary; then lay off from the scale 1400 at crossing the fence, and an offset of 200 to the boundary; then lay down the whole distance from D to E, which is 2700; you may now draw in the boundary from the river to the hedge at 300, from thence to 280, and from 280 to the offset at 90, from thence to the offset at 200, and from 200 to E; then lay off a perpendicular and the distance 500 from E to F, where the line closes; then draw in the brook to F, which finishes the whole of the outline of the farm. If the distance meets, which it will do if all the distances have been right measured, and the lengths taken exactly from the scale with the compasses, you may rest assured all is right, so far as is done.

I now come to point out how to finish the measurement of the enclosures which were not finished in going round the farm.

First, look out for the mark you left at *a*, which is in the corner of the enclosure No. 1, and walk along that hedge, and put up the cross staff along the fence, and try it several times till the perpendicular cuts the corner of the fence at *b*; then measure from the cross staff to *a*, which is 180, and the distance to *b* is 740; both of those distances set down on your eye-draught; then stick up the cross staff near the fence, so as you can see parallel with it as far as the boundary: if you cannot see the angle of the enclosure at *d* at the first trial, move yourself along the fence till you do see it; then measure the distance into *b*, which is 230, and the perpendicular to the cross hedge at *d*, which is 1035; insert these distances; then go to the mark at 1220, on the line C D, and measure up towards the houses, which is 980, to the end of the building, which mark, and also the breadth of the house, which is 30 links, and the breadth of the road 30, and the far end of the house is 1070; insert 1275 to the next house, and 1300 to the upper side of it, and an offset of 190, which is its length; at 1430 crossed a hedge, and at 1820 an offset was taken across the yard of 200; at 2200 crossed a hedge, and 3260 is the length of the whole line from the mark at 1220 to the brook; then go back to the houses, and measure 30 for the breadth of the road, 30 more for the breadth of the house, and 90 for the whole width of the yard, all of which distances being inserted on the eye-sketch.

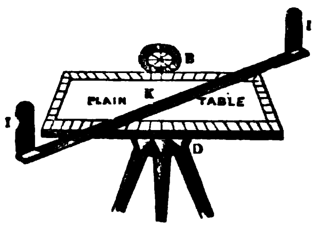
I now come to point out what way to plot upon the plan the enclosures which were not plotted when the outline of the farm was made. Begin at the corner *a* in No. 1; lay off 180 from *a* to where a perpendicular was taken, to the cross-hedge *b*; lay off that perpendicular, and the distance 740 upon it; then lay off the distance from *b*, 230, to where a perpendicular was taken to *d*, which distance, 1035, lay off with the scale; then lay a ruler upon the points *a*, *b*, and *d*, and continue that line to the river; then lay a ruler upon the corner *a*, and the mark upon the line D E at 1400, and draw that line into the angle of the boundary at 200; then lay a ruler upon the point *b*, and the mark left at 700 on the line D E, which will reach to the boundary at the offset 90; lay a ruler upon the point *d*, and see if it answers the former line; if it does, this finishes Nos. 1, 2, 3, and 4; then draw a line from the house, on the opposite side of the road, at 980, to *b*, which finishes No. 7; then draw a line from 1420 above the house, to where the hedge was crossed at 1620, and an offset taken of 50 on the line B C to the river; then lay a ruler on the point where an offset was taken of 120 at 880, on the perpendicular to *a*, from the line A B at 930, and draw the fence through the mark where it was crossed on the line B C at 830: this will finish enclosure No. 9, 10, 11, and 5. Next, lay off 200 from the scale, from the road at the underside of the house towards *b*; then draw a line from the offset of 200, that was taken across the yard from 1820, and draw in the line of the yard, which finishes No. 6; lay off all the short distances about the houses from the dimensions on the eye-draught, and it is completed. The enclosures 8 and 12 were done when the perpendiculars and offsets were taken on the line C D. After having drawn all the fences, the brook, and river, &c., with ink, rub out all the black-lead lines, and you will have an exact outline of your plan.

Particular care should be taken, if the assistants at any time should get into hollow ground, where they may lôse sight of the poles, or station staffs. In measuring a long line—which frequently happens, the cross staff being easily fixed in the ground—by looking back to one pole through the slit, and forward to the other, from a rising ground where the poles are both seen, the assistants measure forward to the cross staff till they perceive the poles. This instrument is of great service to land-measurers who make use of a theodolite, for laying off the perpendiculars in the field, to the bends and angles of fences, which saves them the trouble of inserting the bearing that they otherwise would have to take, had they no cross staff: this instrument invariably gives a right angle. It has another property I have frequently found, that of saving much time in the field: for example, when I have been using a theodolite, a bush or small height sometimes prevented me from seeing a pole which could not be perceived from one station to another; by ordering one of my assistants to go forward, and put himself in a line with the two poles with the cross staff, (which he can soon do, by removing it to the right or left, till both station staffs are seen through the slit,) I then measure the distance which is in a line with the pole I am measuring to. More might be advanced in favour of the utility of the cross-staff, but what has been already explained is deemed sufficient for an attentive student.

ART. III.—OF THE PLAIN TABLE.

This figure, 1, is the representation of a plain table. A, 2, represents the upper side of the table, 20 inches by 14, upon which a sheet of paper is fixed, containing the representation of a reduced plan of figure 4th. B is a compass-box with a magnetic needle, K is the index, and I I the sights, which are fixed at each end, D is the junction of the legs which support the table, fastened by three brass screws at the head to keep them together, but, at the same time, not so tight but to allow the legs to move easily out and in. The head to which the legs are attached is commonly made of boxwood, the table and legs of mahogany. The paper is fixed to the table with a frame. 3 represents the under side of the table, *d* is a brass socket fastened with three screws, the socket projecting out about 2 inches, which goes on to the cone at the top of the legs, and is made fast to them by the screw *e*. The index I I is movable, and only laid upon the view of the table, to represent the method of using it when an angle is taken: it is commonly made of brass, about 18 or 20 inches long; the sights are also of brass, about 5 inches high. From the end of the index, in each of the sights, there is a small and large aperture, or slit, one over the other. If the aperture be undermost in one sight, it will be uppermost in the opposite, and *vice versa*. The plain table can be purchased, with all its apparatus, from any of the mathematical instrument makers, from four to five guineas. The wood-work may be made by any carpenter, and the other work by a brass-founder; the magnetic needle may be had at any of the watch-makers in any seaport town that are in the practice of repairing mariners' compasses; but it is commonly much more complete when had from a mathematical instrument maker. Surveying by the plain table is a very expeditious method, as every angle taken is plotted in the field, and all the distances laid off by scale and compasses. Even the fences may be all drawn with a pençil upon the paper (that is put upon the table) at the same time, and such lines only might be inked as you wish to insert upon the plan.

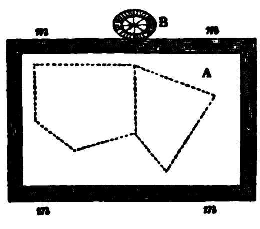
1.



This useful instrument is so simple in itself, that any person with a little practice may survey with it. However, a further explanation, including the method of using it, will be essentially necessary to the young surveyor.

The table is made of a smooth board, in the form of a sheet of demy paper, and sometimes made as large as to hold a sheet of royal. The frame that is made to keep the paper fast upon the table is taken off, and the paper should be made wet with a sponge previous to its being laid upon the table, and the frame put over it, which keeps it tight: when the paper drys, it will be so contracted as to leave it quite smooth upon the table. To keep the frame also fast upon the table, four pieces of brass are fastened to the frame, marked *m m m m*, which goes through four holes through the frame, and again fastened with four pins, that go through holes made in the four pieces of brass, which keeps the frame fast upon the table: the index is chamfered off on one side like a Gunter's scale. In the lower part of one sight is a vertical or upright slit about an inch and a half in length, and in the other opposite is a wide opening, where a hair, or piece of silk thread, is fixed exactly vertical in the centre of the wide opening, to cut the object when you look through the slit next the eye. In the opposite sight, the slit is made on the uppermost part of the slit, and the wide opening in the undermost part of the sight, and the vertical hair placed exactly in the centre; so that the aperture on one sight is at the lower part in one sight, and *vice versa* at the slit. In general, a few scales, of different sizes, are engraved upon the upper side of the index; any of which you choose you may plot the survey by, as every distance must be laid off by scale and compasses on the spot. The legs of the plain table should be a convenient height to support the table, and made so as to move out and in, which allows the table to be planted high or low to the height wanted. I shall now give one general rule how it ought to be placed at each place it is set up at.

2.



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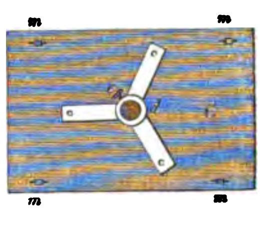
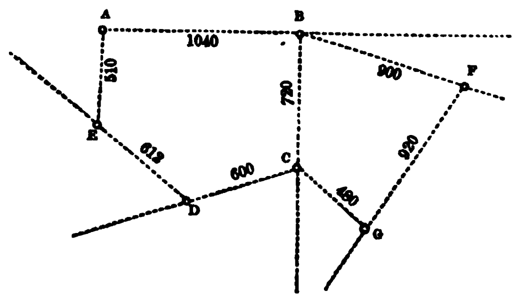


To use the plain table.—I. Place the table as nearly horizontal as you can guess, by moving the legs out or in to the height you want it, and turn the table round by the socket, upon the top of the three legs, till the north end of the needle points over the *fleur de lis*, in the compass box. The long way of the table will be always north and south, the short way always east and west; and before an observation is taken, screw the instrument fast with the screw in the socket to the cone at the top of the legs, which should have a strong ferrule put upon it, made of brass, at least one-sixth of an inch thick, with a groove cut out a little for the screw to go into, which will keep the table from slipping off the ferrule, when it is removed from one station to another.

Place the table as before directed, and observe that the needle settles over the *fleur de lis*; then screw it fast at A, where you begin; lay the chamfered edge of the index upon the station at A, and look through the sight, and find out the pole placed in E and the hair in the sight to coincide, and draw the line A E with a black lead

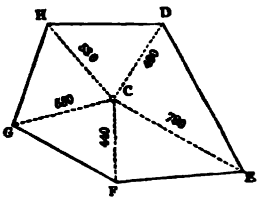


pencil or the point of the compasses, and lay off the distance from any scale you have fixed upon, which suppose 510 from A to E; you then lift the index, and lay the chamfered edge of the index upon the point A, and take a bearing to B, which you will know when you see the hair in the sight and the pole placed in B to coincide: then draw the line A B, and lay off the distance from the same scale from A to B, which is 1040; you then remove the table from A, and plant it at B; loose the screw *e*, 3, a little that holds the table fast to the legs, and lay the thin edge of the index upon the last line you drew upon the paper, which is the line A B, and take a back sight to A; the longer the lines are drawn the better, as you can lay the index with more exactness upon a long line than a short one. Hold the index fast after it is laid exact upon the line B A, and move the table round till you see the hair in the index and the pole in A to coincide; then screw it fast as before with the screw *e*, and turn the index; then lay the thin edge of it over the point at B, and when you see the hair in the telescope and a pole placed in C to coincide, draw the bearing, and lay off the distance with the scale and compasses from B to C, which is 720. Before drawing the line, observe that the index has not moved from the line it was laid upon. When you take a back sight while turning the table, if it should lay it upon the line again, look to the back pole, and turn the table till you see the hair and the pole to coincide exactly; then screw the table fast, and lay the chamfered or thin edge of the index on the point C; next, move the index till you see, through the sight, the cross hair and the pole in D to coincide; lay off the distance C to D, which is 600, from the same scale the rest of the work is plotted by; again, plant the table up at D, and loose the screw in the socket *e* a little, and lay the thin edge of the index on the line C D, and take a back sight to C; here screw it fast; next take a bearing from D to E: if it answers, and also the distance to 612, which is the length from D to E, you are certain of having made no error either in measuring the distances or in taking the angles.

The next thing to be done is to plant the instrument up in B, and lay the edge of the index on the line B A; then turn the table round till you see, through the index, the pole placed in A and the vertical hair in the sight to coincide; screw it fast by the screw in the socket, and the table will be in the same position it was in when it was planted in B as before; you can also know by the needle if it settles over the *fleur de lis*, which it will always do if there are no metallic substances to attract it.

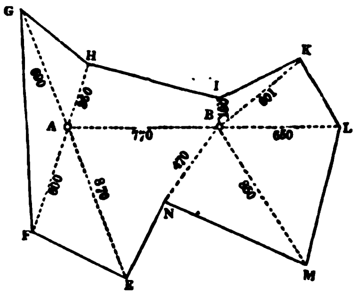
Then lay the thin edge of the index on the point at B, and take a bearing to F, and lay off the distance, which is 900, from B to F; then plant the instrument in F, and lay the thin edge of the index upon the line F B, and turn the table round till you see the pole in B and the hair in the sight to coincide; then screw it fast, and lay the index upon the point at F. and take a bearing to G; then lay off the distance 920 from F to G; again, plant the instrument at G, and lay the thin edge upon the line G F; then take a back sight to F, by turning the table round till you see the hair and the pole in F to coincide; then screw it fast, and lay the thin edge of the index to the point G, and take a bearing to C: measure the distance from G to C, which is 480: if the angle and distance agree, it is what is generally termed an exact closing. From what has been now explained, it is presumed the method of taking the bearings and placing the table up at the different stations will be easily comprehended: great care, however, must be taken to place the table as nearly level as possible, and the centre of the legs immediately above the holes the poles were placed in at each station. The plain table, by using it in the manner above, has an advantage that no other surveying instrument has. If an error has been committed in taking either a bearing or a wrong distance from the scale, the work will not meet. A good method to correct an error is to leave a pole or mark of any kind at any of your stations: by applying the thin edge of the index to that mark and the station you stand at, and looking through the sights to it, the needle will settle over the *fleur de lis* in the compass-box, if no error has been committed. If an error has been made, the needle will settle over some other of the degrees in the compass-box, which should be corrected before you go farther.

This figure is an enclosure where all the angles are seen from the point C which can be measured, and a plan made of the same without moving the table from the spot where it is placed. Suppose the angles D E F G H are all seen from the point C; place the table up at C as horizontal as you can, loose the screw a little that is in the socket at *e*, and turn the table round upon the head of the legs that support it, till the



needle in the compass-box settles over the *fleur de lis;* then screw the table fast to the head of the legs, and lay the thin edge of the index at the point C; then look through the sight or slit next your eye, till the vertical hair in the sight opposite is in a line with the angle D; measure the distance to D, which is 490; take that distance from the scale, suppose half an inch to a chain, and lay it off upon the bearing, and make a mark at D with a black lead pencil; then lay the thin edge of the index on the point C, and look through the sight next the eye, till the hair in the other sight is seen to be in a line with the angle E, and order your assistants to measure from E to C, which is 780; next, lay that distance off upon the bearing line at E, and make a mark; then lay the index upon the point C, and look through the sight next the eye, till the hair in the other sight coincides with the angle at F; draw that bearing, and measure the distance to F, which is 440; lay off that distance from the scale, and make a mark at F; then lay the edge of the index upon the point C, and look through the sight in the index till you see the hair and the angle G to coincide; then cause your assistants to measure from G to C, which distance is 550; lay that distance off upon the line, and make a mark at G; then lay the index upon C, and look through the sight till you see the hair in the other sight and the angle at H to coincide; draw in the bearing, and measure to H, which is 530; lay off that distance, and make a mark at H; lastly, draw in the fences from D to E, E to F, F to G, G to H, and H to D, which will give a plan of the enclosure; then ink in the fences, and rub out all the bearing lines with Indian rubber.

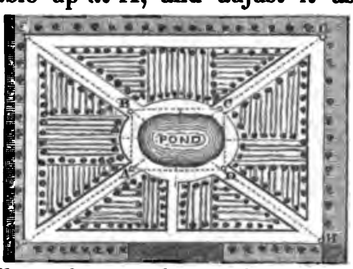
This figure represents an enclosure of nine sides, to which four angles are taken at the point A, and five at B; a line is also measured from A to B, whence a plan may be made on the spot.



First plant the table at the station A, and set it as level as possible, which you can nearly guess; when the needle traverses with freedom in the compass-box, turn the table gently round till the needle settles over the *fleur de lis;* then take bearings from the station A (as directed) to E, F, G, and H; measure the distances to those points, and lay them off from a scale, suppose from A to E 870, A to F 600, A to G 690, and from A to H 360.

These distances being all laid off, draw in the fence from E to F, F to G, and from G to H; then take a bearing to B, and lay off the distance 770; then plant the table at the station B, and take a back sight to A; after laying the thin edge of the index upon the line A B, look through the sight next the eye till you see the vertical hair in the opposite sight on the index to coincide with the pole left in A; here screw the table fast, and draw lines from the point B to the angles I, K, L, M, and N, and measure all the distances, and lay each distance off from the same scale as those laid off from the point A to their respective angles, viz. 180 from B to I, 601 from B to K, 650 from B to L, 850 from B to M, and 470 to N; lastly, draw in the fence from H I, I K, K L, L M, M N, and N E, which will give a true representation of the enclosure of nine sides, which should be drawn in with ink, and the pencil lines rubbed out.

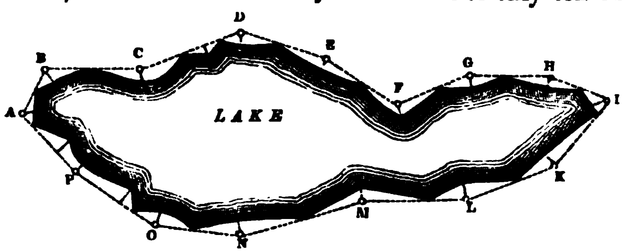
This figure represents a garden, with a fish-pond in the middle of it. To survey this, plant the table up at A, and adjust it as before mentioned, and take a bearing to B, and lay the thin edge of the index to the point A, and look through the sight till you see the hair in the opposite sight and the pole in B to coincide; measure the distance from A to B, which is 110, and lay it off from the scale



you choose to adopt, and also an offset of 50 to the pond, another of 20 on the right, and another on the left of 10 to the edge of the walk; which also lay off. Before moving the table, take a bearing to D; in measuring that line, opposite 100 you have an offset of 18 to the pond, and another to the edge of the walk of 13, and the distance to D is 200; all of which lay off upon the plan, and draw in with a black lead pencil part of the pond, and also the walk from B to D. Before removing the table take a bearing to E; then plant the instrument up at D, and take a back sight to A; then take a bearing from D to C, and another up the walk to H, and measure the line D C, which is 140, which distance lay off, and also an offset of 15 to the pond, and another offset of 15 to the edge of the walk taken at 70 in measuring the line D C; then plant the instrument at C, and take a back sight to D, and screw the instrument fast, and take a bearing up the walk to G, and another to B; in measuring that line at 100, an offset was taken of 10 to the pond, and another of 20 to the edge of the walk; prick off these distances, and also the distance from C to B, which is 200; then draw in the pond, and also the walk round it; you then go to A, and measure up the walk from A to E, which is 280, and the breadth of the walk 26; prick off these distances upon the plan, and draw in the walk from A to E; then place the instrument at E, and take a back sight to A; screw the table fast, and take a bearing to H and another to F; measure the distance to H, which is 630, and take an offset to the front garden wall, which is 60, and 40 to the garden wall on the right. In measuring that line, you were opposite a walk at 310 fronting the hot-house; lay off these distances, and also the length and breadth of the hot-house, which is 100 by 40; then draw in upon the plan the wall from E to H, also the hot-house, and the walk on that side of the garden, which is 26 wide; then measure from E to F, which is 448, and an offset to the garden wall at E, which is 40, and another offset of 40 at F; prick off these distances on the plan, and draw in the garden wall from E to F, and also the walk, which is 26 wide; remove the instrument to F, and take a back sight to E; here screw it fast, and take a bearing down the walk from F to B; if it answers it is right, if it disagrees it must be rectified before any more is done, by going back till you find out the error. Next, take a bearing from F to G, and measure the distance, which is 630, and take an offset of 40 at F, and another of 40 at G, and draw in the garden wall from one offset to the other, and also the walk 26 wide; then plant the instrument up at G, and take a back sight to F; then screw the table fast, and take a bearing to where the pole stood in C: if it answers, there is no occasion for measuring from

G to C. Next, take a bearing to where the pole stood in H, and measure to H, which is 447; if the bearing and distance answers, the garden-wall may now be drawn in, and also the walks 26 wide, from the offset at G of 60 to the offset at H of 60, which finishes the plan; that should be inked in, and the black lead lines rubbed out. For want of room on the cut, I have been under the necessity of using small scales; but it is presumed that, being merely for explanation, they will be understood equally well. It will not be improper to remark, that the larger the scale the work is plotted from, (particularly when the figure is complex,) it renders the less confusion in the multiplicity of lines; it also admits of the figure (however complicated) being protracted with facility. Besides, there is another advantage; very minute parts of a plan may be represented on the large scale, which the small one would not admit of. For making out a drawing of a garden, see *Plate* XVI.

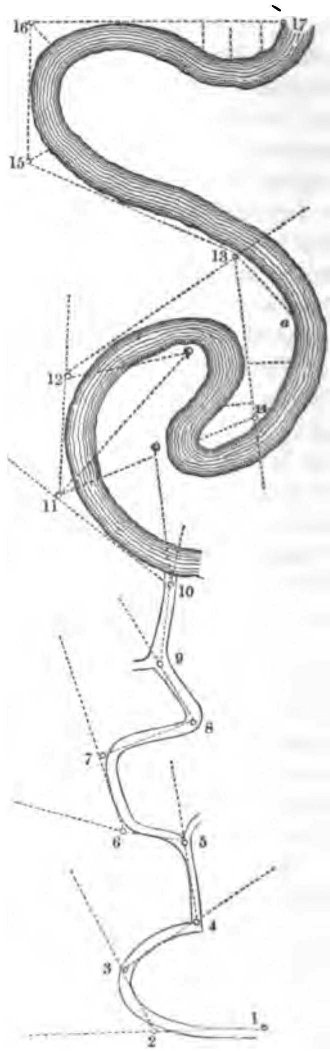
This figure represents a very crooked lake, which, for want of more room, is contracted to a very small scale of only ten chains to



an inch. Plant the table at A, and adjust it as before, and take a bearing to B, and measure the distance 60 to the lake, and also 140 where the chain touches the edge of the water, and the distance to B 285; prick off those distances, and plant the instrument up at B, and take a back sight to A, and lay off 80, the distance that B is from the lake; then draw in the edge of the lake from the offset 60 to where the chain touched the water at 140, from thence to the water opposite B; take a bearing to C, which should be drawn a considerable way past C, for the purpose of laying the thin edge of the index correctly upon the line; when a back sight is taken, all other bearings should be done in the same way as is represented in *Fig. page* 36, or in *Fig. page* 52. In measuring from B to C at 200, the chain touches the water, and the distance to C is 450, and an offset of 80 to the lake; lay off these distances, and draw the outline of the lake from the offset at B to where the water was touched at 200, from thence to the offset of 80; then place the table at C, and take a back sight to B; screw the table fast, and take a bearing to D; in measuring from C to D, the chain touched the water at 280, and an offset was taken at 400 of 60, and again was close by the lake at 500, and the distance to D is 590, and the distance from D to the lake is 60; then lay off all these distances, and draw in the outline from the offset at C to where the water was touched at 280, to the offset 60, from thence to the mark at 500, and from thence to the offset opposite D; then remove the instrument to D, and take a back sight to C, and a foresight or bearing to E; in measuring from D to E, it touches the lake at 200, and the distance to E is 500, and an offset to E to the lake is 80; lay off these distances, and draw in the outline of the lake from the offset opposite D, to where the chain was nearest the lake, from thence to the offset of 80; shift the table to E, and take a back sight to D, and a fore sight to F, the chain touches the lake at 220, and the distance to F is 400; lay off these distances, and draw in the outline from the offset E to 220, from thence to the offset of 60, opposite F; then plant the table at F, and take a back sight to E, and a bearing or fore sight to G; in measuring that line, the chain was near the lake at 220, and the whole distance to G is 430, and an offset of 90 to the lake; draw in the outline of the water to the offset of 90; remove the table to G, take a back sight to F, and a fore sight to H; in measuring the line from G to H, I touched the lake at 170, and the whole distance to H is 400, and an offset taken from H to the lake is 50; lay off these distances, and draw in the outline of the lake from the offset 90 to 170; from thence to the offset of 50, opposite H; then place the table at H, and take a back sight to G, and take a bearing to I. Observe if the needle settles over the *fleur de lis;* if it does, you may measure the line from H to I; if it should not settle exactly over the *fleur de lis*, you must go back and find out the error, which you will soon accomplish by taking a few observations back. In measuring the line HI, the chain touched the lake at 130, and the distance from H to I is 300, where an offset of 70 was taken to the lake; draw in the outline from the offset taken at H to 130, from thence to the offset of 70; plant the instrument up at I, and take a back sight to H, and a fore sight to K, which is 450, where an offset was taken of 80; lay off these distances, and draw in the outline of the lake from the offset opposite I to the offset opposite K; the instrument is then placed at K, and a back sight to I, and a fore sight to L: in measuring that line, the chain touched the lake at 200, and the distance from K to L is 510; lay these distances upon the plan, also the offset of 100 to the lake, and draw in the outline of the water from the offset opposite K to 200, from thence to the offset of 100; then plant the instrument at L, and take a back sight to K, and a fore sight or bearing to M: in measuring that line, the chain was close upon the lake at 200, and the whole distance to M is 530, and an offset taken from M to the lake is 65; these distances being laid down, draw in the outline of the lake from the offset of 65, opposite L, to 200, and from thence to the offset of 65 opposite M; next, plant the table at M, and take a back sight to L, and a fore sight to N; in measuring that line, the chain touches the edge of the lake at 430, and the length of the line to N is 660, and an offset to the lake is 70; lay off these distances, and draw in the outline of the lake from the offset of 65, opposite M, to 200, where the chain touched the water, from thence to the offset of 70, opposite N. Again, take the instrument to N, and take a back sight to M, and a fore sight or bearing to O: in measuring that line, the chain touches the lake at 230, and the length of the line to O is 460, where an offset is taken of 80 to the water; lay off all these distances on the plan, and draw in the outline of the lake from the offset of 70, opposite N, to 230, where the chain touches the water to the offset of 80, opposite O; then, with the instrument at O, take a back sight to N and a fore sight to P: in measuring that line, the chain touches the lake at 200, at 230 an offset is taken to the lake of 80, and the whole distance to P is 510, where an offset of 40 is taken to the lake; lay off these distances, and draw in the outline of the lake from the offset of 80, opposite N, to where it touches the water at 200, from thence to the offset of 80, taken at 230, and also to the offset of 40, opposite P; then plant the table at P, and take a back sight to O, and a fore one to A, where the survey began. In measuring that line, the chain was close by the lake at 330, and the whole distance from P to A is 410. If the bearing from P to A and the distance agrees, you may rest satisfied no error has been made. Lastly, draw in the outline of the lake from the offset of 40 opposite P to where the chain touches the edge of the lake to the offset taken at A, which will finish the plain table plan of the lake.

Now, if the angle and distance had not met in A at the last station, it is evident that some mistake had been made; in that case, it shows the absolute necessity of leaving marks at each station. It is in general done with three small stones laid about the holes the poles stand in; and, when stones are not at hand, some surveyors get a few wooden stakes made, and stick one in each hole where they had a station; and others carry a spade, and make a mark. The best way of correcting the error (should one have been committed) is to measure back, and set up the table as before till it is found out. If the error has been made in taking a bearing, the needle will point it out; as, in that case, it will not settle over the *fleur de lis* in the compass-box; and if a mistake has occurred in measuring a distance, it will not agree with the extent laid off upon the plain table sheet.

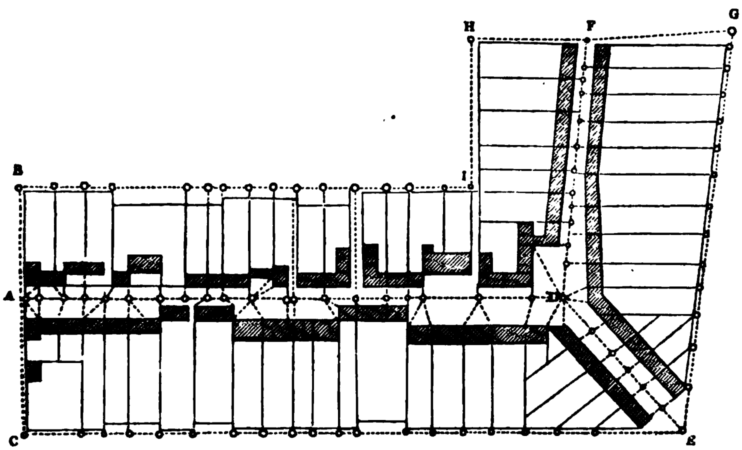
To survey a road, &c., with the plain table.—The figure represents a serpentine road or gravel walk, and supposed to be in the midst of a plantation, from No. 1 to No. 10; also, from No. 10 to No. 17, a crooked river. Plant the table at No. 1, and having adjusted it, begin near the edge of the paper, and first draw a bearing to a pole placed in 2, (with a black lead pencil,) which line should be a considerable length, in order to place the index exact upon the bearing when a back sight is taken; measure from No. 1 to 2, which is 530 links, which take from any convenient scale. Before proceeding further, I must remark, that to enumerate every distance and offset that is necessary to be taken in this survey, would only render the business very complex; but by omitting such a multiplicity of figures as the explanation would require, it will appear very plain. However, I will refer the pupil to the method used in the explanation of the *Fig. page* 44. To proceed: Plant the table to No. 2, and take a back sight to No. 1; then take a fore sight or bearing to No. 3; measure those distances, which lay off upon the plan, and also at what distance the



chain reached when it was nearest the edge of the road; then remove the table to No. 3, and take a back sight to No. 2, and a fore sight to No 4; lay off that distance, and also the distance to where the chain was nearest the edge of the road, also the breadth of the road; then set up the table at No. 4, and take a back sight to No. 3, and a fore sight to No. 5; lay the distance off, and likewise the distance to where the chain was nearest the edge of the road; again, go to No. 5, and take a back sight to No. 4, and a fore sight to No. 6, and lay down that distance, together with the distance the chain was at nearest the road; then go to No. 6, and take a back sight to No. 5, and a fore sight to No. 7; lay off that distance on the plan, also the distance the chain was at when nearest the edge of the road; then go to No. 7, and take a back sight to No. 6, and a fore sight to No. 8; measure the distance, and lay it off, also where the chain reached when nearest the road; next, go to No. 8, and take a back sight to No. 7, and a fore sight to No. 9; measure that distance, which lay off upon the plan, and the distance the chain was at when nearest the edge of the road; examine the needle; if it settles over the *fleur de lis* it is a proof of your having taken all the bearings right; again, lastly, go to No. 9, and take a back sight to No. 8, and a fore sight to No. 10, and lay down the distance, and also the extent to where the chain touched the edge of the road, which will finish the survey of the serpentine road. You should now draw in the road from all the different distances that were laid down, and you will have a representation of all the bendings and windings of the road through the wood.

How to survey the river.—Place the table at No. 10, and take a back sight to No 9, and a fore one to No. 11; from No. 10 take also a bearing to a tree on the opposite side of the river; measure to No. 11, and lay off the distance with an offset at No. 10, and mark when the chain was at the edge of the river, noting the distance from No. 11 to the river; these distances being all marked, sketch in the river from the offset at 10 to that at 11, and the river 120 links wide; then go to No. 11, and take a back sight to No. 10, and a fore sight to No. 12; measure that line, and lay off the distance, also the distance where the chain touched the river, and the offset to the river at No. 12; then draw in the river from the offset at No. 11 to where the chain touched the river, from thence to the offset taken at No. 12; at No. 11, taking a bearing to the tree that was seen from No. 10, where these lines intersect, gives the distance to the tree; take also a bearing from No. 11 to another tree; then go to No. 12, and take a back sight to No. 11, and a bearing to the tree that you observed from No. 11, the intersection is the distance to that tree; next, take a bearing at No. 12 to No. 13. In measuring that line, mark where the chain touches the edge of the water, also where an offset is taken, and the length of that offset, also the whole distance to No. 13. You may now sketch in that side of the river from the offset at No. 12 to the offset taken between No. 12 and 13. Note, in measuring up one side of a river, many objects may be seen, such as a large stone, a bush, or any other mark. If you take a bearing from one station to each of the objects on the opposite side of the river, they may all be intersected by taking bearings from other stations. These intersections give the exact width of the river, which draw in upon the plan from one intersection to another. Again, go to No. 13, and take a bearing and distance to *a*, also take a bearing from No. 13 to 14, and from 14 to the river, in a line with the last tree that was intersected, also some offsets; then draw in the river from the offset taken upon the line between 12 and 13, from thence the one taken on the right hand near the river on the line between 13 and 14, from thence to the mark opposite the tree, and thence to No. 14, and from No. 14 to the point made upon the offset that was taken to the left hand from the line between No. 13 and 15, thence to *a*, and also from *a* to No. 13; take next a bearing from No. 13 to 15, and measure the line to 15 and an offset to the river; lay off these distances, and draw in the river from No. 13 to the offset at the river at No. 15; again, with the instrument at No. 15, take a back sight to No. 13, and a fore sight to No. 16. In measuring that line, note where the chain touches the edge of the river, and also the distance to No. 16 and an offset; lay off these, and draw in the river from No. 15 to where the chain touches the edge of the river; from thence to the offset taken at No. 16: go to No. 16, and take a back sight to No. 15, and a bearing to No. 17. In measuring that line, mark where the chain touches the river, also the distance where offsets were taken, and the distance to No. 17, and all the offsets between No. 16 and 17; next, draw that side of the river from the offset at No. 16 to where the line touches the river; from thence from one offset to another to No. 17. In measuring the different stations, care must be taken to intersect as many objects at the edge of the river on the opposite side as you can conveniently take, for the purpose of ascertaining its breadth, and drawing it upon the plan; the distance from one intersection to another is the width. In taking a survey, represented by the preceding figure, the surveyor should observe the needle frequently, to see that the north end of it settles over the *fleur de lis*, which (as I have before observed) it will do if the bearings and distances have been all right taken, and provided there is no attractive substance near. The reason of taking a back sight at every station is merely to prevent any mistake that the needle would cause, if it was influenced by attraction. If a land-measurer could trust to his needle, the plain table might be only placed at every alternate station. When all is planned upon the plain table sheet, it ought to be inked in, and the multiplicity of black lead lines that are drawn rubbed out.

This figure represents a small town, surveyed and planned upon the spot with the plain table.—This is performed much quicker than



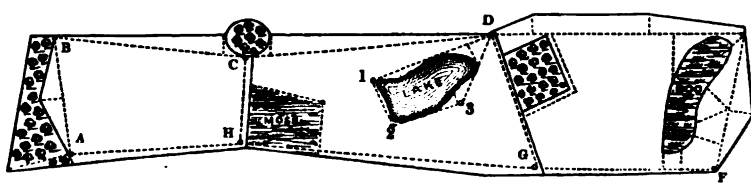
by any other method yet known, even with the most costly instruments, as it spares the trouble of protracting and laying off distances and bearings in the house, which a surveyor is constrained to do, if he uses either a circumferenter, sextant, or theodolite; and although it takes longer time in the field, yet very little time in the house is necessary, as a land-measurer has only to ink in the plan drawn upon the table on the spot.

To take the survey and plan of a town.—This is tedious, and requires great care, patience, and attention. It will be proper to observe, that the table must be set up at every spot where bearings are to be taken, to the angles and corners of houses, &c., which are represented by dotted lines; each of which is measured, as well as the station lines: For example, on the line A D, the table was erected sixteen different times before the observations were finished on that street, and measurements taken, not only to the corners of houses, but also to all the projections, and the distances all pricked off upon the spot with the scale and compasses. This I have inserted, merely to give an idea of the labour to be expected in taking the survey of a barony or domain, most baronies having a village belonging to them.

Suppose, then, the measurement is to begin at A, take bearings to a pole placed at C; then along the street to one at D, and another to B. In measuring from A to B, lay off the distance to the corner of the house, also the length and breadth, which is taken with the offset staff or a tape; then lay off the whole distance to B, and leave a mark; return to A, and measure to C. In measuring that line, mark the distance to the corner of the house, also the length, and take the breadth; also lay off the distance from A to another house, where a garden wall goes off, and also the length and breadth of that house, and the distance to C; all of which distances being laid down, draw in the houses and the garden walls. In measuring the line A D, at every place the table is erected lay off the distance, and lay the chamfered edge of the index upon the bearing line A D, and turn the table round till the cross hair and the pole placed in D coincide, and *vice versa;* let the index remain, and turn yourself to the other sight, and take a back sight to A. If the hair and the pole left in A coincides, you are certain that the table is exactly on the line betwixt A and D. At every station observe that the back and fore poles are in a line with A and D. Should this not be the case, move the table till you perceive they are so: Take bearings from all the stations to the corners of houses, and lay off the distance to where it is placed from A; then measure the length of all the short distances to the corner of the houses, and lay these distances down upon the plan; then draw in, with a black-lead pencil, the front line of each house, as you proceed from one place to another. In many places you can determine the breadth of the houses taken, particularly to those that project, which should be laid down upon the plan and by measuring, taking the bearings and laying off the distances from each station, also those distances taken with the tape, upon the plan, to all the corners of houses on both sides of the street, which will enable you to draw in the whole length of the street from A to D;

then set up the table at D, and take a back sight to A; screw the table fast, and take a bearing to F and another to E. Observe to set up the table opposite where angles are taken, and the distance laid off from D to every station; also all the short distances to the divisions of houses must be pricked off from the scale, and drawn upon the plan till you measure to E; which distance being laid down, leave a mark; then plant the table at E, and take a back sight to D, and a fore sight or bearing to C. In measuring that line, mark the distance to the corner house of the street, and also where the first garden wall goes off, and take bearings, and lay off all the distances upon the plan; erect the instrument at the corner of each garden, and lay off the breadth of each as you proceed; take a bearing along the garden walls, and prick off every distance upon the plan and the bearings, which is soon done by placing the instrument opposite to each; next, take a back sight to E, or a fore sight to C, and lay off the whole distance from E to C. If the distance meets in a point at C, it is a proof that the measure is right; then draw in all the gardens, (and if necessary, distinguish each by inserting the occupier's name;) and if any omission has been made, it must be rectified afterwards; again, return to D, and measure the line D F; set up the instrument wherever you see it necessary, marking the distance from D to where it stands, and take offsets to all the different houses, and lay off the distances upon the plan the whole way to F as you go on, which will give the length of each house both on the right and left; then draw in upon the plan the fronts of the houses on both sides of the street, distinguishing each house by making a short line betwixt it and the next, till the breadth of the houses is ascertained; again, with the table at F take a back sight to D and a bearing to H, also one to G. In measuring from F to G, mark the distance to the corner of the house, also the breadth, and take a bearing along that side next the gardens, which ascertains the width of the houses on that side; then go to G, and take a back sight to F and a fore sight to E. In measuring to E, lay off every distance from G, where the gardens go off, and take bearings by looking through the sights of the index along each of them as you proceed to E. Observe to let the chain always lie stretched on the ground, that you may count the odd links as well as the chains, where each bearing is taken, and lay off all the distances as you go on to E; which lay down upon the plan, and the distance to E. If it agrees, what is done so far is right. Return to F, and measure to H, and lay off the distance; set up the table at H, and take a back sight to F and a fore sight to I. In measuring to I, lay off the breadth of each garden or yard as you proceed, and take bearings of each as before directed; then go to I, and take a back sight to H and a fore sight down a narrow lane; measure the length and breadth of the lane; return to I, and take a bearing to B. In measuring that line, take a bearing along the yards, and also offsets where they occur, and note the breadth of each yard or garden till you come to another lane, which should be measured as far as the street and the length and breadth of the houses. Proceed all the way to B, and be particular in pricking off all the offsets, (the gardens being very irregular on this side,) and lay off the distance from I to B. If it answers, it is right. Ink in the plan upon the table as carefully as you can. If any thing is omitted, such as the breadth of houses, &c., you must return, and measure them and lay off the distances, and insert them on the plan. When all the outline is done, rub out all the black-lead lines, and write in the name of every proprietor or occupier in every yard or garden. Observe that the dotted lines that are inserted in the sketch are merely to show where bearings were taken and distances measured.

This figure represents three enclosures, containing three plantations, a moss, a bog, and a pond, measured and planned with the



plain table. The table was set up at A as level as possible, and being adjusted, a bearing was taken to the far corner of the plantation, the distance to which is 500 links; lay that distance off upon the plan; another bearing was taken to H, and a distance measured of 1010, and an offset of 10; on the right at H a line was drawn from the corner of the plantation to the offset of 10 at H; another bearing was taken from A to B. In measuring that line, an offset was taken of 170 at 350, to the angle in the plantation, which lay off, and the distance to B 780; then draw the fence from A to 170, from thence to B. The table was then taken to B; and a back sight taken to A; also a bearing, with a distance of 160, taken to the low corner of the plantation, and a line drawn from the upper corner to the under corner, which finishes the plantation; a bearing was taken from B at C. In measuring that line, an offset was taken of 130 at 1130 to the corner of a round planting, and the distance from B to C 1300. The fence is drawn in from B to the offset of 130; from thence round to C. The instrument was then taken to C, and a back sight taken to B, and a bearing to H close by the hedge. In measuring from C to H, a moss goes off at 200, and the distance to H is 560. Draw in the fence, which finishes the first enclosure. Again, a bearing was taken from C to D. In measuring that line, an offset was taken of 130 at 120 to the corner of a round plantation, and the distance from C to D is 1600; draw the fence from C to the offset of 130 round, and from thence to D. The table was then set up at D, and a back sight taken to C, and a fore sight to E, and another bearing to G. In measuring the line D G, a planting goes off from the fence at 170 and at 500; the chain is opposite the far side of the planting, and the distance to G is 950; draw in the fence from D to G. In measuring from D to E, an offset is taken of 130 to an angle in the fence at 260, and at 330 the chain touches the angle of the planting. Lay off that distance, and draw that line of the planting in from the mark where it went off on the line D G; erect the table at the end of the plantation, by laying the index upon the line D E, and looking through the sight till you see the pole in E and the hair to coincide, and take a bearing along the plantation, and lay off the distance, which is 400; then draw in the line of the planting to the other mark, where it went off on the line D G, and it is finished. Again, return back, and continue to measure the line D E, where an offset was taken of 150 at 990 and at 1130, another on the right hand to a bog of 200, and the chain touched the edge of the bog at 1330; and the length of the line from D to E, where there is an offset of 40, is 1580; draw in the fence from D to the offset of 130; from thence to the offset of 150; thence to the offset of 40 at E; set up the table at E, and take a back sight to D, and a fore sight to F. In measuring that line, take a bearing to the bog, and lay off the distance 100; set up the instrument at 490, and take a bearing; then lay off the distance 210; take another bearing to the same bog, and lay off the distance, which is 200. You may now draw part of the bog in from where you took the first offset to it on the line D E; from thence to where the chain was nearest to it, from thence to 100, 210, and 200; an offset of 190 was taken to the angle in the fence, where the instrument was placed at 490; an offset was taken of 130 at 720, and the whole length of the line E F 900; draw in the fence from the offset of 40 at E to the angle at 190; from thence to F; then plant the instrument at F, and take a back sight to E and a fore sight to G. In measuring that line, an offset was taken of 130 at 200, and another of 200 at 320. Draw in the bog to where these offsets are pricked off on the plain table sheet; from thence to the first offset that was taken to it on the line D E, and the bog is finished. Continue measuring the line to G, which is 1098. Plant the instrument in G, and take a back sight to F; then lay the index upon the line G D; if it answer, you are certain of having performed the work right. Take a bearing to H. In measuring that line, an offset of 160, at 330 and at 1450, you enter upon the moss, and the whole distance from G to H is 1970; draw in the fence from F to the offset of 160 to the offset of 10 at H, which closes the outline of the enclosures; then return to the mark left on the line G H at 1450, where the instrument is set up, and a bearing taken to a pole placed at the corner of the moss, and a distance laid off upon the plan to the corner of 310; measure from thence to the fence C H, and lay off the distance, which is 450, to the point; this finishes the moss. The next thing to be done is to go to the old mark D, and plant the instrument, by laying the chamfered edge of the index upon the line C D, and take a bearing to No. 1. In measuring that line, take an offset of 190 to the lake at 220, at 450 touched the lake; another was taken of 40 at 690, and the whole distance from D to No. 1 is 850; those distances being all laid off, draw in part of the lake from the offset 190 to where the chain touched the water, from thence to the offset of 40, and thence to No. 1; plant the table at No. 1, and take a back sight to D, and take a fore sight to No. 2, measure the distance from No. 1 to No. 2, which is 330, and an offset of 20; draw in the outline of the lake from No. 1 to 2; then go to No. 2 and take a back sight to No. 1 and a fore sight to No. 3 at 200. In measuring that line, the chain was contiguous to the lake, and the distance to No. 3 is 530, and an offset to the water from No. 3 is 100; draw in the lake from the offset of 20 to where the chain was contiguous to the water, from thence to the offset of 100, opposite No. 3. Place the table at No. 3, and take a back sight to No. 2 and a fore sight to D. In measuring that line, the chain touches the water at 270, and the distance to D 500; lastly, draw in the lake from the offset of 100 to where the chain touched the water at the first offset taken from D to No. 1, which will finish the survey of the three enclosures.

What has been said in this and the preceding pages, with a careful inspection of the foregoing sketches, is presumed sufficient to explain the common use of the plain table. I have now to point out the inconveniences, or rather defects, pertaining to this instrument,—for in fact no instrument yet invented possesses all the requisite advantages for surveying and plotting, some being advantageous in one point, while counterbalanced by inconveniences, more or less, on the other. In the first place, the plain table can only be used in fair weather; and as every thing is laid down and plotted on the spot, it takes a considerably longer time in the field than any other instrument, yet, on the other hand, it gives much less labour in the house. Although the weather should be fair, yet the paper expands on a damp day, and if made use of when damp, the distances (being all laid off upon the spot) will become a little too short when the paper shrinks to its natural size; besides, the plain table is not a fit instrument for taking the measurement of an extensive estate in the common way of using it, particularly as the work runs very soon off the paper that is put upon the table, which will require shifting three or four times in a day. If the scale is large, by joining so many plain table sheets together, it becomes very difficult to get the plan of a large estate laid down correct; yet it is my opinion that, in dry weather, for surveying a small farm or a pleasure-ground, which include a variety of serpentine walks, shrubberies, ponds, curved plantations, or irregular fences, it is preferable to any other instrument.

Hitherto I have only described in what way it is commonly made use of by land-surveyors in the field, and drawing the rough plan upon the spot.

Many surveyors create objections to it, because they cannot determine horizontal distances by it; but this can easily be remedied by carrying a small quadrant, and taking the altitude or declivity, or with the chain, as particularly described in page 29.

The figure, *p.* 64, represents the survey of the Common of Hassendean, containing upwards of 300 acres, surveyed with the plain table, but in a very different manner from that commonly practised by land-measurers. The method I am about to explain removes many of the objections that some have to it, and increases its value even to those who are apt to condemn it. It obviates particularly the inconvenience of shifting the paper, and commencing upon new sheets; it also does away the great objections they have to the paper swelling and shrinking again to its natural size when it dries. In this manner of surveying with the plain table which I am to describe, a field-book is kept, the distances are all inserted in it, then protracted and laid off upon one, two, or more sheets of drawing-paper joined together, and it may be plotted from any scale you choose to adopt in the house.

Fix upon any part of the paper near the centre of the table, and make a mark thus ⊙ to represent the centre, or the last letter or reference when a back sight is taken, or it answers for the letter or reference when a fore sight or bearing is taken. With a pair of compasses draw a circle round the centre as large as the paper on the table will admit, which will be about four times larger in diameter than the one represented on the plate, and the plan will be equally large in proportion. This circle you may call your *protractor*, which has a much larger radius than the brass protractors that are commonly used.

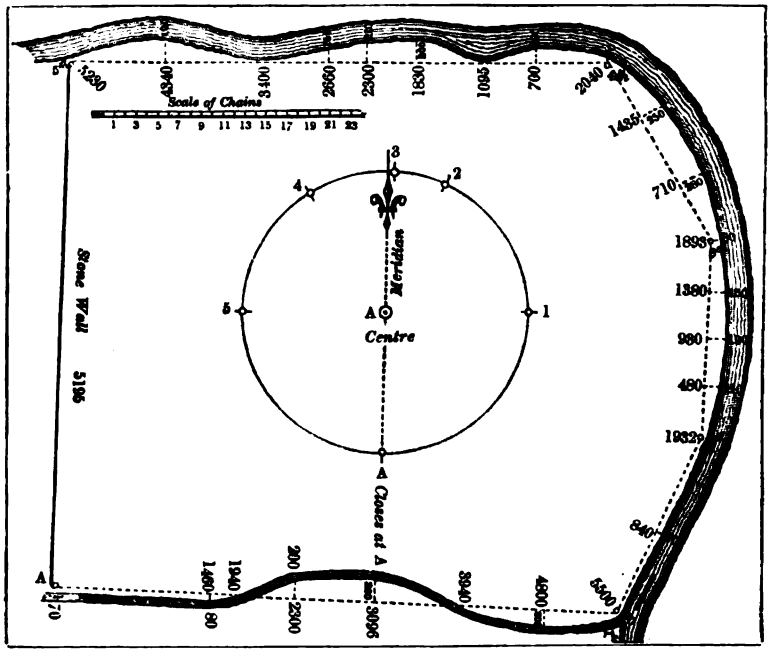
Begin at any part of the common you choose with your measurement, suppose at A. Plant the plain table there, as level as you can guess, and turn it round upon its axis till the north end of the magnetic needle settles over the *fleur de lis* in the compass-box; then screw it fast to the legs with the screw in the socket *e*, and take a bearing to No. 1, by laying the chamfered edge of the index close to the centre, and look through the slit in the sight next the eye till you see the hair in the opposite sight and the pole placed in No. 1 to coincide; then draw the bearing, and where the chamfered edge of the index crosses the arc of the circle mark 1; measure the line from A to 1, and also the offsets, and insert them in the field-book, or an eye-draught, whichever you choose to keep; then plant the instrument at 1, and lay the edge of the index at the mark 1 made across the circle and the centre, and take a back sight to A, which is the centre; then take a fore sight to 2, and measure from 1 to 2, and mark where the bearing crosses the circle at 2, and insert the distances in the field-book and the offsets, and where they were taken at; then plant the table in 2, and take a back sight to 1, by laying the chamfered edge of the index upon the mark 2 made on the arc of the circle and the centre; then take a fore sight to No. 3, and examine the needle if it settles over the *fleur de lis*: if it does, it is a proof that you have done right so far as 2; then mark where the bearing crosses the arc, and write 3; measure the distance from 2 to 3, and insert the distance and the offsets to the river in the field-book, and where they were taken at; then place the instrument at No. 3, and lay the index upon the mark made on the arc at No. 3 and the centre, and take a back sight to 2; screw the instrument fast, and take a fore sight from the centre to the pole placed in 4; then mark where that bearing crosses the arc 4; then measure the distance, and insert in the field-book that distance, and also the offsets to the river, and where they were taken at; then plant the instrument at 4, and lay the index at the mark made on the arc 4 and the centre, and take a back sight to 3; then lay the index upon the centre, and take a bearing or fore sight to No. 5; measure that distance, and all the intermediate ones, inserting them in the field-book; then set up the instrument at 5, and take a back sight to 4; screw the table fast, and take a bearing to A, which mark on the arc, and measure to A, and enter the distance in the field-book, and write "Closes at A." If the needle settles over the *fleur de lis*, after taking a back sight from A to 5, it is a proof that the angles are all right taken.

FIELD-BOOK OF HASSENDEAN COMMON.

Beginning at the top of the column.

	Offsets.	Distances.	Offsets.	
		A	70	Rivulet.
		1500	80	Rivulet.
		1940		Crosses rivulet.
Rivulet,	200	2300		
Rivulet,	230	3096		
Crosses,		3940		Rivulet.
		4800	200	Rivulet.
End of line,		5500	50	River.
		1st.	50	River.
		840	70	River.
		1932	40	End of line.
		2d.		
		480	140	River.
		930	190	Do.
		1380	150	Do.
		1833	60	Do.
		3d.		
		710	280	River.
		1435	250	Do.
		2040	40	End of line.
		4th.		
		700	190	River.
		1095		Touches river.
		1830	200	River.
		2300	210	Do.
		2660	150	
		3400		Touches river.
		4340	190	River.
		5230	10	End of line.
		5th.		
	Closes	5195	at A	

To protract and make out a plan of the Common of Hassendean.—Remove the paper from the plain table on which the circle is



described, and lay it upon a sheet of large drawing-paper; then with a fine point prick off all the marks made upon the circumference of the circle, also its centre; number each point, beginning at A, thus, 1, 2, 3, 4, 5, A; then with a parallel ruler, or the T square and its companion, afterwards described, and as it is used in the farms of Tipperty and Bonnyton. If you begin at A, near the south-west corner, on the drawing-paper lay the T square upon the centre ⊙, and the prick at 1, and slide the square, by the help of its companion, parallel to where you begin at A, and draw the bearing from A to No. 1, and from any scale of equal parts, suppose eight chains to an inch; then lay off the distance 5500 links from the scale, and also all the offsets, and the distance from A, and where they were taken at, which you take from the field-book; next, lay the edge of the ruler on the prick at No. 2 and the centre, and slide the ruler down, by the help of its companion, to 1, and draw the bearing 2, and lay off the distance 1932, and the offset of 70, taken at 840, also the offsets taken at the stations; then lay the edge of the T square on the centre and the prick made at No. 3, and move it parallel, by the help of its companion, to No. 2, and draw the bearing from 2 to 3, and lay off the distance from the scale, which is 1833, and all the intermediate offsets to the river inserted in the *field-book;* then place the T square upon the centre and the prick made at 4, and move it parallel as before to 3, and with a fine-pointed pencil draw the bearing from No. 3 to No. 4, lay off the distance 2040 to 4, and all the intermediate offsets to the river; again, lay the T square upon the prick made at No. 5 and the centre, and move it as formerly to 4, and draw the bearing from No. 4 to No. 5; and from the scale lay off the distance 5230, and all the intermediate distances and offsets inserted in the field-book; then lay the T square upon A and the centre, and move it parallel to No. 5, and draw the bearing from No. 5 to A. If it agrees, you are certain the angles have been right laid off. Lay off the distance 5195. If that answers, you have made an exact closing. Now, with a black-lead pencil, draw in from the offset at 70, where you began, to the offset 80; from thence to where you cross the rivulet; thence to 200; and so on from offset to offset, till you have gone all round; then, with a pen and Indian ink, draw in the whole of the boundary, and also the river Tyne about two chains wide, and it is completed.

Plate I.,

Represents a farm called *Harestanes*, and also the field-book, surveyed with the plain table. This is performed in the same manner as Hassendean Common (fig. p. 64); but is much more intricate, being all inclosed, and consists of upwards of 150 acres; and if measured in the common way, the paper would require to be shifted six or eight times, to lay it down upon a scale of two chains to an inch, which is a scale small enough for inclosed lands. In order to give the learner a better idea, I have made a rough protraction in the plate, taken from the field-book; which I have been under the necessity of doing upon a small scale for want of room upon the copper, and is intended merely to show that a very extensive piece of land may be measured without shifting the paper, and the whole afterwards protracted upon any scale. To proceed then: Set the table up at A, as level as you can, adjusting it as before; and having previously described a circle as large as the plain table sheet will admit of, take a bearing to a pole placed in B, by laying the index over the centre and mark B, where the chamfered edge crosses the arc of the circle; measure to B, and insert the distance as well as the offsets in the field-book; then plant the table at B, and lay the chamfered edge of the index upon the centre and the mark made upon the arc at B, and take a back sight to A; then take bearings to C and D, and mark where the thin edge of the index crosses the arc, noting C and D; measure these distances and the offsets; then erect the table at station D, and lay the index upon the centre and the mark on the arc at D, and take a back sight to B, and a bearing from the centre to the station E, noting E; where the edge of the index crosses the arc, measure to E; plant up the table, and lay the edge of the index upon the centre and the mark upon the arc at E, and take a back sight to D, (observing, every time a back sight is taken, that the instrument is screwed fast;) then lay the index upon the centre, and take a bearing to F; and where the index crosses the arc mark F; measure that distance, which enter in the field-book, and the offsets, &c.; then plant the table at F, and lay the index upon the mark at F on the arc and the centre, and take a back sight to E; again, lay the edge of the index upon the centre, and take a bearing to G; and where the index crosses the arc mark G; measure to G, noting the offsets; then, with the table at G, take a back sight to F, by laying the index upon the mark on the arc and the centre; then take a bearing to H, and where the index crosses the arc mark H; measure to H, and plant up the table at H, and lay the edge of the index upon the mark H and the centre, and take a back sight to G; take a bearing to I, and where the edge of the index crosses the arc mark I; measure from H to I, and then set up the table at I, and lay the index upon the mark at I on the arc and the centre, and take a back sight to H; then take a bearing to K, and where the index crosses the arc mark K; measure from I to K, then plant up the table at K, and lay the index upon the mark on the arc at K and the centre, and take a back sight to I; then lay the index upon the centre, and take a bearing to L, and where the index crosses the arc mark L; measure to L, and enter all the distances, offsets, &c. carefully in the field-book wherever they are taken; then set up the table in L, and take a back sight to K, also a bearing to A, where the survey was begun; measure from L to A, and mark where the index crosses the arc, and write "Closes at A;" insert the distance, offsets, &c.

Return to L, and take a bearing from the centre to M, and where the index crosses the arc mark M; measure to M, and set up the table, and lay the index upon the mark on the arc at M and the centre, and take a back sight to L; then lay the index upon M, and take a bearing to I, and mark where the index crosses the arc, and write Meets at I; measure to I, and insert the distance in the field-book; then take a bearing from M to C, and where the index crosses the arc write Closes at C; measure to C, and enter the distance in the field-book; plant the instrument in C, and take a back sight to M; then lay the index upon the centre, and take a bearing to N, and where the index crosses the arc mark N; measure that distance, which enter in the field-book; plant the table up at N, and take a back sight to C, by laying the index upon the mark made on the arc at N and the centre; then take a bearing from the centre to D, and another bearing to F, also one past the houses to O; mark all these bearings where they cross the arc, and mark D F O; measure the distances, and insert them in the field-book; then plant the table at O, and take a back sight to N, and lay the index upon the centre and the mark made at O on the arc, and take a bearing to I, and write Meets at I, where the index crosses the arc; take another bearing from the centre to G, and where the index crosses the arc write, Meets at G; measure the distances, and insert them in the field-book, also the offsets, and where they were taken at; then take a bearing opposite 1030 upon the line N O, along the north side of Harestanes garden-wall, which lay off from the centre; and where it crosses the arc, mark P, and measure the distances about the houses. The chief matter is to be well acquainted with laying off the bearings on the spot. Care must be taken, when the index is placed over the centre in taking a back sight, that you perceive the station the table was last at; and when you take a bearing or fore sight, it is from the station you are standing at; and if any error has been made, the needle is an excellent check, as it will always settle (if a good one) over the *fleur de lis.*

I must observe, that in taking a survey in this manner with the plain table, it is easily adjusted, and very little time lost in taking the back sight and the bearing to another station; it has also this advantage over other instruments, that you have no degrees and minutes to insert in the field-book, they being expressed on the circumference of the circle by a short line drawn across the arc upon the spot; and if the chamfered edge of the index is laid exactly over the point in the centre, and a line drawn across the arc with a fine pencil close to the chamfered edge of the index, it is equally correct as a protractor, and is much larger than the protractors are commonly made. Although the one represented on the protracted sketch appears small, it, as well as the sketch itself, is owing to the smallness of the scale that the sketch is plotted by, which is only seven chains to an inch, and the diameter of the circle, upon a plain table at least, thirteen inches and a half, which, as before mentioned, is a much larger radius than the common size of protractors. It is well known to surveyors, that the larger the protractors are, the work will in proportion be the more exact.

FIELD-BOOK OF HARESTANES FARM.

	Offsets.	Dist.	Offsets.	
K, . .		1770	0	River
		1530	180	River
		1327	230	River
		1070	235	River
		750	40	River
		550	0	River
		352	65	River
		240	200	to River; 200 to angle in do.
	15	172	435	River
	15	I		
I, . .		1940		Meadow
Meadow, .		1905	..	Crosses hedge
		1700	235	Corner hedge
		1600	20	River
		1490	0	River
		1365	45	River
		1300	150	River
		1110	200	River
		628	30	River
		H		
H, . .		1480	10	Rivulet
		1070	210	Rivulet
		672	6	Rivulet
Northfield, .		20	..	Crosses hedge
		G		
G, .		1280	140	Rivulet
		764	10	Rivulet
		482	50	Rivulet
		340	170	Rivulet
		210	50	Rivulet
House Croft,		30	..	Crosses hedge
		F		
		1318		F
		1170	90	
		1050	100	
		800	0	Rivulet
Crosses hedge,		540	150	Rivulet
		10	15	
		E		
		1900	10	E
		D		in a road
	15	1570	10	Road
	10	50		
Returns to .		B	15	
		880	10	C
		B		
		1320		
	15	A	10	at A

BEGINS HERE.

	Offsets.	Dist.	Offsets.	
		210		Breadth of yard
Far corner, .		110		
Corner house,	40	40		
South side, .		290		
		200		Dike
Corner house,		210		East side
		290		
Harestanes,		90		Farm House, north side
		1680		Meets at G
	218	930		
	200	670		
	130	400		
Crosses hedge		10	..	Into house croft
Returns to .		O		
		1075		Closes at I
		O		In road, 20 wide
		1605		O
		1030	15	Corner garden
Fence,	10	930	15	House
Fence,	10	850	15	House of Harestanes
Fence,	10	805	15	Corner yard
Fence,	15	250	15	
Returns to .		N		Along a road
		1580		Closes at F
	90	1210		
	135	965		
	135	622		
Returns to .		N		
		1150		Closes at D
		N		In a road
		1575		N
		1565		Road
Hedge, .	20	1050		
Wood, .	10	345		Which is in line with the other corner
Crosses hedge,		10	..	
		C		
		830		Closes at C
Corner of wood,	10	30		
Returns to .		M		
	10	1425		Closes at I
	110	800	10	
		M		
	10	1467		M
	110	645		
Corner, .		100	..	Crosses hedge
Returns to .		L		
		1245		Closes at
		L		
	L	1835		
	90	1125		
	100	600		
		K		

CONTINUATION.

To protract the rough plan from the field-book. Take the sheet of paper off the table, and lay it upon a large sheet of drawing-paper; prick through the centre, and also through every mark upon the arch, and insert the letters with a black lead pencil upon the paper, and make a mark thus ⊙ to represent the centre; adopt any scale you choose, suppose two chains in an inch, which will require a very large sheet to hold the plan of the farm of Harestanes. If it had been surveyed in the common method, and laid down upon that scale, the paper would require to be shifted six or seven times, and then to join them altogether, which is very troublesome to do correctly; whereas, by this method, every thing is protracted from one centre, and on one sheet of paper.

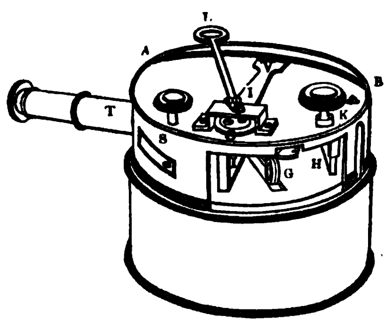
Having begun the survey at A, fix upon any part of the paper to begin at, and mark ⊙ A on the west side, near the bottom of the paper; then find out the prick B; lay the T square upon the centre and B, and slide it parallel to A, which is the point fixed upon to begin at, and draw a line with a pencil; then from the scale lay off the distance, which is marked in the field-book, 1320, and also the offsets taken to each side of the road, which ascertains its breadth; then lay the square upon the centre and C, and run it parallel to B; and from the scale lay off 880, which is the distance from B to C; then lay the square upon the centre, and the mark at D on the arc, and move it parallel to B, and draw the bearing to D, and lay off the distance from the same scale, which is 1570, also the offsets to each side of the road: again, lay the square upon the mark on the arc at E and the centre, and slide it down to D, and draw the bearing to E; lay off the distance from D to E, which is 1900, and also the offsets taken on each side of the road; then lay the square upon the mark on the arc at F and the centre, and slide it parallel to E, and draw the bearing to F; lay the distance off from E to F, which is 1318, and all the intermediate distances and offsets to the rivulet marked in the field-book; then lay the square upon the mark at G and the centre, and slide it parallel to F, and draw the bearing from F to G; lay off the distance, which is 1280, and all the intermediate distances and offsets to the rivulet; then place the square upon the mark at H and the centre, and slide it parallel to G, and draw the bearing to H, and lay off the distance from G to H, which is 1480; also the intermediate distances and offsets to the rivulet to H; then lay the square upon the mark I and the centre, and slide it parallel to H, and draw the bearing to I; lay off the distance, which is 1940, and all the intermediate distances and offsets marked in the field-book to the river;

then lay the square upon the mark at K and the centre, and slide it to I, and draw the bearing from I to K; lay off that distance 1770, and all the intermediate distances, offsets, &c., to the river, and draw it in about 100 links wide; again, lay the square upon the mark at L and the centre, and move it parallel to K, and draw the bearing from K to L; lay off the distance with the scale from K to L, which is 1835, and all the intermediate distances, offsets, &c., to the boundary; next, place the square upon L, and the mark where it closes at A, and move it parallel to L, and draw the bearing to where the survey was begun at A. If it joins, you may rest satisfied that you have not only measured the distances right, but the bearings also, particularly if the distance from L to A answer, which is 1245. Then lay the square upon the mark at M and the centre, and move it to L, and draw the bearing from L to M; lay off that distance, which is 1465, and the offset to the fence; then lay the square upon I and the centre, and slide the square parallel to M, and draw the bearing from M to I. If the bearing and distance, which is 1425, answers, you are certain it is right. Again, place the square upon the mark where you close at C and the centre, and move it parallel to M, and draw the bearing from M to C, laying off the distance, which is 830; then lay the square upon the mark at N and the centre, and slide it parallel to C, and draw the bearing from C to N; lay off the distance, which is 1575, and also the intermediate distances and offsets; next, lay the square upon the mark where a meeting is made at D and the centre, and move it parallel by the help of its companion to N, and draw the bearing to D. If the distance, which is 1150, and the bearing answer, you are certain that no error has been committed. Then lay the square where a meeting is made at F and the centre, and move it parallel to N, and lay off the bearing. If it and the distance, 1580, answers, it is right; lay off also the offsets as marked in the field-book. Again, lay the square upon the mark where a meeting is made at I and the centre, and move the square to O, and draw in the bearing to I. If the distance answers to 1075, you may rest satisfied it is right. Then lay the T square upon the mark made upon the arc where it closes at G and the centre, and move it parallel to O, and draw in the bearing to G. If it answers, and the distance to 1680, it makes an exact closing. Next, lay the square upon the mark at P and the centre, and move it parallel to the north-west corner of the garden wall, and lay off the distances of the length and breadth of the garden, also the length and breadth of the houses, as marked in the field-book; draw in the whole, first with a black-lead pencil from offset to offset, as before directed on other surveys; then ink it carefully in with Indian ink, rub out all the black-lead lines and marks, and the protraction is finished.

Many more examples might be given in the surveying estates, &c., with the plain table by this method; but it being merely a repetition, I shall now only take notice, that if a large survey, such as the common of Hassendean and the farm of Harestanes, can be surveyed with it in a damp day, although the paper should expand, a few inclosures can be equally well done with it by describing a circle upon the sheet of paper fixed upon the table, and the bearings marked upon the arc by letters or references, the same as is done upon Harestanes, which can be protracted at home upon any scale you choose to adopt. The theodolite has now, however, nearly superseded the plain table.

ART. IV.—OF THE SEXTANT.

This figure represents a box sextant, which I have found very useful in taking an angle with great exactness in the field: it is so nicely divided, every degree and minute can be read upon it (with the assistance of a magnifying glass L,) and is so convenient, it may be carried in the pocket, being only two inches and a half in diameter, and only about two inches deep. It is a great improvement on the cross staff; for, by setting the index to 90 degrees, it gives an

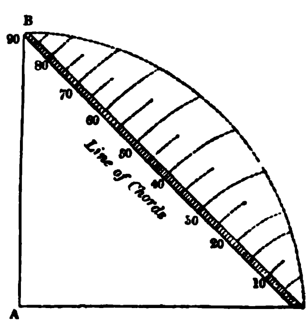


exact perpendicular, by reflecting a mark or pole to an angle of 90 degrees to a bend or angle of a fence, where a perpendicular is required; it also answers the same purpose as an *optical square** used in military observations for taking right angles in the field. The sextant is of essential use in trigonometrical observations, as any angle can be taken with it, either vertical or horizontal. To take an observation with it in the field, set the index to *o*, and look through the telescope or a small hole on the side of the box at T;

* The optical square is of the same form as the box sextant, having the index and horizon glass permanently fixed at half a right angle, and is now frequently employed in place of the offset staff, being more easily carried.

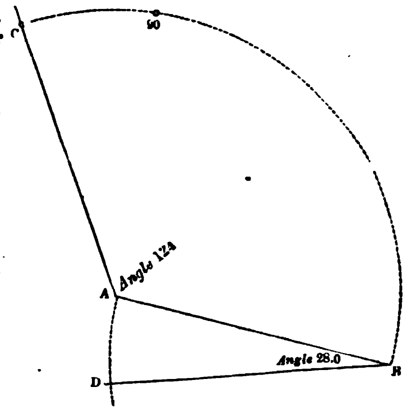
hold the sextant in the left hand, and turn the pinion near S with the right hand towards the left, and it will cause the index to move towards the right, at same time looking through the hole at T to a pole, or any other mark you want, to know the angle where you stand to another pole or mark: the reflecting glass in the box carries one pole to the other; and when they coincide exactly, one over the other, the index points out the degrees and minutes, which is read with the help of the magnifying glass L. If the angle should be larger than a right angle, it can be taken at two observations; and by adding together the degrees each makes, gives the angle required. If a third pole is set up at any distance nearly opposite where the observer stands, and the index set to *o*, look through the hole at T, and the reflecting glass will carry the first pole to the pole placed nearly opposite; mark the number of degrees and minutes cut by the index; again, set the index at *o*, and look through the hole at T, and by turning the pinion as before, the reflecting glass will carry the pole opposite to the other pole; and when the one pole it seen exactly over the other, the index will point out the degrees and minutes; which, added to the former observation, gives the angle required; and it may either be plotted by a protractor or a line of cords. This little instrument is very useful in taking small surveys, and supersedes the necessity of taking larger instruments to the field. For nicer purposes, a box circle has been contrived by the editor, having three verniers, each reading to one minute, and measuring the angle to right and left alternately, thereby obviating the effects of any index error. It is provided with clamping and tangent screws, and is indispensable in marine surveys for observations on board.

This figure shows the method of constructing a line of cords, which may be laid down upon any scale you please, (the larger it is so much the better.) Raise a perpendicular upon the line A C at A to B, of the same length of A C; put one foot of the compasses in the point A, and with the other describe an arc from B to C, and divide that quadrant into nine equal parts; then draw a line from B to C, which is to be divided into a line of cords; and is done in the following manner,—by putting one foot of the compasses in the point



C, and, with an extent to the first division on the quadrant, describe an arc from it to the line of cords; also describe an arc from the second division on the quadrant from the same point C to the line of cords; in like manner, do the same with all the other divisions, which will divide the line into nine parts, all unequal: each of these large divisions should be again divided into tens, as represented on the figure.

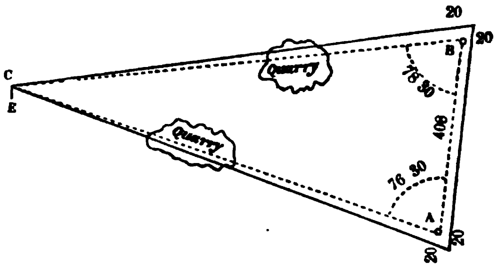
To lay off any angle by the line of cords. — Let the angle A B D, 28 degrees, be laid off with a pair of compasses. Set one foot on the point C, and extend the other to 60 on the line of cords; with that extent put one foot in B, and describe the arc A D; then, with one foot in C, on the line of cords, extend the other foot to 28 degrees on the line of cords, and with that extent put one foot in A, and the other foot will extend to D; lastly, draw a line from the point B through the mark made on the arc A D, which gives the required angle A B C.



Let an angle of 124 degrees be laid off from A on the line B A; take 60 degrees as before between the points of the compasses, and describe the arc B C from the point A. First, take off 90 degrees from the line of cords, and lay it off from the point B upon the arc B C, which mark; then take 34 degrees from the line of cords, and put one foot of the compasses in the mark at 90 degrees, and the other foot will extend to a mark upon the arc at C; draw the line A C, which will form the angle B A C, containing 124 degrees (being the sum of 90 and 34;) or lay off half the angle, 62 degrees, twice in succession.

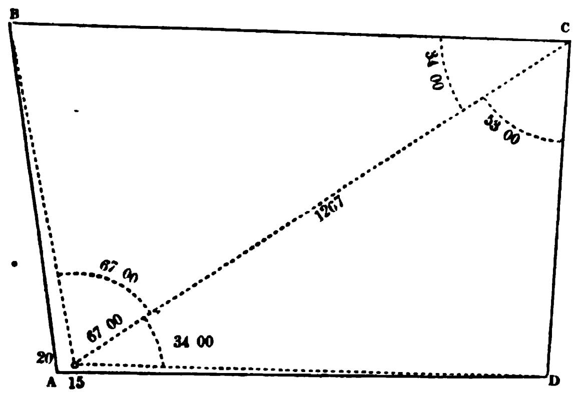
Angles may be laid off very accurately by a table of cords and a good plane scale, as was practised by Bird, in dividing his large astronomical instruments.

This figure is a triangular field A B C, where the short fence can only be got at, having deep quarries and other impediments, otherwise it would have been more certain, if the other fences could have been got at, to have measured both, or even one of them. A pole being placed at C, another at B, 20 links from the fence on both sides, in the inside of the inclosure, you stand at A; measure the offsets to the fence on each side, 20 links, which insert in an eye-sketch; then look through the telescope T in the sextant to the pole at B, and turn the index with the right hand till you see the pole B and the pole C exactly over one another in the reflecting glass; then look what degrees and minutes is cut by the index, which is 76 degrees 30 minutes; you then measure the distance from A to B, which is 408 links; then set the index to 0, and stand at B, and look through the hole to the pole in C; turn the index round as before till the pole at C is seen by the reflecting glass immediately above the pole that was left at A, and note the degrees and minutes cut by the index, which is 78 degrees 30 minutes, and insert the angle in the field-book.



The inclosure may be plotted by drawing a line at pleasure, and laying off 408, the distance from A to B, from any scale you choose to adopt; then take 60 degrees from a line of cords, and put one foot of the compasses in A, and sweep an arc with that extent; then take off 76 degrees 30 minutes from the line of cords, and with that extent put one foot of the compasses on the line you drew at pleasure, and make a mark with the other foot on the arc, and draw the line A C through the point on the arc; then, with the extent of 60 degrees, taken from the line of cords, put one foot of the compasses in B, and describe an arc with the other foot, and take 78 degrees 30 minutes from the line of cords, and lay that extent off upon the arc from the line A B, and make a mark with the point of the compasses; draw a line through that mark from the point B, and the intersection of the line A C at E ascertains the distance, which is 939, and is known by applying the length of the intersected line to the scale from A to C, as well as from B to C. The angles might have been equally well laid off with a protractor of large radius. The next thing to be done is to lay off the offsets, which is 20 each way from A, and 20 each way from B; lay a ruler upon the point of intersection C and the offset at A, and draw that fence; then lay a ruler upon the point made at the offsets at B and A, and draw in the short fence; then lay the ruler upon the point of intersection at C and the prick at the offset made at B; draw in that fence, and it is finished. *Note.*—The most correct method of ascertaining the distance of so acute an angle as B C A is to calculate it by logarithms.

This figure shows a method of taking the survey of an inclosure in a much quicker way than any that has hitherto been taken notice

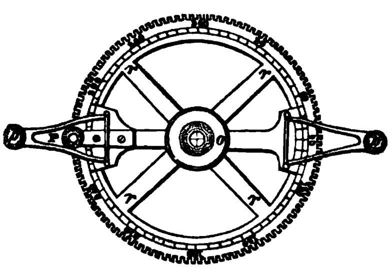


of, and is done as follows: let this quadrilateral inclosure be laid down by measuring only the diagonal and the angles taken with the sextant. Suppose the diagonal is 1267 links, place up poles (with red flags upon them, that they may be more conspicuous) at B, C, and D; look from A to D through the little eye-hole, or telescope of the sextant, and carry the pole at D till you see it coincide exactly in the reflecting glass in the box immediately above the pole in C; insert that angle, which is 34 degrees, in the field-book; then look through the hole, after having put the index to o, and then turn the index round till you see the pole in C and the pole in B to coincide, and set that angle down in the field-book, which is 67 degrees; then go to C, and look through the hole to the pole at B; turn the index round till you see it and the pole left in A to coincide exactly (the one appearing immediately, as it were, above the other in the reflecting glass); then set down in your field-book the angle, which is 34 degrees; set the index to o, and look through the hole in the side of the box as before, and turn the index round till you see the pole in A and the pole in D to coincide so as the one appears immediately above the other; lastly, insert that angle, which is 53 degrees.

To make out a plan of the inclosure, draw a line at pleasure with a black-lead pencil to represent the diagonal, and lay off the distance, which is 1267, from any scale you choose (the figure is laid down by a scale of one-fourth of an inch to a chain); but, to be very correct, you should use one much larger; then with a protractor, or the line of cords, lay off the angle C A B, which is 67 degrees, and draw a line from A through the point made in B; then lay off the angle A C B, 34 degrees, and draw the line from C through the prick or point made upon the line C B, the intersection is the distance to B; then lay off the angle C A D, 34 degrees, from A, and draw in the line A D; also lay off the angle A C D, 53 degrees, and draw the line C D; and where these lines intersect, ascertain the distance from C to D, as well as from A to D; then lay off the offsets 30 and 15 from A, and draw in the fences, and it is done.

ART. V.—OF PROTRACTORS AND PARALLEL RULERS.

This figure represents a protractor, wherewith as many angles or bearings may be pricked off as you choose to lay off at a time: *o* is the centre, made of a piece of glass with lines drawn across it at right angles, on purpose that the centre may be seen through the glass when laid exactly upon the stations, thereby avoiding the effects of parallax: *p p* is a movable index with a vernier scale: it is the



kind of protractors that is acknowledged the best by those that use it, and most accurate in practice: the outer circle on the limb is generally made about seven or eight inches in diameter, and is either divided into 360 degrees, or into twice 180 degrees; each degree is subdivided by a vernier scale into minutes, (or less if required,) which, with the index, is moved by a rack and pinion, P, round the glass centre, and the limb and vernier points out the degrees and minutes; at *p p*, near the end of the index, are fixed very fine steel points to prick off the angles or bearings by pressure; the points at the extremity of the index being in a direct line with the centre of the glass on the protractor, on purpose that the centre, when laid upon the station point and the point at the end of the index, may coincide; then the protractor is laid upon the meridian, which is a line drawn upon the protracting paper, one end representing the south, the other the north, with the reading index at zero, generally to the north: when the protractor is fixed as above, prick off as many bearings or angles at a time as you choose; during the time you are turning the index round with one hand, with the other hand hold the protractor fast to the paper, to prevent its shifting. This is better accomplished by fine steel pins permanently fixed in the brass circle, or by a cylindrical lead weight, with a circular hole in its centre, through which the intersection of the lines upon the glass may be distinctly seen. This protractor is allowed to be more expeditious in laying off bearings than those that are made finer, with a rack and pinion to move the index; but a practical surveyor in general prefers a semicircular one, upwards of a foot in diameter, with each degree divided into quarters, which answers the same purpose, and can be made at one-fourth of the expense, besides being more expeditious: when its centre is once laid upon the station, and the chamfered edge laid exactly on the meridian line, which should be kept steady and firm to the paper by a weight laid upon it, with the point of a pair of compasses, a protracting pin, or a fine needle, prick off the bearings, counting each of the four small divisions that each degree is divided into fifteen minutes, which you can again divide with your eye into five minutes, or less if you choose. A land-surveyor need be at no loss although he should have no brass protractor, as he can make a semicircular one of large dimensions with a pair of compasses, as large as the protracting paper will hold; fix upon any part of the paper you choose for a centre, which mark thus ⊙. As it will often have to be resorted to when you have to protract your bearings, the larger you make the protractor the better. In the Ordnance Map Office large engraved paper protractors are used, which have the property of expanding and contracting by the state of the atmosphere nearly in the same ratio as the drawing-paper on which the plan is being constructed. These are sometimes divided to show ten minutes, and five or less may be easily estimated. For the easier understanding it, I have laid one down on the farm of Bonnyton, Plate III., whereon the angles on that farm were all protracted; which, although done upon a small scale, you may suppose to be upon a sheet of large drawing-paper, at least six times the diameter of the one on Plate III. Some surveyors reckon this troublesome, and without doubt it is to make the first; but I shall only observe here, that after one has been made, you may make as many more as you please with very little trouble, by only laying the one you have made upon four, five, or six sheets of paper, and pricking off every degree with a very fine needle or protracting pin, as you will observe is done by short lines on the farm of Bonnyton, Plate III. In place of those short strokes, you have only to prick through every sheet of paper with a needle or protracting pin, which always remains visible. The only trouble that now remains is to mark the figures, representing the degrees, with a black-lead pencil, which are rubbed out when the survey is protracted, similar to those that are marked on the farm of Bonnyton; so that, when one is made, you may make as many more as you please, or think you will want, by the additional trouble of numbering every fifth degree, as is done on the Plate. A protractor of this kind can never shift like those that are made of brass, which they sometimes do in the hands of very careful land-measurers. If you want to be very nice, I have inserted a small scale, marked A. Supposing each degree upon your protractor to be three times larger than the degrees upon the farm of Bonnyton, you can easily make one similar upon any blank corner on your paper the size of the degrees on your protractor, and divide one degree into six equal parts, which is equal to 10 minutes each division: if you choose to take the minutes, you may do it within 5 minutes with a pair of compasses applied to that scale; for example, you want to lay off 11 deg. 25 min., set one foot of the compasses in 10 deg. marked on the protractor, and let the other foot extend 2 divisions and a half past 11, which will be 11 deg. 25 min.; or if you wanted to lay off 90 deg. 40 min., you can with the compasses take off 4 more than 90, which is equal to 40 min. If you find it troublesome to take off with the compasses 4 of these small divisions on the scale A, you can put one foot in 10, and let the other foot extend 4 divisions past 11; then apply that extent to the protractor, and put one foot in 89, and the other foot will extend 40 minutes past 90, which mark, and draw the bearing from the centre through the mark. A practical surveyor can estimate upon the protractor the minutes very near, without applying the compasses to any scale. Not to enlarge, I shall only say that I give the preference to a protractor of large dimensions, of a surveyor's own making, to any other, and that all who have used them have uniformly approved of them. In making use of one, you have no numbers or bearings to mark, where the bearings are pricked off with other protractors, nor the bearing, or a number referring to the bearing, to set down upon the protracting paper with a black-lead pencil, as every 5 degrees are marked upon it, and the bearing you may want to lay off is instantly found out on the paper protractor; the odd degrees can be easily traced with the eye; for example, you want to lay off 38 degrees, find out 35, and reckon 3 degrees more, which is 38 degrees; if the bearing should be 38 degrees 30 minutes, count off 38 as above, and guess the half of another degree. Your protractor being of large dimensions, you can estimate, within a minute or two, which is as near as most protractors made of brass can lay them off, although ever so finely divided with an index and vernier; the breadth of the mark that is made with the point that is placed at the end of the index of those protractors covers more than 5 min. Lay a parallel ruler or the T square, which will be afterwards described, upon the centre, and on the angle or bearing you want to lay down; then move it parallel up to any station from which you want to lay off that bearing; draw a line from the station; lay off the distance upon that line by the plotting scale, or from a scale of equal parts, and a pair of compasses; you then look for the next bearing that is on the field-book, and find that bearing on the paper protractor, and lay the edge of the ruler upon the centre, and the degree and minute, and move it parallel up to the station that was last laid off, and draw that bearing, and prick off the next distance. If you have ten or twenty bearings and distances, they are all laid off in the same manner.

It will be proper to observe here, that one line drawn parallel to another has the same bearing from the meridian, whether drawn from the centre of the paper or any other part of it; so that if a parallel ruler or the T square is laid upon the centre, and the degrees and minutes upon the edge of the semicircle where a bearing was taken to a pole or any other mark from the centre, and if the parallel ruler or T square is moved parallel to any station you have to lay off the bearing from, it is the same bearing as if laid off from the centre, corresponding to the degrees and minutes on the protractor.

Fig. page 86, represents a square protractor, with the outline of the farm of Broad Meadows drawn upon it, which was protracted by it on a small scale of 40 chains in an inch; but you may suppose it to be on a large sheet of drawing-paper, or at the edge of a drawing board. If you will be at the trouble of making it, one will serve for numbers of plans, and will save the time of drawing protractors on paper, which some surveyors object to on account of the trouble of making them; but, in my opinion, any trouble they give is but trifling.* It is well known to most of them, that the radius of

* This additional trouble may be saved almost entirely by keeping a paper protractor, either drawn or engraved, which, placed on the sheet intended to receive

a circle is the sixth part of the circumference; and if a line is drawn through the centre, and cuts the circle in halves, then with the radius between the compasses divide the circle in 6 parts, each of these divisions being 60 degrees; again, divide each of these divisions by 6, which is soon done by taking, as near as you can guess, a short distance between the compasses, and run along the arc from one division to another till it is divided into 6 equal parts. This divides the circle into 36 equal parts, each division being 10 degrees; then divide each of these divisions into 5 degrees, or taking one half, which divides the circle into 72 parts; lastly, divide each of the 72 divisions into 5, and the circle will be divided into 360 equal parts called degrees; then mark every 5, 10, 15, 20 degrees, till you have gone all round from the left to the right; also mark 180. Begin again, and write 5 after 180, and so on as in the other semicircle, 10, 15, 20, 25, to 180. If you choose to divide the whole circle into 360 degrees, in place of beginning anew at 180, write 185, 190, 195, 200, till you have gone all round; when this is finished, you may make as many more as you please, as explained in page 78.

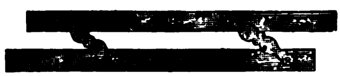
I have now to point out what way a square protractor is made, after you have made a round one upon paper of large dimensions. Get a drawing-board, made at least 30 inches by 3 feet, and paste a slip of white paper round the edges of it, or, what is better, a piece of white wood about half an inch broad, sunk into the drawing-board, on each side of it near the edge, and draw two lines on each side as on the plate; if it is done on paper, draw the line with a drawing-pen; but if done on wood, use a sharp point, which will leave a scratch or impression on the wood; then take one of the paper protractors, and lay its centre upon the centre of the drawing-board, and let the diameter be laid parallel with the sides of the board; when thus laid, fasten the paper to the board with drawing-pins or wax, so as it will not shift; then apply a rule or straight edge to the centre, and the divisions on the paper protractor, and where the straight edge crosses the square at the edge of the board make a scratch with the steel point; do the same at every division till you have gone all round; when that is done, mark every 5th degree as directed above. When you have any thing to protract, lay a sheet of paper upon the drawing-board, and fix it upon the board with drawing-pins, wafers, or wax. The centre of the white paper is easily found out by applying the straight edge to the meridian, and drawing a line with the point of the compasses close to

the plan, may have each degree and fifteen or ten minutes pricked through by the point of a fine needle.

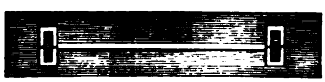
the edge of the ruler; do the same, by crossing the meridian line at right angles by laying the straight edge upon 90 degrees, and 90 degrees, if divided into twice 180; but if divided into 360, let the straight edge be laid to 90 on the east side and 270 on the west side, and draw a line close to the edge of the straight edge; and when that line bisects the meridian line, it fixes the centre upon the plotting-paper, which should be marked thus ⊙.

Parallel lines to a land-measurer, &c. occur so often in plotting, it is no wonder that so many different kinds of parallel rulers have been invented to expedite and facilitate his work. Many ingenious improvements have been made, and to give an account of each would be unnecessary; let it suffice to describe first the simple parallel ruler.

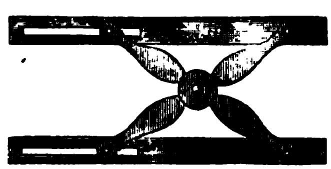
This figure represents a parallel ruler, acting on the principle of the parallelogram. It consists of two straight rules, which are so connected together as to keep in a parallel position by two equal and parallel bars, which move very freely on riveted pins, by which they are fastened to the rules.



This figure is a rolling parallel ruler, so called from the easy manner in which it runs, being supported by two wheels, which are connected together by an axis; the wheels are made the same size, and their rolling surfaces being parallel with the axis, when they are rolled backwards and forwards, the axis and rulers move in a direction parallel to one another; the wheels project a little on the under side of the rule, and are chamfered and grooved to prevent them from sliding. In using it, the finger should be placed nearly in the middle, that the one wheel may keep pace with the other. The wheels should only touch the paper when the ruler is moving, and the surface of the drawing-paper should be smooth and flat upon a table or drawing-board. These are best when made of brass entirely, which, by weight, gives steadiness to the instrument.



This figure is what is known by the name of a *cross bar parallel ruler*, and is made of two straight rulers joined by two brass bars, which cross



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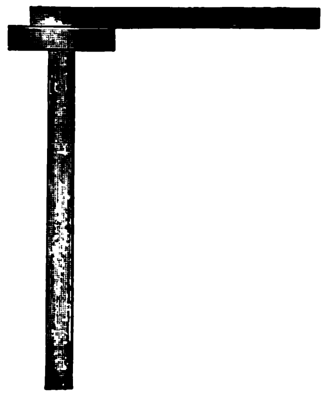
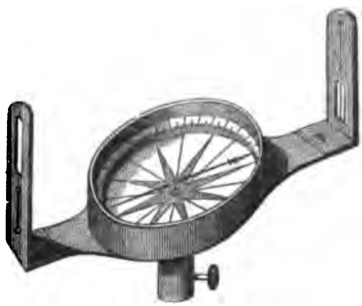
each other on a centre. One end of each bar moves upon a centre in each, the other moves in a groove as one rule recedes from the other. This, as well as the simple parallel ruler, is used in the same way as most other parallel rulers. I will only further observe that, when working with them, you should press the edge of the lower rule tight with one hand, and with the other move the ruler up or down till it coincides with the given point, through which a line is to be drawn.

This is a well-known square, amongst architects called a *T square;* which undoubtedly answers the purpose of a parallel ruler to a land-surveyor, with the addition of a separate piece of wood about sixteen inches in length, an inch and a half broad, and half an inch thick, smoothly plained and made straight to let the head of the T square run easily along it, and is called the T square's *companion.* The T square requisite for a land-surveyor should be at least two or two feet and a half in length, and the head of it six or eight inches, fixed exactly at right angles. By mere accident the T square and its companion was found to answer much better than any of the above described parallel rulers, or any others yet invented for the use of land-surveyors, in facilitating their protractions; and not one surveyor that I am acquainted with has ever used any other parallel ruler since they made a trial of it. All of them allow that it is not only the most exact, but the most expeditious: and I may venture to say, that all other land-measurers who survey with a theodolite, after having once made a trial of it, will give it the preference to all other rulers that they ever used, both for expedition as well as facility and cheapness.



ART. VI.—OF THE CIRCUMFERENTER.

This figure represents a circumferenter, which is a compass-box with two plain sights, like those on the plain table index, and a magnetic needle, which points out the bearings. The one sight is placed over the *fleur de lis*, or north point, and the other is placed exactly opposite, over the south point. To take an angle or bearing with it, set it up upon the legs which support it as level as you can, which you will easily know by the needle's traversing freely in the compass-box. If you look through the sight that is over the north point, the south end of the needle will point out the bearing when it settles. The divisions in the box are numbered in the compass-box the contrary way you



look—east being put where west should be, and *vice versâ*. This is the reason of the south end of the needle pointing out the bearing. It is best to get it divided into 180 degrees, and either end of the needle will point out the bearing. In this way the protractor is divided the contrary way that the needle goes; therefore the degrees in the compass-box are divided in the opposite way of the protractor. Some surveyors that use this instrument have it divided into four nineties, and reckon so many degrees from the north to the east, and so on from east to south, and from south to west, and from west round to north. Whatever way it is divided, the protractor must be divided the contrary way to the degrees in the compass-box, otherwise the work will not protract; in short, those that use this instrument, divided into 360 degrees, must protract their work by a protractor diametrically opposite the degrees in the compass-box. By taking bearings with the circumferenter, every thing is trusted to the needle; and the method of protracting and laying off the bearings is done in the same way as in the farm of Tipperty. It is needless to give here a particular description. Since the great improvements that have been made on theodolites, this instrument, as well as the semicircle, is gone very much out of repute in Britain; but no instrument has yet been invented equal to it for taking surveys through woods and uncleared grounds, such as in America; and it is still very much used in that country, particularly where it is intricate. It is found by experience that the circumferenter, when placed at every other station, by taking a back sight and then a fore sight, providing the degrees in the compass-box be divided into twice 180 degrees, is not only the most correct method, but the most expeditious to survey with.* It is also used in coalpits and mines

* Mr Ainslie seems to have had a peculiar partiality to have all his instruments

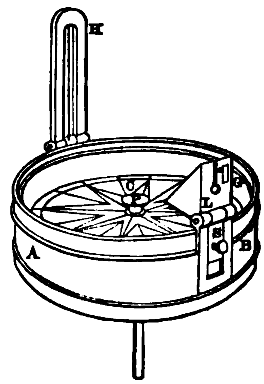
in taking bearings, which are protracted in the same way as particularly described on the farm of Tipperty, page 94. Much more might be said of this instrument; but this will be deferred to the surveying of land with the theodolite. The way colliers use this instrument below ground is by planting it at the bottom of the pit, and taking a bearing, suppose in a zigzag mine, and measuring the distance to a candle, which is placed as far along the mine as the candle can be seen, and the distance and the bearing inserted in a book. The instrument is planted up immediately above the mark where the candle stood, and the candle again placed as far as it can be seen through the sights, and another bearing and a distance measured, which should be also marked in the book. In this manner you go on to as many different stations as the mine is in length, setting down every distance and bearing to the end of the mine. The same operation is repeated above ground as was done below. Plant the instrument as near the pit as you can get it placed, and order your assistant to go forward with a pole; in the mean time, set the needle to the same bearing it was at below ground in the bottom, to the first candle that was placed in the mine, screw the circumferenter fast with the screw that fixes it upon the legs, and look through the sights to your assistant: if he is not right, cause him to move to the right or left till you see him exactly on the line, and sign to him to place up his pole, which is in a line with the same bearing that was taken below ground on the first line; then examine your book, and measure the same distance above ground you measured below, in a line to your assistant, and place the instrument up at the end of the line; then order your assistant to go forward while you place the needle exactly over the second bearing you took below ground, and sign to him to place his pole; when he is seen exact on the line, you look your book for the length of the next line, and measure the length above ground; go on in this manner till you have taken the same bearings and the same distances above ground as you measured below, which determines the spot for digging a new pit to reach a certain vein of coal.

An experienced land-measurer, in place of using the circumferenter, would take all his bearings with a theodolite, and protract and lay down all the zigzag angles and distances carefully that were taken below ground upon a plan, and draw a line upon

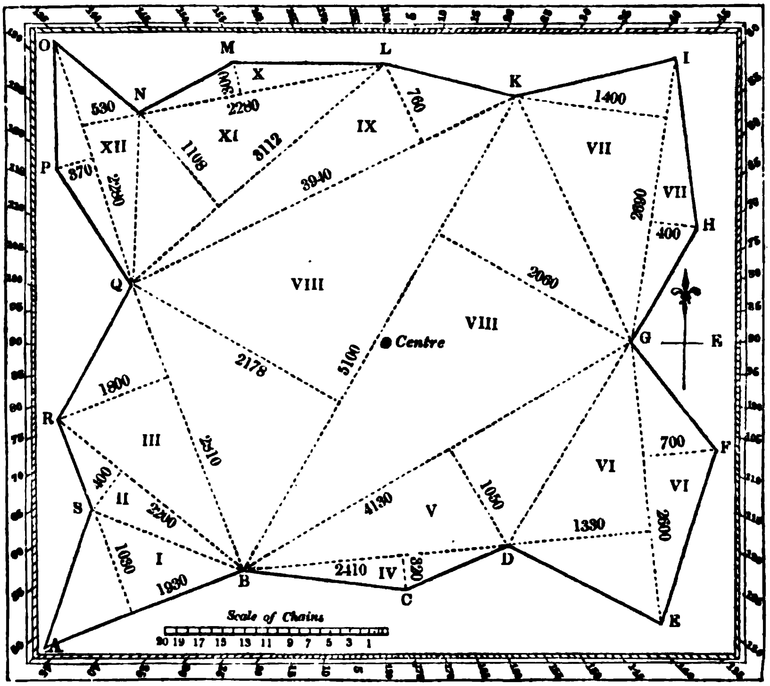
divided into twice 180 degrees. In some cases it may be convenient, but its universal adoption seems to us unwarranted by general experience.

the plan from the pit mouth to the end of the mine; he would then apply the protractor, and lay it upon the plan in the same way it lay when he laid down the zigzag angles, and see how many degrees and minutes the line cuts upon the edge of the protractor; he then goes to the pit mouth, and sets his theodolite in the same position it was below ground, and then puts the index to the same degrees and minutes as were cut upon the protractor, and orders one of his assistants to go forward and place a pole, by directing him to the right or left till he is seen through the telescope or sights. The surveyor then measures the length of the line upon his plan with a pair of compasses, and applies them to the same scale he protracted the bearings, by which he ascertains the distance to be measured from the pit mouth in a line towards the pole, which determines the place for digging a new pit by measuring one line only.

Schmalcalder's surveying compass will supply the place of the circumferenter very advantageously, and is much more portable. It is more frequently denominated the prismatic compass. A B is the prismatic compass of brass, containing the card, having its exterior edge divided into degrees. In some, these are subdivided into 30 or 20 minutes. H is an upright stem, into an opening of which is inserted a silk thread. S is the sight-vane, movable to suit the focus of the eye of the observer, placed near L, in which there is a diagonal reflector, to enable the observer to see the divisions bisected by the thread, and cutting the object observed at the same time. G is a coloured glass, to be turned over the eye-slit when the sun is observed. Below is a pin to be fixed in a staff, when steadiness is necessary.



This figure is the outline of the farm of Broad Meadows, which was surveyed with the circumferenter, and each bearing taken with the needle by its pointing to the degrees marked in the compass-box, and protracted by a square protractor, which is engraved round the plan of the farm of Broad Meadows, and which is divided into triangles and trapeziums, to give the learner an idea how it and other grounds are commonly divided before the area is obtained. (See the calculation in Areas.)



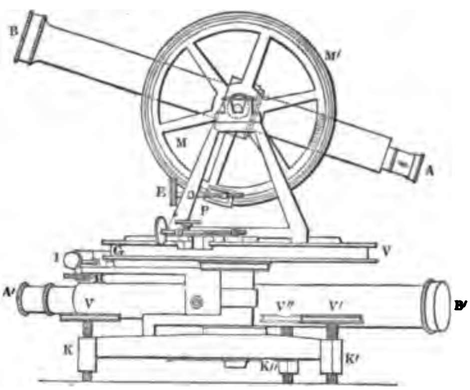
The field-book of Broad Meadows was kept as follows :—

BEARINGS.				DISTANCES.
Deg.	Min.			Links.
70	30	Easterly,	from A to B	1930
96	24	Do.	... B to C	1460
65	30	Do.	... C to D	1000
115	0	Do.	... D to E	1520
16	20	Northerly,	... E to F	1620
140	0	Do.	... F to G	1240
27	0	Do.	... G to H	1218
174	0	Do.	... H to I	1550
76	50	Westerly,	... I to K	1450
101	0	Do.	... K to L	1212
90	30	Do.	... L to M	1326
60	0	Do.	... M to N	1000
131	0	Do.	... N to O	915
3	20	Southerly,	... O to P	1080
146	15	Do.	... P to Q	1250
28	0	Do.	... Q to R	1380
156	20	Do.	... R to S	860
20	25	Do.	... S to A	1300, which makes a close.

N. B.—In surveying with the circumferenter, every bearing or angle is intrusted to the magnetic needle.

ART. VII.—OF THE THEODOLITE.

This figure is the representation of a repeating theodolite, which is now the most common instrument that is made use of by experienced practical surveyors, and has many advantages over all other surveying instruments, particularly for taking the surveys of large estates. To say any thing here of the invention would be superfluous, as most mathematical instrument makers claim having made improvements upon it.



Mr Sisson was the first person that made any great improvements. His first theodolites were made with plain sights, like those on the circumferenter; two of which were fixed, one over the north point, and the other over the south point. There were also two movable sights that went round the centre, which pointed out the degrees and minutes upon the limb, with an index and nonius for reading the minutes. Mr Sisson afterwards made his theodolites with a telescope, and divided the limb into twice 180 degrees, which made it answer a semicircle as well as a theodolite. Messrs Adams, Jones, Ramsden, Cary, Troughton, and Adie, lay claim to having made great improvements; and indeed it may be safely allowed, that the united abilities of those gentlemen have brought the theodolite to a state of the highest perfection. Since the invention of Mr Ramsden's dividing machine, instead of dividing the limb into twice 180 degrees, they now divide it into 360 degrees, and many of them have two telescopes; one of which is always in a line with the north and south division on the limb, or may be put in any position so as to repeat the measurement of the angle as often as required, and the other is movable round the limb, and points out the degrees and minutes by a fine graduated vernier. Theodolites for fine purposes have generally two verniers, each reading to 20 or even 10 seconds. Those for the Ordnance and Admiralty surveyors have three or four verniers, each reading to 15 or 10 seconds,—a degree of accuracy indispensable, in great distances, in the present state of science. The French geographical engineers use the repeating theodolite, of which the preceding figure is a representation, very successfully. A B is the upper telescope; A′ B′ the lower; V V′ V″ the levelling screws; P the clamping screw, attached near the tangent or slow motion screws of the horizontal circle G V; near E, the same for the vertical circle M M′; I G, the same for repeating the angle as often as necessary. The best size for common use is about four inches and a half and five inches in diameter, with a telescope and two spirit levels fixed to it at right angles, having a vertical arc divided on one side into degrees, and the other side divided so as to point the number of links to be deducted for each chain's length in ascending or descending a hill. To reduce the length of the chain to the horizontal distance, great care is required, before a theodolite is used, that it is properly adjusted, and to observe if the cross hairs in the telescope be exactly in the centre of the tube, and that the level be exact.

To use a theodolite, place it as firm as you can on the ground where you intend to begin, and also as level as possible; by moving the legs out and in till within the limits of the level screws, you may then level it exactly by the help of the four screws between the brass plates, one of which is fixed to the head of the legs; you then loosen the screw a little that holds the theodolite fast to the brass plates at the head of the legs. With both hands turn the theodolite round upon its axis, till the north end of the needle settles over the *fleur de lis* or letter N in the compass-box, and observe that the vernier is exactly over 180 on the limb, if it is divided twice into that number; if the theodolite is placed over 360, bring the letter N in the compass-box, or the eye-glass and vernier, to 360, and the south end of the needle will point out the same degree as the vernier does on the limb, if the degrees in the compass-box be divided the contrary way that they are divided on the limb. The screw that holds the instrument fast to the brass plates on the head of the legs is sometimes placed below the brass plates; on other instruments it is fixed in the socket.* This screw must be made very fast when the theodolite is placed, so as the needle in the box and the limb is set to correspond; and before an observation can be taken, the screw must be loosened or unlocked a little, which holds the telescope and arc or circle fast to

* The method of adjusting and using the theodolite is fully given in the article Description and Use of the usual Instruments, in a following portion of this work.

the limb, so that the telescope may be easily turned round with the thumb and forefinger, till you observe the pole placed in the second station to coincide exactly with the cross hairs in the telescope; then apply your thumb and finger to the screw which clamps the telescope and arc fast to the limb; insert the bearing to the pole in the second station, and measure the distance. If the theodolite has no pinion, or clamping and tangent screw, recourse must be had to turn the telescope or plain sights gently round with the hand, and then fix it fast with the screw, and mark the bearing—that is to say, the number of degrees and minutes pointed out by the vernier on the limb; then plant the theodolite at the second station, as before directed. The centre of it, by a plummet, should be exactly placed over the hole where the pole stood. Unlock the screw a little that holds the instrument fast to the legs, and turn the theodolite round till you see through the telescope the pole left at the first station; here screw it fast again, and unlock the other screw a little which holds the telescope and quadrant fast to the limb, and turn the pinion round to the third station, and mark down the degrees and minutes of that bearing. If the magnetic needle cuts the same degree in the compass-box that it does on the limb, it is a proof that no error is made, and that the needle has not been attracted, which it sometimes does 1 or 2 degrees; but this you can allow for in circumstances where the needle is influenced by attraction.* Notwithstanding, in such cases, little dependence can be placed on the needle, yet it is of great use, as it affords an excellent check. If it should be found wrong at one station, it will come right again at another; and if it has nothing to attract it, the degrees in the compass-box will coincide with the degrees on the limb of the theodolite, if no error has been made, and no undue attraction at the primary station. What is said, I trust, will be sufficient for a pupil to understand the manner in which he is to use his theodolite at each station.

There is another method, much practised by a number of land-surveyors, by setting the needle and limb of the theodolite to correspond with the north point, and the eye-glass being brought to 360 on the limb, and the north end of the magnetic needle to be made to settle over the south point in the compass. Turn the upper plate round till you see a pole placed in station second, and

* In localities of peculiar sub-strata, such as iron ore, &c., the deviation of the needle may be very great, in which cases no dependence can be placed upon it. Care should be taken, in commencing surveys, to avoid iron gates, smithies, &c., if the plan is to have the compass put upon it.

insert the bearing in the field-book, and set down the degrees and minutes cut by the index on the limb; measure the distance from station first to station second; then plant the instrument as before at station second, by placing the north in the compass-box to correspond to 360 degrees on the limb, and turn the instrument round till the south end of the needle plays over the north point in the compass-box. Here screw the theodolite fast, and loosen the screw a little that holds the instrument fast to the limb, and turn the pinion till you see the cross hairs and the pole placed in station third; set down the bearing cut by the index on the limb, and measure to the third station. If you have taken all the bearings right, the south end of the needle will correspond with the limb and vernier. Go on in this manner till you have finished the survey. By this method of measuring, every angle depends on the needle alone, and is liable to error, although the bearings are set down from the limb and index; as the needle, being set at every station over the south point in the compass-box, if there is any thing to attract it, the bearing will be false. The above is supposed to be taken with a theodolite divided into 360 degrees, and the degrees in the compass-box divided into the same number, but the reverse way. Few accurate surveyors practise this method.

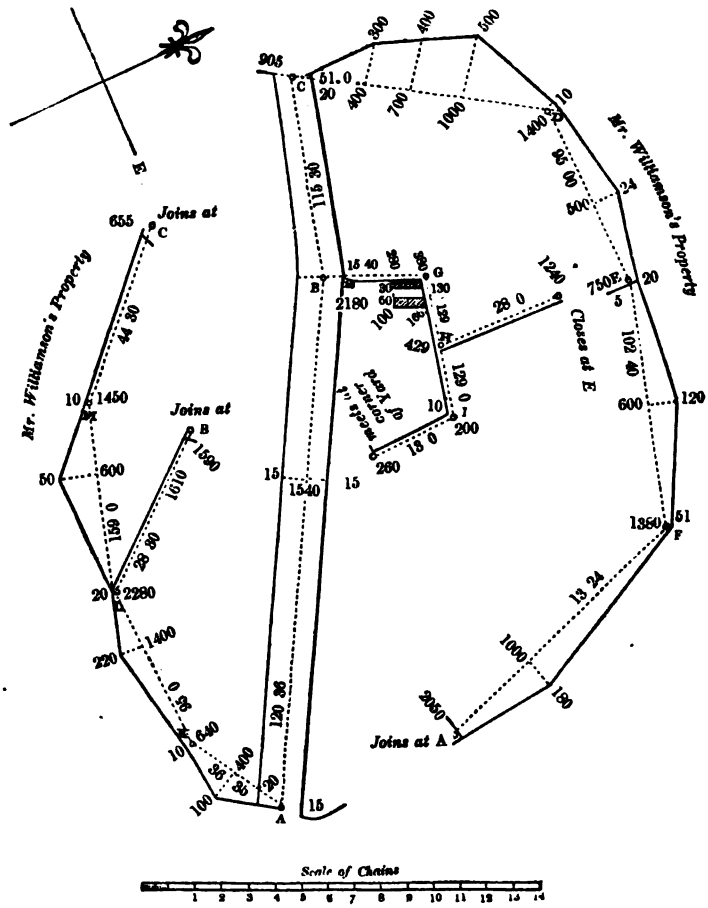
Many land-measurers, who trust to the magnetic needle, have their theodolites divided into twice 180 degrees, which in this case is preferable to dividing it into 360 degrees all round; for this particular reason, if you set up the theodolite, and turn it round upon its axis till the north end of the needle settles over 180 degrees, the south end of the needle will be over the opposite 180 degrees; so that, whenever a bearing is taken, both the fore sight and back sight will cut the same degree and minute on the limb. To use the theodolite in this manner, plant it only every other station; leave a pole at station first, and measure the distance to station second, which distance mark on an eye-draught or field-book; then plant the instrument at station second, and turn the instrument round till the north end of the needle settles over 180 degrees in the compass-box, and observe that the index is exactly at 180 on the limb: here screw the instrument fast to the legs, and unlock the screw a little, and take a back sight to station first, by turning the telescope round with the pinion till you see through it the cross hairs and the pole left in station first to coincide: observe what degree and minute is cut by the index on the limb, which mark either on an eyé-draught or a field-book, whichever you choose; then turn the telescope round by the pinion till you see the pole in station third and the cross hairs in the telescope to coincide exactly, and observe the degrees and minutes cut by the index on the limb, which insert on your eye-sketch; measure the distance from station second to station third, and also to station fourth, where the instrument is again planted; set the index to 180 degrees, and also the needle to 180; here screw it fast, and take a back sight to station third, by turning the pinion till you see the pole and the cross hairs in the telescope to coincide: observe the degrees and minutes cut upon the limb by the index; then turn the pinion round till you see the pole placed in station fifth; and when the cross hairs and it coincide, mark the degrees and minutes in your eye-sketch: go on in this manner from station to station, till you have finished your survey, by placing the instrument up at every other station. This is a very quick method of working with the theodolite; but at every other station the needle is liable to be attracted, particularly if the country is mountainous and rocky. In many parts of England you may work with great safety, without the least apprehension of danger of the needle being attracted, unless you are near a gate with iron hinges, or a blacksmith's smithy, where I have observed the needle very much influenced by attraction.

This figure is a field-sketch or draught of the farm of Tipperty, surveyed with a theodolite, and divided into 180 degrees from north to south, and into 180 degrees from south to north, by setting the magnetic needle at every station over the *fleur de lis*, and taking the bearings or angles from the meridian.*

1st. A bearing was taken from A to a pole placed at B in the road of 120 deg. 36 min. In measuring that line, an offset was taken of 15 links to the right, and another of 20 to the left; at 1540 an offset of 15 on the right to the corner of a yard, and another of 15 on the left; the whole distance from A to B is 2180, from which an offset of 20 on the left and 10 on the right, to the corner of the yard; another bearing was taken from B to C of 115 degrees 30 minutes; the distance to C is 905; a bearing was taken from C to a pole placed at D of 51 degrees: crossed the hedge at 20; at 400 an offset on the left of 300, at 700 an offset of 400, and at 1000

* The method of dividing the circumference of the horizontal circle of the theodolite into *twice* 180 degrees, instead of 360 degrees, was formerly much in use; but the latter method is the most convenient, as it avoids errors, and introduces advantages in taking rounds of angles. Ramsden's great theodolite has been objected to by the Royal Engineers, on account of its being divided into twice 180 degrees.—See Account of the measurement of the base of Loch Foyle.

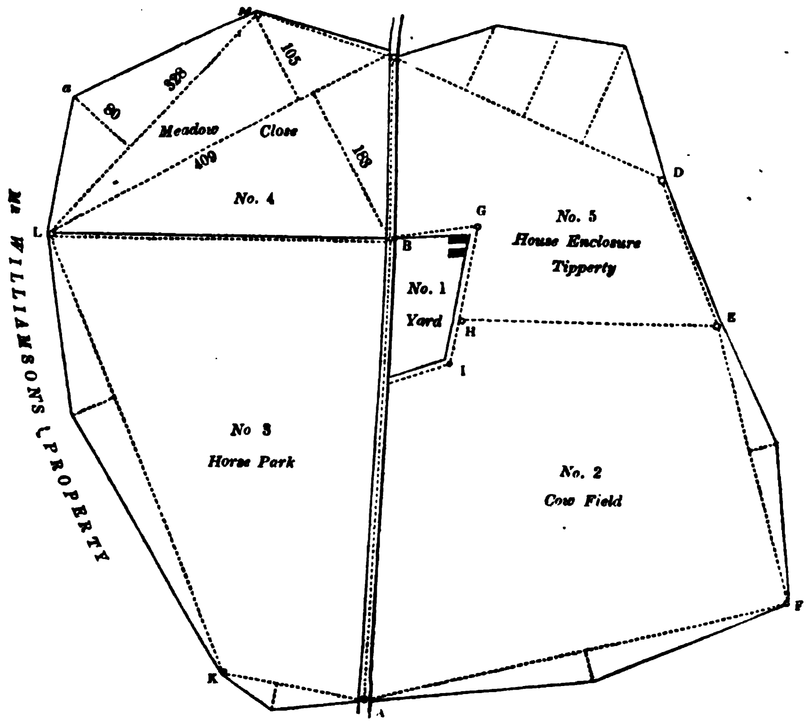
another of 500, both on the left; and at 1400, which is the whole distance from C to D, an offset of 10 to the fence or boundary: a bearing was taken from D to E of 95 degrees. In measuring that



line, an offset was taken of 24 at 500, and the whole distance to E is 750, where an offset of 20 was taken to the boundary of Mr Williamson's property; a bearing was taken of 102 deg. 40 min. from E to a pole placed in F. In measuring that line, crossed a hedge into the cow-field at 5 links; at 600 an offset was taken of 120 on the left to an angle in the fence; and the whole distance from E to F is 1380, where an offset of 15 is taken to Mr Williamson's boundary. A bearing was taken from F to a pole placed in A, where the survey was begun of 13 degrees 24 minutes. In measuring that line, an offset of 180 was taken at 1000 on the left to an angle in the hedge; and the whole distance is 2050 to A. Began again at B, where a bearing was taken to a pole placed in G of 15 degrees 40 minutes. In measuring that line, was close by the end of the house of Tipperty; and the whole distance from B to G is 380. A bearing was taken from G to a pole placed in H of 129 degrees. In measuring that line, was close by the end of Tipperty house at 26, which ascertains the breadth of the house at 130; close by the corner of the barn at 166, close by the other corner, and the length of the house is 100; and the whole distance from G to H 429. A bearing was taken from H to a pole placed in E of 28 degrees, and the distance measured to E of 1240, which finishes the house inclosure; another bearing of 129 degrees was taken from H to I, and the distance to I is 200; a bearing was taken from I of 13 degrees to the corner of the yard, where an offset was taken to the corner on the line A B at 1540; the distance from I to that corner 260 links, which finishes the cow-park.

Began again at A, where a bearing was taken of 35 degrees 36 minutes to a pole placed in K. In measuring that line, crossed the fence at 20, and at 400 an offset was taken of 100 on the left to the boundary of Mr Williamson's property; and the whole distance from A to K 640, where there is an offset of 10. A bearing was taken at K to a pole placed in L of 95 degrees. In measuring that line, an offset of 220 at 1400; and the whole distance from K to L 2280. A bearing of 28 degrees 30 minutes from L to a pole placed in B at 1590; crossed the fence to the road; and the whole distance from L to B 1610; which finishes the horse-park. Again, an offset of 20 at L to the fence on the left, and a bearing from L to M of 159 degrees. In measuring that line, an offset of 350 at 600, and another of 10 at 1450; which is the length of the line L M. Lastly, a bearing of 44 degrees 30 minutes was taken from M to C, and the distance to C is 655; which finishes the survey.

This figure is the protracted plan of the farm of Tipperty, which is laid down upon a scale of 7 chains in an inch. The plotting of which is done in the following manner: Being provided with a large sheet of plotting paper, at least twenty inches by two feet, first draw a meridian line, to represent the magnetic north and south on any part of that line; near the centre make a mark ⊙, which is called the centre. The protractor used for this plan was a semicircular one about six inches in diameter, and was divided into 180 degrees, which answered to the divisions of the theodolite, and which is by far the best way of dividing a theodolite, as every time it is turned round with the pinion, either end of the magnetic needle will cut the same degree in the compass-box that the limb and



vernier points out on the instrument, if no error has been made in taking a bearing.

Lay the protractor upon the meridian, and the centre of it upon the mark ⊙ made on the paper; then prick off 120 deg. 36 min., and mark B, 115 deg. 30 min., C, 51 deg., D, 95 deg., E, 102 deg. 40 min., F, 13 deg. 24 min., 15 deg. 40 min., G, 129 deg., H, 28 deg. 0 min., E, 129 deg., I, 13 deg., 35 deg. 37 min., K, 95 deg., L, 28 deg. 30 min., B, 159 deg., M, 44 deg. 30 min., C. The above bearings being all pricked off from the protractor with a protracting pin or a fine needle, with a black-lead pencil either write in the figure which denotes each bearing, or the letter of reference where the prick is made,—it does not matter which you insert on the plotting-paper, but let them be as near the point as possible. When all is done, take up the protractor, and with a parallel ruler, or the T square and its companion, (see fig. page 82,) lay the edge of the T square on the centre ⊙, and the prick made at

FIELD-BOOK OF TIPPERTY FARM.

Begins at Bottom.

	Offsets.	Bearings and Distances.	Offsets.	
FINISHED . . .	10	655		Closes at C.
Returns to . . .		44.30		M.
	10	1450		M.
	350	600		
		159.00	L	
		1610		Meets at B.
		28.30	L	
	20	2280		L.
	220	1400		
		95.00	K	
	10	640		K.
	100	400		
		 15	....	Crosses hedge.
Returns to A . . .		35.36		
		260		Meets at corner of yard at road.
		13.00	I	
		300	10	At corner of yard.
		 5	....	Crosses hedge into Cow-field.
Returns to H . . .		129.00		
		1240		Meets at E.
		28.00	H	
		429		H.
		146		Breadth of house; 100, length of do.
		130		Corner of a house.
		30		Breadth of house.
		129.00	G	
		380		G. far end of house.
		280		End of house.
		 10	....	Enters house park.
Returns to B in the road		15.40		
		2050		Joins at A.
	180	1000		
		13.24	F	
	10	1380	10	F. Cow-field.
	120	600		
		 5	....	Crosses hedge.
		102.40	E	
	20	750		E.
	24	500		
		95.00	D	
	10	1400		D.
	500	1000		
	400	700		House park.
	300	400		
		 20	....	Crosses hedge.
Leaves the road . .		51.00	C	
	20	905	20	C.
		115.30	B	
	20	2180	10	B. Road.
	15	1540	15	Corner of yard.
BEGAN IN A ROAD . .	20	120.36	15	A

The mark thus shows the bearing inclines to the right of the last bearing.

This mark shows the bearing inclines to the left of the last bearing.

120 deg. 36 min.; then lay the companion close to the head of the T square, and slide the T square parallel to that part of the plotting paper where you begin at A, and draw that bearing with a sharp-pointed black lead pencil or the point of the compasses, and upon that line lay off the distance from A to B 2180 links, from the scale you think most convenient to adopt, either with a feather-edged scale, page 33, or with the compasses from a scale of equal parts, page 14, and mark each station ⊙ round the prick or point made with the protracting pin or the point of the compasses; then lay the T square as before on the centre, and its edge over the prick made at 115 deg. 30 min., and lay its companion close to the head of the T square, and slide it parallel to the mark made at B, and with a black lead pencil draw the bearing B C, and lay off the distance from the scale 905, and mark it exactly on the black-lead line or bearing from B to C; then lay the edge of the square upon 51 deg. 30 min. and the centre, and slide the T square parallel to the point made at C, and draw in that bearing, and prick off the distance 1400 to D; then lay the edge of the T square upon the prick made at 95 deg. and the centre, and lay the companion close to the head of the T square, holding the companion fast to the paper while you are sliding the T square parallel to the prick made at D; then draw the bearing from D to E; then lay off the distance to E 750; again, lay the edge of the square upon the prick made at 102 deg. 40 min. and the centre, and slide it parallel to the mark made at E; draw in that bearing, and lay down the distance to F 1380; then lay the square upon the mark made at 13 deg. 24 min. and the centre, and draw in that bearing, and lay off the distance 2050 to A. If it closes at A, where the survey was begun, it is right. Next, lay the T square upon the centre and the mark made at 15 deg. 40 min., and slide it parallel to B, and draw in that bearing, and lay off the distance 380 to G; then lay the edge of the square upon the prick made at 129 deg. and move it parallel to G; draw in that bearing, and lay off the distance to H 429; then lay the edge of the square upon the mark made at 28 deg. and the centre, and slide it parallel to H; then draw in that bearing, and lay off the distance 1240 to E. If the bearing and distance answers, you may rest satisfied that you have not only taken the angles (or bearings) right, but measured the distances correctly. Then lay the edge of the T square on the mark made at 129 deg. and the centre, and slide the square parallel to H; draw in that bearing, and lay off the distance 200 to I; then look for the next mark made at 13 deg.

lay the T square upon that mark and the centre, move it parallel to I, and draw the bearing to the corner of the yard, where an offset was taken on the line A B of 15 at 1540. If it and the distance 260 answer, that part of the farm is finished that lies on the north side of the road. Again, lay the edge of the T square upon the prick made at 35 deg. 36 min. and the centre, and slide it to A; then draw in that bearing, and lay off 640 to K; then lay the edge of the square upon the mark at 95 deg. and the centre, and slide it parallel to K, and draw in that bearing with a sharp-pointed pencil; then lay off the distance 2280 to L; then with the T square upon the mark made at 28 deg. 30 min. and the centre, slide it parallel to L, and draw that bearing, and lay off the distance 1610 to B. If it closes right, both the bearing and distance will meet in a point. Next, lay the T square at the mark at 159 deg. and the centre, and slide it parallel to L, and draw in that bearing, and lay off the distance 1450 to M. Lastly, lay the edge of the T square upon the mark at 44 deg. 30 min. and the centre, and slide it parallel with the help of its companion, which must always be held fast with one hand while the T square is moving parallel to M; draw in the bearing from M to C, and lay off the distance, which is 655. If both the bearings and distance answer, it is a proof that no mistake has been made.

Then lay off all the offsets wherever they were taken, which are inserted both in a field-book of this survey, page 95, and in the eye-sketch, page 92. The manner of laying down the offsets is particularly described in the figure, page 22.

The above method of surveying with the theodolite is more or less liable to error, as the whole depends on the needle, which is very apt to be attracted. In some parts of the country, as some hidden magnetic power is frequently met with, and particularly in a rocky country, I have known it in a very short distance vary from 8 to 10 degrees. Such methods should be practised only in cases of necessity. In taking military plans for the march of armies, this method cannot be trusted, as was the case in Spain by the officers of the army under the Duke of Wellington, who lost a whole day's surveying by the action of a substratum of iron ore, and might have been productive of serious consequences.

Plate II. shows another method of surveying with the theodolite, which many land-measurers practise in preference to any other, as no dependence is left to the needle farther than to form an idea where to draw a meridian line to fix a compass in some blank corner of the plan. The method I am to mention removes the objections which are apt to arise from the needle's variability. In this method, every angle is supposed to be read off on the limb of the instrument; and in this manner the farm of Bonnyton is surveyed. First, plant the theodolite at A near Bonny Bridge, and set the index to 360 or 180 (according as your instrument is divided) on the limb, and look through the telescope to a pole placed in B; then turn the telescope round till you see the other pole placed in C; mark the degrees and minutes cut by the index on the limb on an eye-sketch, which is 75 deg. In measuring the line to B, insert all the offsets to the road, noting its breadth, also the distance where each offset was taken at, and the whole distance from A to B, which is 900 links. In measuring the distance from A to C, which is 560, enter in your field-book all the intermediate offsets to the river, and where they were taken at; then plant the theodolite immediately above the hole where the pole stood at station C, and set the index to 360 on the limb; loosen the screw a little that holds the instrument fast to the brass plates fixed upon the legs, and take a back sight to A and a fore sight to D and E, and set down the angle cut by the index on the limb, which is 132 deg. 30 min.; measure the line first to D 395, and then to E 300; then plant the instrument at E, and set the index to 360 on the limb, and look through the sight till you see the pole in C and the cross hairs in the telescope to coincide; then screw the instrument fast, and turn the telescope round till the cross hairs and the pole in B coincide; then look what degrees and minutes are cut by the index on the limb, which is 72 deg. In measuring to B, an offset was taken of 105 at 450, and the whole distance to B 870. Again, plant the theodolite at B, unlock the screw a little that holds the theodolite to the legs, and put the index to 360 on the limb, and look back to E; screw the instrument fast, and take an angle to A, which is 80 deg. 30 min. which insert on your eye-sketch. You may prove upon the spot if you have taken all the angles right, by adding them together; and if the sum amounts to 360, you are certain no error has been made in taking the angles; you then return to D, and set the index to 360 on the limb, and plant the instrument, and turn it round till you see the cross hairs in the telescope and the pole left in E to coincide. Here screw it fast, and turn the telescope round till you see the pole placed in F; set down that angle, which is 88 deg. 30 min. which enter in your eye-sketch, including all the offsets, and where they were taken at to the river, and the distance to F, which is 630; set the instrument up at F, and place the index at 360, and turn the instrument round till you see the pole in D, and screw it fast, and turn the telescope round till you see the pole placed in G, and mark the angle, which is 71 deg. and the distance to G 778; then plant the instrument in G, and set the index to 360 on the limb, and turn the theodolite round till you see through the telescope the pole in F, and screw it fast, and take an angle, by turning the telescope round till you see the pole placed in E; mark the angle on your eye-sketch, which is 59 deg. and also the distance from G to E, which is 572, and also the offsets, and where they were taken at in measuring that line. Again, plant the theodolite at E, and put the index to 360, and take a back sight to G; then turn the telescope round till you see through it the pole placed in D; mark that angle, which is 141 deg. 30 min. You may again prove the angles by adding them up; if the sum amounts to 360, you are certain of having made no error.

Again, plant the instrument at G, and put the index to 360, and turn the theodolite round till you see the pole in E, and screw it fast; then turn the telescope round with the pinion till you observe the pole placed at H, insert the angle on your sketch, which is 130 deg. 20 min. and the distance from G to H, which is 550; also the offsets to the river; then go to H, set the index to 360, and turn it round till you see the back pole at G; there screw it fast, and turn the telescope round with the pinion till you see the pole at I: note the angle, which is 86 deg. 20 min. also the distance from H to I, 760, likewise the offsets taken to the river. Next plant the theodolite at I, put the index to 360, then turn the theodolite round till you see the back pole at H; then turn the telescope till you see the pole placed in E; mark the angle on your sketch, which is 73 deg. 20 min. also the distance from I to E, 928; then with the instrument at E, turn it round till you see the pole placed in I; then turn the telescope till you perceive the pole placed in G; mark that angle 70 deg.; return to I, and plant the instrument as it was before, and turn the telescope round with the pinion till you see the pole in K; insert the angle in your sketch, which is 106 deg. 40 min. and also the distance to the pole at K, 420, and an offset to the river at O, which is 128; then plant the instrument at K; placing the index to 360, take a back sight to I, and turn the telescope round till you see a pole placed in L; mark the angle, which is 73 deg., and the distance from K to L, 420; next plant the instrument in L, and put the index to 360, and take a back sight to K; then turn the telescope round till you see the pole placed in M; mark that outward angle, which is 90 deg. 22 min., and the distance to M, which is 400, also all the offsets and small distances about the houses of Bonnyton; then set the instrument at M, adjust it as before till you see the pole placed in the old mark at B, and mark the angle 95 deg. 6 min. in your sketch, also the distance to B, which is 430: then plant the instrument at B, look back to M, and turn the telescope round till you see the pole placed in the old mark in E; insert that angle in your eye-sketch, which is 99 deg. 30 min.; then return to M, and take a back sight to L, and turn the telescope round till you see a pole placed in N; mark that angle in your sketch, which is 85 deg. 32 min. and also the distance from M to N, which is 550, likewise all the offsets and measurements mark on the sketch near the houses of Bonnyton. Again, plant the instrument at N; it being directed, look back to M, and turn the telescope till you see the pole placed in the mark that was left at O; insert the angle, which is 92 deg. 30 min. also the distance from N to O, which is 365, which closes the survey.

Lastly, plant the instrument at O, and put the index to 360 as before; take a back sight to N, and turn the telescope round with the pinion till you see a pole placed in L, which is in a line with the mark left at K; mark the angle in the field-sketch, which is 91 deg. 30 min., all the angles, distances, and offsets, and where they were taken at, being carefully marked in the field on the spot in the field-sketches or a field-book. I shall only recommend to those who use a theodolite, in a survey taken in the above method, to be very attentive, wherever it is set up, to place it as nearly level as possible with the levelling screws, and to erect it over the hole where the poles or station staffs stood. Although a practical surveyor is seldom so nice as to the levelling, imagining it sufficiently correct when he sees the needle get free play in the box, (and, in practical surveying, this is near enough, in general, for taking horizontal angles,) yet, in taking angles of elevation or depression from the horizon, the instrument must be levelled to a great nicety. At every station where the theodolite is placed, when the index is put at 360, or if divided twice into 180 deg. on the limb, the screw that holds it fast, which is commonly placed between the legs below the brass plates, must be unscrewed a little, to let the head of the instrument run easily round upon its axis: with both hands turn it round till you see the cross hairs in the centre and the pole that was left at the back station to coincide exactly; then screw it fast: you then loosen the screw marked M a little, in the figure representing the theodolite in our description of instruments, that holds the telescope and arc fast to the limb, which gives liberty to turn the telescope round to the next pole you intend to take an angle to; and when that pole is seen to coincide with the cross hairs, screw the telescope and arc fast with the screw M to the limb, and then mark what angle is cut by the index on the limb. This must be particularly observed at every station. To take the survey of an estate in the manner above described is more certain than trusting to the needle, but is more tedious, not only in taking the survey, but in laying off the angles.*

I shall now point out the mode of plotting or laying off the observations contained in the sketch of the west inclosure of the farm of Bonnyton. To give an explanation of each of the inclosures throughout the whole farm would be extremely tedious, and contain frequent repetitions: a pupil comprehending the protraction of one inclosure, can perform the whole with facility; and an inspection of the sketch will give him a very good idea of it. The best protractor for this purpose is a whole circle divided into 360 deg. Draw an obscure line at pleasure, to represent the line A B, and lay off the distance from A to B, which is 900 links; then apply the protractor to the line A B, lay its centre upon the point A, and 360 on the line towards B; prick off the angle 75; then lay a straight edge or ruler upon the point A and the point at 75, and draw a long line with a black-lead pencil; then lay off the distance 560 to C, and make a mark round the point thus ⊙; then lay the centre of the protractor upon that point and 360 on the line C A, and prick off the angle 132 deg. 30 min.; then apply the ruler to the point C and the prick, and draw a long line, and lay off 395 to D, and 300 more to E, and make a mark at E; then lay the protractor upon the line E D C, and the centre on the point E, and prick off the angle, which is 72 deg., which should meet in B; lay off also the distance from E to B 870. If the distance answers, you may then lay off the offsets, by applying the feather-edged scale to the line A B, and prick off all the distances where offsets were taken at 330, 369, 540, 734; then 122 from 330 to the road; then 122 and 60 across the road from 369, the offset 150 to the road, and 59 more across it from 540; then 98 and 70 more across the road from 734, and from B 70 across the road:

* Many practical surveyors, who use a theodolite in the above manner, are not at the trouble of taking more than two or three angles in measuring an inclosure, but make use of the chain to finish the other lines, which saves them some time in the field; but it is not so satisfactory, as they cannot prove their angles in the way mentioned in page 98.

you then draw in the road from one mark to another; then prick off 40 to the road from A, and 60 more for the width; then draw in the road to those offsets; from A an offset was taken of 35 to the fence, and 50 more to the water at 240, on the line A C and opposite it an offset of 100 to a fence, and 50 more to the water; draw in the fence, and also the river from the bridge, and prick off 20 to the fence, 70 more to the water at C, and 100 wide; draw in the fence, and also Bonny river; then lay off the distance from C to D, which is 395, also the offset of 60 opposite 182, and mark where the hedge was crossed at 382; then draw in the fence from C to the offset at 60, from thence to where the hedge was crossed; next, draw in the fence from D to where it was crossed at 12 from D; lastly, lay off the offset of 105 opposite 450; then draw in the fence from E to 105, and from thence to an offset of 20 at B, which finishes the west field: the other inclosures are all laid down in the same way. Observe particularly, when you lay off an angle with the protractor, that the centre of the protractor is exactly laid upon the mark at the station, and that 360 and 180 is exact upon the line, which should be produced a considerable way past both stations, on purpose to have more scope for the protractor.

Plate III. is a plan of the same farm of Bonnyton, surveyed with the theodolite in a very different and more expeditious manner than the method described in Plate II., as every bearing is observed and reckoned by the index and limb, and no regard paid to the magnetic needle, farther than being a check to know if any error has been made in shifting the instrument from one station to another, which it is apt to do if not made fast with the screws. The reading on the theodolite may be advantageously checked by the needle indicating the same angle nearly, which should be frequently, if not always, read and recorded. This method is generally denominated surveying by the back angle, or by traversing.

To avoid repetition, I will give one general rule how the theodolite ought to be used at every station. Fix upon any part of the grounds for your first station, and set the theodolite as level as you can, first, by means of the legs and the levelling screws; then set the index exactly over 360 or 180, according to the method of division of the limb,* and unlock the screw a little that

* The division of the limb into 360 deg. is now almost universal, and that into twice 180 deg. has generally fallen into disuse; but as some surveyors still possess the latter, the text has been retained nearly as before. I prefer 360 deg., as no

holds the instrument fast to the legs, and take both hands and turn the head round till the north end of the needle settles over the south point in the compass-box. Here screw the instrument fast to the brass plates, then loosen the clamping screw a little that holds the telescope and vertical arc fast to the limb; now turn the telescope round till you see the pole placed in the second station and the cross hairs in the telescope to coincide; then make it fast with the clamping screw, which will keep it from shifting in carrying it from one station to another, and set down the bearing cut by the index on the limb. The same operation must be performed at each place the theodolite is planted at, except setting the needle in the compass-box, which is only done the first time to ascertain the bearing. In this method of surveying every angle is taken from the meridian. This keep in mind; then plant the instrument at A in the west inclosure near Bonny bridge, and take a bearing to B, which is 62° 30′, and the distance to B is 900, which mark on an eye-draught or field-book, with all the intermediate distances on that line—that is to say, not only the offsets, but the distance must be marked where they were taken at, which insert either in an eye-sketch or a field-book; then return to A, and take a bearing to C 138°; then screw the telescope and arc fast to the limb, and measure to C, which is 560; which insert on your eye-draught or field-book, also all the intermediate distances and offsets to the river. Again, plant the instrument at C, and unlock the screw a little that holds the head of the theodolite fast, and turn the theodolite round till you see the pole placed in A and the hairs in the telescope to coincide, (this is called taking a back sight); then screw it fast, loosen the screw M a little, and turn the telescope round till you see the pole placed in D and E, which are in a line; mark the bearing, which is 89°, also the distance to D 395, and from D to E 300, and where the offset was taken of 60 at 182, and where the hedge was crossed at 382 near D; all of which being entered on your sketch, set up the instrument in E, and take a back sight to C and a fore sight or bearing to B, which is 162° 48′, and the distance to B is 870; which mark on your eye-sketch or field-book, also the offset of 105 at 450, and mark Meets at B, which will keep you in remembrance that you made a close at B.

You then return to the mark left at D; erect the instrument, and put the index to 89° on the limb, and take a back sight to E,

mistake can happen with regard to the direction and its opposite, which may, without due caution, occur in those theodolites divided into twice 180 deg.

which is the same bearing as was before observed; then turn the telescope to F, and make it fast with the screw M, and mark the bearing, which is 177° 36′, and the distance from D to F is 630; which mark in the sketch or field-book, likewise all the intermediate distances taken to the river. Again, plant the instrument at F, and take a back sight to the pole left in D; then turn the telescope about till you see a pole placed at G; insert the bearing 68° 48′, also the distance to G 778, and all the intermediate distances on that line; then plant the instrument at G, and take a back sight to F, and turn the telescope round to E, and mark the bearing 127° 12′, and also the distance 572, and all the intermediate distances and offsets taken on that line, to prove if the angles have been all right observed. Plant the instrument in E, and take a back sight to G; then turn the telescope round till you see the pole placed at the old mark D. If the bearing is 89°, it is the same as it was before, which makes you certain that no error has been made in taking the angles. Return to G, and put the index to 127° 12′, and take a back sight to E, and turn the telescope about till you see the pole placed in H; mark the bearing 77° 24′, also the distance 550 to H, and the offsets taken to the river. Plant the instrument in H, and take a back sight to G, and turn the telescope about to I; insert the bearing 164° 44′ on your eye-sketch or field-book, also the distance from H to I 760, likewise the intermediate distances and the offsets to the river; next plant the instrument in I, and take a back sight to H, and turn the telescope about till you see the pole placed in E, and insert the bearing 57° 12′, also the distance 928 to E, and the offsets, and mark Closes at E; then take a bearing to K 164° 44′, and mark also the distance from I to K 420; then plant the instrument in K, and take a back sight to I; turn the telescope round, and take a bearing to O; mark the bearing 57° 30′, and the distance to O 128°, both of which enter in the eye-sketch or field-book; then take a bearing from K to L 57° 30′, also the distance 420 from K to L, and the short distances and offsets about the houses of Bonnyton; then go to L, and take a back sight to K, and a bearing to M, which is 147° 6′, and the distance from L to M 400. Plant the instrument in M, and take a back sight to L, and turn the telescope about till you see a pole placed in B. If it answers to 62° 30′, and the distance 430 from M to B, it is right. Then turn the telescope round from M to N, and mark the bearing 62° 30′ on the eye-sketch or field-book, and also the distance from M to N 550, and all the intermediate distances and offsets about the houses of Bonnyton. Lastly, go to station N, and turn the telescope about till you see the pole placed in the old mark at O, and mark the bearing 147° 30′, and the distance from N to O, which is 366: the angle and distance being noted down, write Closes at O; then plant the instrument in O, and take a back observation to N, and turn the telescope about till you see a pole placed in L. If the bearing answers to 57° 30′, which it will do if no error has been made, you may be certain your survey will close, if no mistake has been made in measuring the distances with the chain: if an error has been made, the protracting will not close; which will oblige the surveyor to go out to the field again to find out where the mistake has been made.

I have no doubt but some land-measurers, who have uniformly used the theodolite by setting the index and limb to 360°, and taking the angles in the field at each corner of the fences, will not allow this method to be so good as that which they have been particularly accustomed to; but I can with safety inform them that a land-surveyor can, by the method just described, take the bearings with equal expedition in the fields, and can (by using a T square or parallel ruler) protract them from one centre in half the time usually taken by using a protractor or line of chords.

To protract the observations from the field-book on the farm of Bonnyton, I will refer the learner to the same method as particularly described in laying off the meridian and distances in the farm of Tipperty, page 94, by protracting all the angles from one centre, and using a parallel ruler or the T square—which undoubtedly is the best parallel ruler that was ever made use of by a land-measurer for expediting his protractions, either from a field-book or eye-draught, which should be kept as regular as possible, something in the manner of the field-sketch of Tipperty or of Hardacres. The sketches must be made much larger than those used on the plates. A land-measurer need not be particular as to the proportion of either length, breadth, or size of his sketches in the field; but only to make them so as he may have room upon his paper to insert all his figures, fences, boundaries, &c., and may use as many pages on his book as he chooses, but to be sure to mark where he leaves off on one page, and where he begins upon another. If the surveyor prefers keeping a field-book, specimens are shown in different parts of this work. In writing a field-book out of doors, it cannot be expected to be kept very clean and regular; only observe to make your figures as regular and legible as possible,

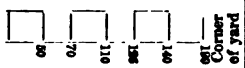
FIELD-BOOK OF THE FARM OF BONNYTON.

	Offsets.	Bearings and Distances.	Offsets.	
H		550		
		280	93	+ 100 over river
		77° 24′		
	Returns	to G		
		572		Meets at E
		360	58	
		· 260	80	
		127° 42′		
G		778	40	+ 30 to river
		480	150	+ 40 + 90 over river
		362	170	+ 43 to river
		250	160	+ 106 to river
		165	90	+ 140 to river, 115 wide
		68° 48′		
F		630	12	+ 50 to river, 120 wide
		530	65	+ 80 to river
		385	130	+ 36 to river
		258	70	+ 70 to river
		177° 36′		
	Returns	to D		
		870	20	Closes at B
		450	105	
Crosses hedge, .	......	...... 5	......	
		162° 48′		
E		300		
		89° 00′		
D	4	395		
	......	...... 382 ..	......	Crosses hedge
		182	60	+ 60 to river Bonny
		89° 00′		
C		560	20	+ 70 + 100 wide
		380	88	+ 55 to river
		240	100	+ 50 + 110, Crosses river
		138° 00′	35	+ 50 to river, 100 wide
	Returns	to A		
Width of road, 70 +	4	900		B
... Do., 70 +	98	734		
... Do., 59 +	150	540		
... Do., 60 +	122	369		
... Do., 60 +	122	330		
Width of road, 60 +	40	62° 30′		A

BEGINNING.

Field-Book for the Farm of Bonnyton begins at A.

FIELD-BOOK OF THE FARM OF BONNYTON—*continued.*

	Offsets.	Bearings and Distances.		
Closes at M, . .		430		Where the survey ends
		410		Crosses wall
Width of road, 60	78	260		
	....	..... 20 ...	.....	Crosses hedge
		62° 30′		
	Returns	to B		
O		366		Meets at Mark O
		147° 30′		
Over road, 56 + .	10	550		N
	28	425		
To road, . .	90	340		
Crosses wall, . .	60	180	170	Corner of yard
	....	.... 20 ...	....	Crosses wall
		62° 30′		
M	10	400		
		300	20	
		235	20	
	10	147° 06′		
L	15	420		
		57° 30′		
K		420	128	To a mark at O
	....	.... 406....	.....	Crosses hedge
		164° 44′		
	Returns	to I		
		928		Closes at E
		878	80	
		57° 12′		I
		760	52	+ 90 over river
I		490	170	To river
		290	205	+ 70 over river
		164° 44′		

CONTINUATION.

and to mark every thing minutely down in such a manner as you can clearly understand it. At night, protract what has been measured during the day; and if an error has been made, it must be rectified the first thing that is done in the morning in the fields.

The only difference in protracting the above farm of Bonnyton from the method used in Tipperty farm is, that the latter was laid off with a *brass* semicircular protractor, and the former laid down by a semicircular one, supposed to be drawn upon a large sheet of *paper*, at least three times the diameter of the one engraved upon Plate III. In the farm of Tipperty, every angle has to be pricked off with a protracting pin, and has to be properly numbered or figured; whereas by a protractor drawn upon your plotting paper, you have only to look for the degrees you want, which are instantly found out, they being regularly marked round the arch. I shall here give a short description of laying off the bearings and distances of the principal lines that were measured on the farm of Bonnyton; and the bearings laid off with a protractor I shall suppose drawn upon paper of large dimensions.

1st. Lay the T square upon the centre ⊙ and 62° 30′, which is the bearing from A to B; and then lay the companion to the head of the T square, and move it parallel up to A; then draw in that bearing, and prick off the distance, which is 900 links to B. Again, lay the T square upon the centre ⊙ and 138°, which is the bearing from A to C; then lay the companion to the head of the T square, which hold fast with one hand till the other hand slides the T square parallel to A; draw the bearing, and lay off upon it the distance, which is 560 links from A to C.* Again, apply the T to 89° and ⊙, and move it parallel to C; then draw the bearing, and lay off the distance 395 to D, and 300 more to E: apply the T to ⊙ and 162° 48′, and slide it parallel to E; draw the bearing, and lay off the distance 870 to B, which makes a close. Lay the T on ⊙ and 177° 36′, and move it parallel to D; draw in the bearing 177° 36′, and lay off the distance 630 to F; lay the T on ⊙ and 68° 48′, and move it parallel to F, and draw in the bearing, and lay off the distance 778 to G; lay the T upon 127° 12′ and ⊙, and slide it parallel to G; draw in the bearing, and lay off the distance 572 to E, which makes another close. Lay the T on ⊙ and 77° 30′, and move it parallel to G, and draw in that bearing, and lay off the distance 550 to H; lay the T upon ⊙ and 164° 44′, and move it parallel to H; draw in that bearing, and lay off the distance 760 to I; lay the T on 57° 12′ and ⊙, and move it parallel to I: draw that bearing, and lay off the distance 928 to E, where it should meet; lay the T upon 164° 44′ and ⊙, and move it parallel to I: draw in that bearing, and lay off the distance 420 to K: lay the

* To shorten the description, ⊙ stands for centre here, T is the T square and companion; which will avoid repeating these words at full length.

T on 57° and ⊙, and move it parallel to K; draw in that bearing both to the right and left of K, and lay off the distance 128 to ⊙ on the right, and 420 to L on the left along the road; lay the T upon 147° 6' and ⊙, and draw it parallel to L; draw in that bearing, and lay off the distance 400 to M; lay the T upon 62° 30' and ⊙, and move it parallel to M, and draw in that bearing both ways from M; lay off the distance 430 to B and 550 to N; lay the T upon 147° 30' and ⊙, and slide it parallel to N; draw in the bearing and lay off the distance 366 to O, where it should meet if no error has been made. The laying down the offsets, and where they were taken, is done in the same manner as those described in fig. page 22.

Plate IV., fig. 1, represents the field-sketch of the farm of *Hardacres*.—The survey was begun near the bridge of Allan with the chain, and a theodolite divided into twice 180°, which, as I have observed before, is the best way of dividing the limb of that instrument,* as the needle and the limb will always coincide when there is nothing to attract it from its natural polarity. Enough has been, I hope, already said to describe the method practised in measuring, taking bearings, offsets, &c., &c. This being the case, it would only be a repetition of what has been done in the farms of Tipperty, Bonnyton, &c. The first line measured, and a bearing taken, was from Allan bridge-end down the river, and all the necessary offsets taken to the river to a mark left at B at 1450; I returned again to the bridge-end, and took a bearing up the river to a mark left at C, distance 1200; I returned again to the bridge-end, and took a bearing up the road. In measuring that line, I made a mark at 800, where an offset was taken on the left of 26, and another on the right of 30, where a hedge went off to the right and another went off on the left; both of which were straight, and continued the line to a pole placed at D; and the whole distance is 1569, as may be seen both in different field-books and eye or field sketches. The instrument was planted at D, and a back sight taken to a pole left at A at the bridge-end, and a bearing taken on the right of 21° 20' to a pole at E, and the distance to it is 1330, where the instrument was again planted, and a back sight taken to the pole left at the cross roads, and a fore sight of 97° taken to B to a pole placed in the old mark made at the river. In measuring that line, crossed the straight hedge at 610; and the whole distance is 1390, which is

* See note, pages 83 and 91.

marked in the eye-sketch as well as in the field-book, page 110. Returned back to the cross roads at 1569, where the instrument was erected, and the index put to 102° 6′, on the limb which is the former bearing from the bridge-end; the telescope was then turned round,

FIELD-BOOK OF HARDACRES FARM.

	Offsets.	Bearings and Distances.	Offsets.	
		1300		Meets at E
		1260		Crosses hedge
	60	960		
	50	910		
Fence	0	710		
		550		Crosses hedge straight to house
	36	500		
	110	145		
	30	118° 24′	G	
	30	1290	30	G
	110	1050		
	170	800		
Fence	180	600		
	90	270		
		40		Crosses hedge
		F 30° 00′		
		1500		F
Fence,	30	830	27	Fence
		780	27	Corner of house + 30 breadth
		750	27	30 breadth of house
		660	27	Corner of house + 30 breadth
	30	30	30	
		120° 00′		
	Returns	to D		Road
		1390		Meets at B
		1360		
		616		Crosses fence
	10	30		
		E 97° 00′		
	10	1330		E
		D 21° 30′		
		1569		D
Corner,	30	1550	30	
Fence,	26	800	30	Fence
		102° 6′		
	Returns	to A		Along road
	20	1200		C
	80	1080		
	311	910		
River	290	620		
	120	310		
		36		Crosses hedge
		52° 30′		
	Returns	to A		
		1450	5	B + 80 to river
		1390	40	
		1260	70	River
		960	240	
		650	230	
		390	190	
		30		Crosses hedge
		13° 35′		A BEGINS

FIELD-BOOK OF HARDACRES FARM—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
		330		Diagonal
		440		Whole length of yard
		190		Breadth of ditto
		160		Length to house
	Begin	again at	Low end	of houses
		200		Breadth of yard
		430		Corner of yard
		180		Corner of house
		150		Corner of house
		30		Breadth of house
		25° 00′		Along yard
	Returns	to	Houses.	
		500		Joins at F
Crosses hedge,	 5	470	....	
	25	68° 00′		
	10	1060		M
	25	10° 06′	L	
	15	990		L
	....	.... 540 ..	....	Crosses hedge
	20	137° 24′	K	
	30	780		K
	0	620		
	104	440		
	220	330		
	12	120		
	10	10		
		80° 00′		
	Returns	to I		
		1200		Meets at D
		25° 30′		I
	Along	Road I		
	30	1100		I
	....	.. 1050 ..	....	Crosses hedge
	122	630		
	165	490		
	208	400		
	....	.... 320....	....	Crosses straight fence
	270	240		
	110	90		
	20	90° 30′		
	Returns	to C		

and a bearing of 120° taken up the road. In measuring that line, mark the distance to Hardacres houses, and also to the cross hedge; also insert the whole distance to F 1500, where the theodolite was again planted, and a back sight taken to the last station, and a bearing to the right of 30°. In measuring that line, insert all the offsets to the boundaries in the eye-sketch, as well as in the field-book, and the whole length of the line 1290 to G, from which a back sight was taken to a pole left in the road at F, and a bearing of 113° 24′ taken to the old mark in the road at E. In measuring that line, crossed a straight hedge in a line with the houses of Hardacres at 550, which had a bearing of 25°, and the whole distance was 1300 to E, where another close was made. Returned to C to the mark at 1200 at the side of the river Allan, where, after placing the index to 52° 30′ on the limb, took a back sight to a pole placed where the survey was begun at A, and turned the telescope and took a bearing of 90° 30′ to I, to a pole at the end of the cross road. In measuring that line, crossed the straight hedge to the houses at 320; the whole length of the line is 1100 to I, where the instrument was set up, and a back sight taken to a pole at the last station, and a bearing taken to a pole on the right of 25° 30′, along the road to a pole which was set up at the old mark at D, and insert the distances in the sketch, and also the whole distance to the middle of the cross roads, which is 1200, which made another joining. Returned again to I, and setting the index to 25° 30′ on the limb, which is the former bearing to the cross roads, took a bearing to a pole of 80° to K, and measured the distance 780, and the offsets to the river. The instrument was again planted, and a back sight taken to the pole left in I. In measuring that line, crossed a straight hedge, which runs in a line towards the houses at 540, and the whole distance to the pole 990, where a back sight was taken to the last station, and a fore sight of 10° 6′ to a pole at M; the distance to which is 1060. The instrument was again placed at M, and a back sight taken to a pole left at 990, and a bearing of 68° and a distance measured of 500 to a pole placed in the old mark at the head of the road at F, which makes another close or joining. Returned to the houses, which were all measured, and also the yards, as may be seen by the sketch which finished the survey.

I shall now again refer the learner to the method of protracting used in laying down the angles and distances in the farm of Tipperty or the farm of Bonnyton, where the bearings were all laid off by a large protractor drawn upon paper, and he may either lay off his parallels with the T square or a parallel ruler, and lay off his distances either with the scale and compasses or a feather-edged scale, as it suits conveniency. If he prefers keeping a field-book to an eye-draught, see page 110, &c.

Plate IV., fig. 2, is the farm of Hardacres protracted from a small scale, on purpose to give the learner an idea how one part bears from another. On it I have only inserted the principal lines that were measured, which are dotted, and the length of each station.

To have put in all the bearings and offsets upon so small a scale would have made it confused and unintelligible. The pupil may make a plan upon a large scale for improvement, either from the eye-sketch, fig. 1, or field-book, page 110, &c.

Plate V. is a sketch of the farm of *Dundaff*, where all the measured lines are represented by dots, also the offsets, and where taken at—likewise the bearings—and is protracted from a scale of four chains in an inch, and is partly measured within the grounds and partly without, and the bearings taken with a theodolite divided into 360°, and made use of in the same manner as described in the farms of Bonnyton and the farm of Hardacres. By beginning the survey with setting the north end of the needle over 180° in the compass, and the index over 360° on the limb of the theodolite—the needle corresponds with the limb of the index only every other station: whereas in the above-mentioned farms of Bonnyton and Hardacres both ends of the needle always correspond with the limb, if no error is made, and no hidden magnetic powers to attract it. The only difference is, that, in taking a bearing one way, I shall suppose 90°, in looking forward to a station, and when you go up to that station, and take a back sight to the pole left at the last station, the bearing is 270°; whereas, when the limb of a theodolite is divided into twice 180°, the fore sight and the back sight is always the same, which requires only a semicircular protractor to prick off the bearings, but the other requires a whole circle, which must be divided into 360°. Should you not make use of it in taking a fore sight, and then a back sight, it must be used in the same way as described in the farm of Bonnyton, with the additional trouble of putting the index to 360° on the limb at every station to take the back sight, which is very tedious to protract, as the protractor must be removed from one station to another, to lay off every angle that has been taken at the different stations. Although this method of surveying is practised by several land-measurers in the United Kingdom, it is neither so quick in the field, nor so soon protracted as by clamping the theodolite at every bearing that is taken, and remains fixed till you go to another station and take the back sight; indeed, some land-measurers that make use of two telescopes, the one below the limb, and the other above the arc, may be as quick in the field, the under telescope being always fixed under 360°, and the upper one that is above the arc is moved round the limb by the rack and pinion. A theodolite of this description is called *the new improved theodolite;*

but few practical surveyors make use of the under telescope; besides, in surveying with it, when you come to lay off the angles, the protractor must be removed from one station to another, as described in the farm of Bonnyton, as I have taken notice of above, which is extremely tedious. But for preferring one method of surveying and protracting to another, I am aware of exposing myself to censure; for well do I know that every surveyor is partial to his own method of working, both in the fields and in the house. It is not uncommon for a land-measurer who has been accustomed to measure with the chain alone, to condemn all other instruments whatever; but this may be owing to his being ignorant of the use of any other. What I have principally been attempting is, to describe the *different* methods of using the theodolite in the field, pointing out the various methods of protracting the observations, keeping the field-book, and taking eye-sketches in the field, that a learner may adopt whichever method he is partial to.

Page 115 is the field-book of *Dundaff.*—The survey of Dundaff was begun at A, and a bearing of 72° and a distance measured of 824 to B; returned to A, where a bearing of 110° was taken to a pole placed in C. In measuring that line, all the intermediate distances were inserted, which are marked on the sketch, and the distance 1600 to C, where the instrument was again planted, and a back sight taken to A: the screw was loosened a little, and the telescope turned round by the pinion to B, and the bearing 318°, as also the distance 1070 to B, inserted in the field-book, with all the intermediate distances and offsets; a bearing of 50° was taken, and a distance of 1340 from C to D, with all the intermediate distances. The instrument was planted at D, and a back sight taken to C, and a bearing of 330°, and a distance of 600 to E. The instrument was set up at the corner of the planting at E, and a back sight taken to D, and a bearing of 116° 30′, and a distance of 790 to F, where the instrument was again planted, and a back sight taken to E, and a bearing of 218°, also a distance of 1200 to B, which makes a close; another bearing was taken from F of 130° 24′, and a distance of 995 to G. The instrument was set up at G, and a back sight taken to F, and a bearing of 51° 30′, and a distance of 666 to H, where the instrument was again planted, and a bearing of 218° 30′, and a distance of 750 to I; where the instrument was again set up, and a back sight taken to H, a bearing was taken up the brook of 118° 6′, and a distance of 1090 to the corner of a hedge at the road near the houses; another bearing was taken from I of 50° 00′ and a

FIELD-BOOK OF DUNDAFF.

	Offsets.	Bearings and Distances.	Offsets.	
		430		Joins corner of yard
Far corner, . .		120		
Corner of house, .		25		
		135° 00′		Yard dike
		1200		Closes at B
		1010	65	Corner of meadow
Corner of house, 100 +	25	990	40	
Gateway, . 30 +	25	940		
Corner of house, .	25	915		
Corner of house, .	25	860		
		290	34	Planting straight
		218° 00′		
		790		F
Corner of road, . .	10	760		
		116° 30′		
	12	600		E
Corner of wood which	80	280		runs straight
	10	10		
		330° 00′		
	10	1340		D
	10	1000		
	80	960		
	15	870		
	10	700		
	80	560		
	90	440		
	20	350		
Wood, . . 40 +	12	80		
	....	 30	....	Crosses hedge
		50° 00′		
	Returns	to C		
		38° 00′		Bearing up road from B
		1070		Closes at B
		1040		Crosses hedge
		890	30	+ 110
		700	10	+ 130 + 220
		542	10	+ 30
		440	50	
		300	120	Corner of wood
		200	40	Wood
		50	30	Corner of wood
		318° 00′	30	
		1600	30	C
		1470	150	Far corner of wood
		1200	5	Angle of wood
		775	250	Corner of wood
		380	195	
	....	.. 42	....	Crosses hedge
		110° 00′		
	Returns	to A		
	62	824		B
Meadow, . . .	60	560	25	
	30	72° 00′		A

DUNDAFF BEGINS HERE.

FIELD-BOOK OF DUNDAFF—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
	30	608		Closes at A
	15	580	28	which finishes
	10	110° 00′		
	10	250	30	L
	55	100		
		330° 00′		
		642		K
	60	580		
	40	350		
	30	40		
		50° 00′		
	Returns	to I		
		1090		Closes at hedge near B
	40	1009		
	20	940		
Meadow, 198 . .	50	880		
To the brook	20	812		
	60	760	104	
Meadow, 200 . .	45	712	100	
	40	630		
		590	90	
	66	490	110	
	30	430	110	
	4	380		
	40	320	92	
	60	250		
	10	180	104	
	44	110	80	
	......	20	....	Crosses hedge
		118° 06′		I
	20	750		1
Meadow, . . .	15	520		
		218° 30′		
	10	636		H
Straight wood, . .	70	340		
	20	20		
		51° 30′		
	20	995		G
	42	840		
	22	772		
65 +	10	625		
	90	480		
	10	428		
Corner of road, . .	10	30		
		130° 24′		
	Returns	to F		

CONTINUATION OF DUNDAFF.

distance of 642 to K. The instrument was again set up at K, a back sight taken to 1, a bearing of 330°, and a distance of 250 to L, where the instrument was again planted, and a bearing of 110° and a distance of 608 to A; which makes a close. Returned to the houses, and took a bearing of 135° along the yard dike, and measured the length, breadth, &c. of the yard and houses; which are all inserted in the field-book, as are also all the offsets and intermediate distances on each of the lines, whether in the farm or out of it.

Great care is required in protracting a farm measured in the way that Dundaff is, with a theodolite divided into 360°; when a back station is taken, it reverses the degrees—that is to say, the first station from A, the bearing was 110° to C, when the instrument is placed at C and a back sight taken to A; and if another pole was put in the same line, the bearing to it would be 290°. The only way that I know to rectify this is, by putting the index to the limb to coincide with 290°, which is known by adding 180° to 110°, or subtracting 180° from the number of degrees between that and 360°, and protract the remainder. For example, in looking down, the bearing from F to B is 218°, in looking up from B to F the bearing is 38°; now, if 180° is subtracted from 218°, the remainder is 38°, which the index should be placed at on the limb in taking a back sight from F to B. This may be thought troublesome, which no doubt it is. I know of no other method to go regularly on, but by placing the theodolite at every station, and setting the magnetic needle over the *fleur de lis* in the compass-box, and taking the bearings and angles from the meridian in the same way as described in regard to the fig. 1, farm of Tipperty, which was surveyed by a theodolite divided into twice 180°, and which is preferable to a theodolite that is divided into 360° to a practical surveyor; yet I must acknowledge I give the preference to a theodolite that is divided into 360° for taking a range of bearings in taking the survey of a county. To protract and lay off the angles, I refer the pupil to the same method as described in planning the farm of Bonnyton or Tipperty.

The most tedious and troublesome survey a land-surveyor generally meets with, (except a large town,) is taking the measurement of common fields, in some places called *Borough-acres*, and in other places *Run-rigs*. The method I have hitherto practised has been, by first measuring and taking offsets where necessary, and marking every distance, and inserting the name of every proprietor on an eye-sketch, which I make as large as I have room for, as there are a great number of short distances to set down, as may be seen on Plate VI. fig. 1.

The survey was begun at A at the corner of Robert Brown's ridges, where the theodolite was properly adjusted and planted, and a bearing taken to D of 26° 48′, and another to B of 155° 30′. In measuring the line from A to D, entered upon David Rennie's property at 300, upon Thomas Smellie's property at 397, upon Mrs George's at 560, upon Gabriel Peacock's at 640, upon Robert Thomas's at 1000, upon John Dice's at 1162, upon John Wilson's at 1290, and crossed John Wilson's far boundary at 1432; and the whole distance to D is 1500; which is inserted both in a field-book and an eye-sketch, also all the offsets and intermediate distances, and where they were taken at: returned to A, and measured towards B; entered upon Robert Brown's property at 793, and left it at 1400, and measured on to B 1560, where the instrument was planted. After having inserted in the field-book and eye-sketch all the intermediate distances and offsets, a back sight was taken from B to A, and a bearing of 43° taken to C. In measuring that line, I inserted every proprietor's name where the chain entered their property, also the distances, in the same manner as marked on the line A D, and the whole distance 1740 to C, likewise the offsets, and where they were taken at. The instrument was again set at C, and a bearing of 142° 24′ taken to the old mark left at D, after taking a back sight to B. In measuring that line, entered upon John Wilson's property at 100, and left it at 1162, and the whole distance to D is 1900; which made a close. That distance, as well as the intermediate ones and the offsets, and where taken at, were entered upon the sketch, as well as the field-book kept by one of the assistants by way of a check.

Plate VI. fig. 2, is the eye-draught, representing where the fields were crossed to ascertain the different breadths, which are very irregular, as may be seen by fig. 3. The field was crossed, as may be observed, at six different places, at the distance of 250 links from one crossing to another, except the last, which was only at 200: a bearing was taken of 43° at each crossing, on purpose to go parallel with the line B C, as no back sight could be got: the needle in that case was trusted to, by setting it over the north point in the compass-box and the index to 180° on the limb, and turning the pinion round till the index cut 43° on the limb. One of the assistants was sent forward with a pole to John Wilson's boundary, and a sign is made to him to move to the right or to the left till he is seen through the telescope, where he sticks in the pole, which is parallel with the line B C. I began to measure across the different properties at Robert Brown's boundary, being the line *b b*, to the outside of John Wilson's boundary, which is 1808, and marked each property at entering upon them, and also the name of every proprietor, as was done upon the line A D. I returned again to Robert Brown's boundary, and measured the line *c c*, where a bearing of 43° was taken across the properties as before on the line *b b*, and marked where every proprietor's land was entered upon, as also their names, and the whole distance, which is 1904. I returned again to Robert Brown's boundary, and measured the line *d d* across to John Wilson's far boundary, which is 1605; I returned again to Robert Brown's boundary, and measured the line *e e* to John Wilson's far boundary, which is 1464; I returned again to Robert Brown's boundary, and measured the line *f f*, which is 1360: lastly, I went to the old mark in A, which is only 200 links from *f f*, and measured across to John Wilson's far boundary, which is 1322. The distances were all carefully set down on an eye-sketch, and also on a field-book, with every distance where each proprietor's land was entered upon, and also their names on each of the crossings. A land-surveyor that has had much practice, instead of returning always back to Robert Brown's boundary, after having finished one line, would measure 250 links from John Wilson's boundary across to that of Robert Brown's, after taking the bearing 43° across the properties; and so on alternately, by measuring across one way and returning the other. But this method is not so distinct as the way described above, although it would save a great deal of time in the field. It is not material whether you cross the properties at 100, 200, or 250 links; but it is absolutely necessary that you know the distance of one crossing from another.

Plate VI. fig. 3, is the protracted plan of the common-field, laid down upon a small scale. The lines that were measured are represented by dots; each angle is inserted, and the whole length from one station to another; but on account of the smallness of the scale, being only eight chains to an inch, there is not sufficient room for inserting all the intermediate distances of each person's property, nor the offsets, or where they were taken at: however, it is apprehended it will give the learner an idea of plotting, either from a field-book or eye-sketch of the ground.

In plotting this field, draw a line at pleasure to represent a meridian, and lay a protractor upon that line: first prick off the centre ⊙, and also the bearings 26° 48′, 155° 30′, 43°, and 142° 24′; then apply the T square, or a parallel ruler, to the centre and the prick made at 26° 48′, and slide it parallel to any part of the paper you intend to begin at, and with the compasses take off the distance 1500 from a large scale, and lay it off from A to D; then lay the T square upon 155° 30′, and the centre ⊙, and move it parallel to A; draw that bearing, and lay off the distance 1560 to B; then lay the T upon ⊙ and 43°, and draw the bearing from B, and lay off the distance 1740 to C; then apply the T square to the ⊙ and 142° 24′; draw in that bearing from C, and lay off the distance 1900, where it ought to close; which it will do if the angles have been all rightly taken, and the distances measured correctly. Again, lay off all the intermediate distances and offsets round the field, draw in the boundary from one offset to another; and when you lay off the cross lines, lay down the bearing 43°, which will always be parallel with the bearing laid down from B to C, and prick off every intermediate distance from Robert Brown's boundary to John Wilson's far boundary, which was measured across the field, and make marks with the point of the compasses where you crossed every property; do the same upon all the other lines that were measured across the field; then draw in with a black-lead pencil from one mark to another, humouring the natural bends of the curve of each property which are very crooked, which ink in without loss of time, and insert each proprietor's name upon his ridges.

Page 121 is the field-book of the common-field, run-rigs, or burrow-acres.

Plate VII. fig. 1, is an eye-sketch of a survey taken of a harbour, which is reckoned a very difficult survey, as so many objects have to be taken notice of. This survey was taken with a theodolite divided into twice 180°, which was first planted at the farther part of the east new pier, where a bearing was taken of 50°, and a distance measured of 460 feet to a pole. The instrument was again set up at 460, and the theodolite turned round, by taking a back sight to the last station and a bearing to the east head of 110°, and the distance to the end of the east head was 210; another bearing was taken from 460 of 90° 30′ to a pole placed at 140; the instrument was again planted at 140, and a back sight taken to the last station, and the telescope turned round to the corner of a house next the harbour, which bearing is 135°, and a distance of 153; another bearing was taken from 140 of 90° 30′ to a pole placed at 125; from whence a bearing was taken of 144° along the side of the dry dock, which measured 244; another bearing was also taken from 125 of 22°, and the distance measured 100 feet; a bearing was taken from 100 of 114° to a short distance of 33, and from 33 a bearing was taken of 151° to the end of a house on the north side

FIELD-BOOK OF THE COMMON-FIELD.

	Offsets.	Bearings and Distances.	Offsets.	
		1808		John Wilson's far boundary
		1570		John Wilson
		1447		Jos. Dice
		1330		Robert Thomas
		1010		Gabriel Peacock
		740		Mrs George
		528		Thomas Smellie
		118		David Rennie
Bearing . . .		43·00	. . .	Across the lands
	Returned	to	Robert	Brown's boundary
	250	below	B C	
		1900		Meets at D
	40	1800		
	50	1490		
Leaves J. Wilson,		1162		
		1000	34	
		800	63	
		504	63	
		260	40	
Enters J. Wilson,		100		
		142·24		
		1740		C
		1704		J. Wilson's far boundary
Enters,		1520	30	John Wilson
Enters,		1420	40	Jos. Dice
Enters,		1300	48	Robert Thomas
		1090	50	
Enters,		1070		Thomas Peacock
		800	40	
Enters,		780		Mrs George
		635	40	
Enters,		604		Thomas Smellie
Enters,		235		David Rennie
		43·00		
		1560		B
Leaves R. Brown,		1400		
		1300	140	
		1180	204	
		1000	122	
Enters,		792		Robert Brown
	40	520		
	30	230		
		155·30		
	Returns	to	A	
		1500		D
		1432		John Wilson's far boundary
	30	1400		
	36	1330		
Enters,	40	1290		John Wilson
	42	1190		
Enters,	40	1162		Joseph Dice
	40	1020		
Enters,	40	1000		Robert Thomas
	90	940		
	110	870		
	103	800		
	80	740		
Enters,	16	640		Gabriel Peacock
Enters,	16	560		Mrs George
	12	428		
Enters,	14	397		Thomas Smellie
	15	330		
Enters,		300		David Rennie
	10	220		
	30	126		
		26·48		A

BEGINS AT A.

FIELD-BOOK OF THE COMMON-FIELD—*continued.*

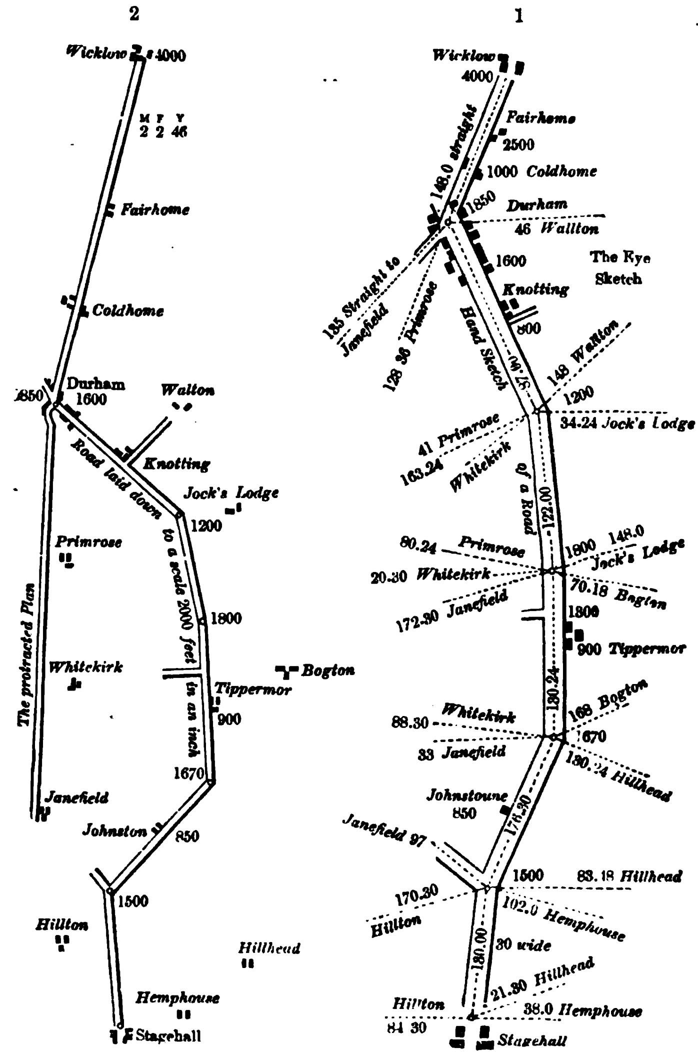
	Offsets.	Distances and Bearings.	Offsets.	
		1322		Far boundary
Enters, . . .		1096		John Wilson
		1000		Joseph Dice
		902		Robert Thomas
		580		Gabriel Peacock
		446		Mrs George
		300		Thomas Smellie
		234		David Rennie
		43·00		
Returns to	Robert	Brown's	boundary	200 below last crossing
		1360		Far boundary
Enters, . . .		1128		John Wilson
		1040		Joseph Dice
		950		Robert Thomas
		620		Gabriel Peacock
		420		Mrs George
		240		Thomas Smellie
		160		David Rennie
		43·00		
Returns to	Robert	Brown's	boundary	250 below last crossing
		1464		Far boundary
Enters, . . .		1280		John Wilson
		1187		Joseph Dice
		1075		Robert Thomas
		737		Gabriel Peacock
		515		Mrs George
		322		Thomas Smellie
		168		David Rennie
		43·00		
Returns to	Robert	Brown's	boundary	250 below last crossing
		1605		Far boundary
Enters, . . .		1410		John Wilson
		1326		Joseph Dice
		1220		Robert Thomas
		822		Gabriel Peacock
		620		Mrs George
		430		Thomas Smellie
		230		David Rennie
		43·00		
Returns to	Robert	Brown's	boundary	250 below last crossing
		1904		J. Wilson's far boundary
Enters, . . .		1680		John Wilson
		1590		Joseph Dice
		1458		Robert Thomas
		1075		Gabriel Peacock
		860		Mrs George
		630		Thomas Smellie
		278		David Rennie
		43·00		
Returns to	Robert	Brown's	boundary	250 below last crossing
		CONTINUATION.		

of the dry dock, which measured 180 feet. It will be proper to observe, that many offsets were taken, and the distances set down in the sketch, which the pupil must refer to. Another bearing was taken from 100 of 22° to a short distance of 40, from whence a bearing of 148° was taken, and a distance measured of 160 feet to the harbour; another angle was taken from 40 of 65° to a pole at 120; from thence a bearing was taken of 15° to a pole placed at 163, and a bearing taken of 178° 20′ to a pole placed in 460, and offsets taken on the left to the harbour, and also on the right to the houses; at 460 a bearing of 85° 30′ to the right hand up the main street, and a distance measured of 300; another bearing to the left of 97° 36′ was taken at 460, and a distance measured of 300 to the corner of the herring pier, which measured 300; from whence a bearing was taken of 51° to the south corner of the herring pier, which measured 160 feet; another bearing was taking from 300 to a pole placed at 138; from thence a bearing was taken of 104° to a pole placed at 630. To enumerate all the different offsets, and where taken, I refer the pupil to the sketch, which will give him a better idea than lengthening the description to several pages. A bearing was taken of 28° at 630 along the west shore pier to a pole placed at 700; from whence a bearing was taken of 158 to a pole placed at 390; from thence a bearing was taken of 96° 30′ along the west head, and a distance measured of 250; another bearing was taken from 390 across the mouth of the inner harbour to the east head of 99°; also another bearing was taken from 390 to a pole of 166° 30′, and a distance measured of 220; from thence a bearing was taken to the end of the new west pier, and a distance measured of 150.

To plot the observations taken of this survey will require a considerable time, on account of the numerous distances and offsets. To give an explanation here, would merely be a repetition of what has been already fully explained of the farm of Tipperty, page 94, also of Plate III. farm of Bonnyton.

Plate VII. fig. 2, is a small protracted plan of the harbour of Burntisland, evidently laid down from the rough eye-sketch, upon a scale of one fourth of an inch to 100 feet in length, including the principal measured lines set down upon it from one station to another only, which, it is thought, will tend to give the learner an idea what he will have to do if he thinks proper to lay it down upon a larger scale.

Land-surveyors are frequently employed to take the survey of a road, and to ascertain the distances of such farm-houses as are near to it on each side, as well as those places that are adjacent to it. The figure on p. 124 is an eye-sketch of a road, the dimensions of which are taken in feet, and the bearings taken with a theodolite divided into twice 180°. The survey was begun at Stagehall, where the instrument was planted, and a bearing on the right taken of 38° to Hemphouse; the next bearing is 21° 30′ to Hillhead, and another bearing was taken on the left of 84° 30′ to Hillton, and a



bearing taken to a pole along the road of 130°, and the distance of 1500, to the pole where the theodolite was planted, and a back sight taken to a pole left at Stagehall; a bearing of 102° taken to Hemphouse, also another to Hillhead of 83° 48′, and another on the left to Hillton of 170° 30′. In protracting these bearings, where they intersect will ascertain the distance they are from the road. An angle was also taken up a by-road of 97°, in a line with the house of Janefield, and a bearing along the road to a pole of 176° 30′, and the distance of 1670 measured to the pole where the instrument was planted, and a bearing taken of 130° 24′ to Hillhead, when you protract that bearing. If the intersection answers by all three bearings meeting in a point, it is a proof that no error has been made. Another bearing was taken on the right to Bogton of 168°, also one of 33° on the left to Janefield, likewise another of 88° 30′ to Whitekirk, and a bearing of 130° 24′ along the road to a pole, and a distance of 1800 measured to the pole. In measuring that line, past by the house of Tippermore on the right at 900, and a by-road on the left at 1300. The instrument was next erected at 1800, and a bearing of 70° 18′ on the right to Bogton, and another of 148° to Jock's Lodge, also a bearing on the left of 172° 30′ to Janefield, and another of 20° 30′ to Whitekirk, likewise another of 80° 24′ to Primrose, and a bearing along the road of 122° to a pole at 1200, where the instrument was planted, and a bearing of 34° 24′ to Jock's Lodge; also another of 148° to Wallton, and a bearing on the left of 163° 24′ to Whitekirk, and another of 41° to Primrose, and a bearing along the road of 87°, and a distance of 1850 to the pole. In measuring that line, past a by-road on the right and the house of Knotting at 800; and at 1600 entered the village of Durham, the whole distance to the pole being 1850 in the middle of the village, where the instrument was planted, and a bearing taken on the right of 46° to Wallton, another on the left to Primrose of 128° 36′, another of 135° along a straight road to Janefield, and another of 148° along the road to Wicklaw. In measuring that line, past the house of Coldhome on the left at 1000, and the house of Fairholm on the right at 2500, and left off at 4000 at Wicklaw. A pupil will observe to be very careful in protracting his distances and bearings, wherever the bearings intersect one another, as observed before. If right taken, they will be the exact distance from the road, also the precise distance from one another; and if three or more bearings to any one place intersect in a point, it is a proof that the distances, as well as the angles, have been all right taken. If the road was 50 or 60 miles in length it must be all done in the same way. A surveyor ought always, if possible, to have three or even more bearings to an object, in order that he may be completely certain of the intersection.

The figure 2 is a protracted plan of the road laid down from the eye-sketch upon a scale of 200 feet in an inch, which is introduced to point out the length of the road, and how the different houses stand, and also how they are situate one from another. It was laid down from one centre, and the bearings laid off by a paper protractor, as described in Plate III. and the bearings drawn with the T square and its companion, as particularly described in Plate III. which it is needless to repeat here, as the method of using the theodolite, the protractor, and T square, have been so often taken notice of in this work. An eye-sketch for a survey of this kind is preferable to keeping a field-book; for to set down all the distances, bearings, and names of places, &c. in a field-book, would make it so complex, that it would not easily be understood; besides, a sketch gives a much better idea how to protract the road.

Plate XVIII. fig. 9, represents the section of two hills that the boundary of an estate goes over, where the hypotenuse can only be measured, which is much longer than the level line A E. As a convex surface cannot be laid upon a sheet of drawing-paper, the difference must be found by reducing the hypotenuses to horizontal measure, which is commonly done on the spot, if the arc on the theodolite is divided, as is observed it should be in the description of the theodolite, which shows at once what number of links to allow in each chain's length in ascending or descending a hill at a certain angle. If it is not divided in that way, the following table must be applied to when you are plotting your survey, otherwise great mistakes will occur in calculating the adjoining lands, as it will give the measure of them shorter upon your plan than what they really are: for example, if you take a piece of thread, and put in pins at A B C D and E, and apply the length to the level line A E, it will reach to F, which is 392 links too long. The figure being laid down upon a scale of ten chains to an inch, this makes it evident that the line measured across the hills must be reduced to horizontal measure, that every field on the plan may lie in its true situation; which they will not do if no allowance is made; and will not only displace the next fence, but overrun a great space into the next field, and make it too little.

The following table shows the number of links to be deducted from each chain's length in ascending or descending a hill, or any uneven ground, to reduce the hypotenuse or inclined plane to a level.*

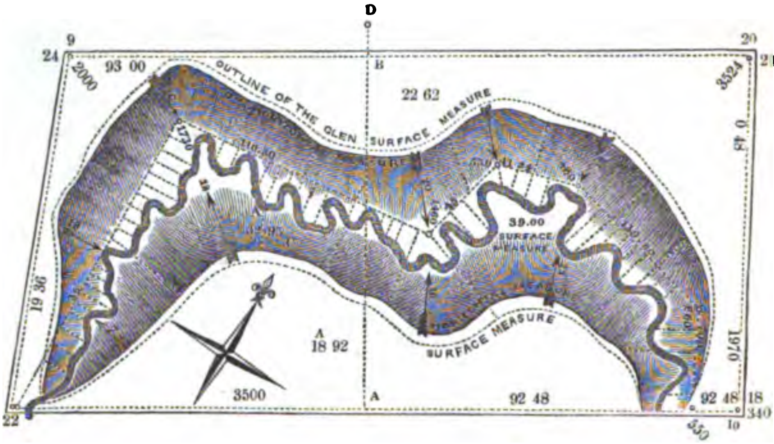
If the ascent or descent of a hill be nearly as below—

Deg.	Min.	Links.	Deg.	Min.	Links.	Deg.	Min.	Links.
4	3 deduct	¼	28	21 deduct	12	40	59 deduct	24½
5	44	½	28	57	12½	41	25	25
7	1	¾	29	32	13	41	50	25½
8	7	1	30	7	13½	42	16	26
9	56	1½	30	41	14	42	42	26½
11	29	2	31	14	14½	43	7	27
12	50	2½	31	47	15	43	32	27½
14	4	3	32	20	15½	43	57	28
15	12	3½	32	52	16	44	21	28½
16	16	4	33	23	16½	44	46	29
17	15	4½	33	54	17	45	10	29½
18	12	5	34	25	17½	45	34	30
19	5	5½	34	55	18	45	58	30½
19	57	6	35	25	18½	46	22	31
20	46	6½	35	54	19	46	46	31½
21	34	7	36	23	19½	47	9	32
22	20	7½	36	53	20	47	33	32½
23	5	8	37	21	20½	47	56	33
23	48	8½	37	49	21	48	19	33½
24	30	9	38	17	21½	48	42	34
25	11	9½	38	45	22	49	5	34½
25	51	10	39	12	22½	49	28	35
26	30	10½	39	40	23	49	50	35½
27	8	11	40	6	23½	50	12	36
27	45	11½	40	32	24	50	35	36½

Explanation of the Table.—For example, the length of the line A B, fig. 9, Plate XVIII., is 1200, and the angle of declivity is 17° 15′, which shows that 4 links and a half are to be deducted from every chain, which shortens the distance 54 links to reduce A B to A I; the length of the line B C is 830, and the angle of declivity is 16° 16′, which shows that 4 links are to be deducted from each chain, which shortens the line 33 links; the length of the line C D is 800, and the angle of declivity 34° 55′, which shows by the table that 18 links are to be deducted from each chain's length, and that the distance must be shortened 144 links; the length of the line D E is 700 links, and the angle of declivity is 39° 40′, which shows that 23 links must be deducted from each chain—which shortens the

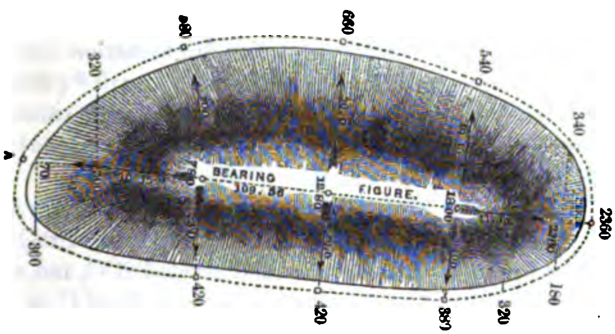
* This is merely a table of natural versines to a radius of 100, the number of links in a chain, by which, if necessary, it may be extended or interpolated. It has been recomputed, corrected, and extended in this edition. It is occasionally graduated on one side of the vertical arc of the theodolite.

distance 161 links; whence A B is reduced to A I, B C to I H, C D to H G, and D E to G E.



This figure represents a large inclosure of 74 acres 49 perches, the outline of which is upon a gentle declivity, but has a very serpentine brook or rivulet running through it, with very steep banks on each side of the brook. The proprietor wished to know the difference betwixt the horizontal and the surface measure. Angles of declivity were taken on various parts of the bank, to find out the difference of the hypotenuse and level. It was found that the difference was 6.05 acres; which, when added to the amount of the inclosure by the first measure, would have made the park 80 acres 54 perches, in place of 74 acres 49 perches, as before mentioned. Those deep ravines, glens, or gullies, are frequently met with in large surveys. Great care ought to be taken by land-measurers to make the allowance for ascending or descending hills and steep banks by the table for shortening the hypotenusal lines in plotting, otherwise the lines will not meet upon paper when the distances are laid down upon the plan. For example, if you were to take the breadth of the inclosure across the middle from A to B, it would extend the line to D, which makes a difference of 170 links in width. This park is merely introduced to show the difference betwixt horizontal and surface measure; which by some surveyors is disputed, on account that hilly ground is not so productive as level land. I certainly agree with them; but it is not a land-measurer's business to mind whether one part of the ground is more fruitful than another, but that of a valuator, who places such value as by experience he thinks the land is worth per acre; and the land-measurer's duty is to do his business correctly. The additional trouble in measuring and plotting hilly ground is undoubtedly their objection for not giving the surface measure the preference to that of the horizontal, which every surveyor ought to allow for in laying off the distances upon their plans, otherwise the lines will not meet, and the adjoining land will have too little measure.

This figure represents a hill; and the difference is required between horizontal and surface measure. The surface measure is 20 acres



76 perches, and the horizontal measure only amounts to 17 acres 76 perches, which makes three acres of difference. In measuring up the hill from whence the survey was begun at A, an angle was taken up the hill of 16° 25′ and the distance to the pole is 780, which shortens the line 31 links and a half at 780; a line was measured down the hill on the right of 420, and the angle of declivity is 23°, which shortens the line 33 links; a line was also measured from the same place on the left of 580, and the angle of declivity is 20°, which shortens that line for plotting 34 links; a line was measured from 1280 on the right of 420, and the angle of declivity is 26°, which shortens that line 44 links; a line was also measured on the left from 1280 of 660, and the angle of declivity is 20°, which shortens that line 43 links; at 1800 a line was measured of 380 on the right, and the angle of declivity is 26°, which shortens that line 40 links: another distance of 540 on the left, and the angle of declivity is 16° 15′, which shortens that line 21 links: continued the line to the far side of the hill, which is 2360; deduct 1800, there remains 560, and the angle of declivity is 11° 30′, which shortens that line 11 links for plotting. *Note*—The black line round the hill is the horizontal, and the dotted line is the surface

line for calculating the contents of the hill. It must be observed, if this hill had stood in the middle of a plain, and no allowance made for reducing the hypotenuse to a horizontal level, it would encroach upon the plain, and make it too little by three acres.

Plate VIII. is a plan of the farm of *Jamesfield*, the boundaries of which lie very flat; but there are four enclosures near the centre of the ground, which are situate upon a hill. After having measured round the farm, and protracted the angles, and laid off the distances, they answered very exact; but in protracting the angles, and laying off the distances across the hill from station 1st to station 6th, in place of meeting in a point, the distance over-stretched as far as *, which a surveyor calls a *bad closing*. A line was then protracted across the ground from station 4th to station 9th over the hill, which overlapped station 9th as far as the mark *; from which it is evident the surface measure over the hill was the cause of the protracting not meeting. The ascents and descents of the hill were then tried to find out these errors.

The theodolite was planted at A, and set level, and an angle of acclivity taken of 20° 40′ to B, and the distance from A to B, which measured 548; the instrument was then planted at C, and an angle of declivity taken of 19° 55′, and the distance from C to D measured 575; the instrument was then set level at E, and an angle of acclivity taken of 23° to F, and the distance measured from E to F 417; the theodolite was then set level at H, and an angle of declivity taken to I of 18°, and the distance measured to I 465.

By the table, the line A B being 548, and the angle of declivity 20° 40′, shows that the line must be shortened 6 links and a half each chain, which in the whole is 35 links, leaving the plotting line only 508, the line C D being 575, and the angle of declivity 19° 55′, shows by the table that 6 links must be shortened per chain, which in the whole is 35 links, and leaves the plotting line but 540; which makes a difference of the whole length from station 1st to station 6th of no less than 70 links. The line E F being 417 and the angle of acclivity 23°, must be shortened 8 links in each chain, in the whole 33 links, and leaves only 384 for plotting. The line H I being 465, and the angle of declivity 18°, must be shortened 5 links in each chain, in the whole 23 links, and leaves but 442 links for plotting; which makes a difference of the whole length in crossing the hill from station 4th to station 9th of no less than 56 links.

After having made the above allowances in plotting, it turned out that the closing came very near the truth. But the surveyor, when he calculates his survey, must be particularly careful to cast up the contents by the surface measure, and not by the distance given in plotting the four enclosures on the hill, but must make his triangles and trapeziums extend to the dotted line *a b c d e f g h* when he casts up the contents of the four enclosures; and he must observe, when he calculates the surrounding fields, to make his triangles and trapeziums to extend to the plotted line or hedge.

This plan is introduced principally to show the bad effects of making no deduction in plotting, as the four enclosures on the hill, and each of the adjoining enclosures in the valley, would have been several poles too little; which points out the necessity of making the deduction betwixt the hypotenuse and level in plotting the survey of an estate, where hills or rising grounds are met with; otherwise it is impossible that the angles and distances will meet, and consequently the calculation will be false.

On this plan I have only inserted the length of each line in going round the farm, and also the angle that each bearing makes with the meridian. To have inserted all the intermediate distances and offsets would have made it too complicated, and not answered so well the purpose intended. The plan is laid down from a scale of four chains in an inch, and the angles laid off by a semicircular protractor, the same as represented on Plate III., farm of Bonnyton.

Having hitherto treated of taking the surveys of enclosed lands, I now come to show the method of taking the survey of a large unenclosed farm of upwards of 190 acres, with a theodolite divided into twice 180 degrees.

Pages 132, 133, and 134, contain the *Field-book* of Greenside Hill farms.

Plate IX. is the grazing farm of *Greenside Hill*, the survey of which was begun at A, and a bearing of 113° 30′, and a distance measured of 1080 to B, to a mark at Red river; returned to A, when another bearing was taken of 92° 48′, and a distance of 2060 to C. In measuring these lines, several offsets were taken to the north brook, which were entered in a field-book, and on the plan also where those offsets were taken, which are represented by dotted lines, as well as the bearings from one station to another, there not being sufficient room upon so small a scale to insert upon the plan all these short distances, and where they were taken at, without creating confusion. An eye-sketch is much preferable to a field-book for a survey of this sort, as upon it a land-surveyor can sketch in the hills upon the spot, and can also make his sketches so large as to allow every figure to be legibly inserted. The theodolite was planted at C, at the edge of a morass, and a bearing inclining a little to the right hand of the last line A C of 94°, and a distance of 3800 to D; also all the offsets to the north brook, and where the morass was left, regularly entered in the field-book in measuring the line;

FIELD-BOOK OF GREEENSIDE HILL.

	Offsets.	Bearings and Distances.	Offsets.	
		700	10	South brook H Tynehouse lands
		470	150	
		260	60	
		107° 30′		
		1500	10	G
		138° 48′		
		1400		F
		153° 30′		
		2360	10	E
		1300	60	
		171° 30′	24	
		3800	108	The north brook D
		3600	110	
		3250	90	
		3080	10	
		2920	20	
		2730	100	
		2600		
		2450		
		2270	190	Lands of Hyndles
		2100	180	
		2020	100	
		1750		
		1580	12	
		1500	140	
		1380	20	
		1230	30	
		1150	18	
		980	180	
		800	80	
		660	180	
		380	220	
		94° 00′		
		2060	10	C
		1800	100	The north brook + 100
		1280	70	
		1100	70	
		910	170	
		630	10	
		380	170	
		100	10	
		92° 48′		
	Returns	to A		
		1080		B
Water	300	880		
	280	770		
	80	700		
	2	500		
Junction of brook	200	200		With the water
		113° 30′		A

BEGINS HERE.

FIELD-BOOK OF GREENSIDE HILL—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
		800 640 400 200 156° 00′	100 110 110	T
		550 71° 00′	5	S Water
Crosses brook . .		380 130 5° 00′	60	R Water edge
		725 570 130 115° 00′	70 10 230	Q
		770 660 400 62° 30′	10 30	P
		900 520 240 140° 30′	200 10 50	O Red water or river
		980 810 400 60 24° 00′	5 90 140	N Crosses road, edge of water, and brook foot
		1720 1540 1400 1140 930 710 520 340 140 85° 30′	10 260 180 240 180 140 80 140	M
		510 360 230 110 72° 40′	140 10 110 150	L Along the south brook
		1180 1000 870 600 340 280 97° 12′	80 60 100 10 70	K
		310 220 80 40° 30′	100 20 106	I

CONTINUATION OF FIELD-BOOK OF GREENSIDE HILL BEGINS HERE.

FIELD-BOOK OF GREENSIDE HILL—*concluded.*

	Offsets.	Bearings and Distances.	Offsets.	
		2000		Closes at I
		2° 38′		
	Returns	to V		
		1500		Meets at C
		62° 00′		
Along a sike . .		1620		Y
		140° 00′		
	Returns	to U		
		2150		Closes at D
Brook head . .	300	1120		
	240	880		
	80	580		
	90	380		
		180	~~~~~~	Crosses brook
		141° 00′		
		1370	18	X
		1240	90	Mid Brook
		870	90	
		600	110	
		400		
		180	70	
		137° 40′		Returns to W
Closes at F . .		1540		
		1200	~~~~~~	Crosses brook
		980	80	
		660		
		500	100	
		370	~~~~~~	Crosses brook
	70	120		
		53° 30′		
		720		
		600	10	W
		400		Mid Brook
		270	10	
		91° 06′		
		720	20	V
		480	120	
		300		
		200	60	
Heath . . .		49° 40′		
		2580	10	U
		2480		Crosses brook
	10	2380		
		2270		Crosses brook
		2140	70	
		1960		Crosses brook
Opposite a sike . .	10	1600		
		1500		Crosses brook
		1380	80	
		1000		
		900	100	
		570	10	
		500	70	
		220	100	Corner yard + 200
		40		Crosses road
Returns to Q		92° 30′		
		625		Closes at B
		500	40	
		280		
		67° 00′		
				T

CONTINUED.

the instrument was again taken to D, and a bearing on the left of the last line C D of 171° 30′, and a distance measured of 2360 to E, and also an offset of 60 at 1300; the theodolite was planted at E, and a bearing of 153° 30′, inclining a little to the left of the last line D E, and a distance measured of 1400 to F: the instrument was then planted at F, and a bearing inclining a little to the left of the last line E F of 138° 48′, and a distance of 1500 measured to G; the theodolite was then placed at G, and a bearing of 107° 30′, inclining a little to the left of the last line F G, and a distance measured of 700 to H, also all the offsets to the south brook or burn; the instrument was then placed at H, and a bearing at a very acute angle on the left of the last line G H of 49° 30′, and a distance measured of 310 to I; the instrument was then planted at I, and a bearing of 97° 12′ on the right, and a distance measured of 1180 to K, and also all the offsets to the south brook, and where they were taken at, and entered in the field-book; the theodolite was taken to K, and a bearing of 72° 40′ inclining to the left, and a distance of 510 measured to L; the instrument was then planted at L, and a bearing of 85° 30′, inclining a little on the right of the last line K L, and a distance measured of 1720 to M, also all the intermediate distances and offsets taken to the south brook entered in the field-book; the instrument was then placed at M, and a bearing of 24° on the left down the river, and a distance measured of 980 to N, and all the offsets to the river, and where they were taken at, and entered in the field-book; the instrument was planted at N, and a bearing of 140° 30′ taken on the left, and a distance of 900 to O; the theodolite was carried to O, and a bearing of 62° 30′ sharp on the right, and a distance of 770 to P; the instrument was then planted at P, and a bearing of 115° on the left, and a distance of 725 to Q; the theodolite was then taken to Q, and a bearing on the right of 5°, and went close by the farm-house, and a distance of 380 to R; the instrument was placed at R, and a bearing of 71° on the right down the river, and a distance of 550 to S; the instrument was planted at S, and a bearing of 156° on the left, and a distance of 800 to T; the instrument was set up at T, and a bearing taken down the river of 267° to the mark that was left, and the distance to B measures 625; which makes a close. All the distances and intermediate distances, with the offsets, were all regularly inserted in the field-book from M to B to the river, which the pupil is referred to. Returned to the mark near the houses at Q, where the theodolite was planted, and the index and the limb was set to 115°, which is the same bearing as before, and a back sight was taken to P, and bearing of 92° 30′, and a distance of 2580 to U, and also the intermediate distances and offsets, and where the line crossed the brook, were entered in the field-book; a bearing was then taken from U on the left of 49° 40′, and a distance of 720 to V; a bearing was taken on the right from V of 91° 6′, and a distance of 740 to W; a bearing was then taken from W to F of 53° 30′, and a distance of 1540 up the small branch of the brook; which made another close at F. Returned again to the mark at W, where a bearing was taken up the mid brook of 137° 40′, and a distance of 1370 to X; from X a bearing was taken, inclining a little to the right, of 141° to D, and a distance of 2150; which makes another closing at D. Returned to U, where a bearing was taken up a sike or small run of water of 140°, and a distance of 1620 to Y, where a bearing was taken of 62° along the side of the morass, and a distance measured of 1500 to C; which makes another joining. Then returned to the mark at V, and placed poles on the line from V to I; as the pole placed in I could not be seen from V, a bearing was taken from V of 2° 38′ to a pole placed in the same line with I, and a distance measured along the side of the dry ground and the moss of 2060 to I, but is obliged to allow 60 links for the rise of the hill, which makes another closing. It is to be observed, wherever a bearing is mentioned to be taken, that the theodolite is exactly placed over the marks where the poles were placed, and a back sight taken to the station it was last set up at.

After a careful perusal of what has been already said, it is hoped the pupil will find no difficulty in plotting Greenside Hill farm, with a theodolite divided into twice 180°. To cast up the contents, the plan should be laid down upon a large scale, and the quantity of heath, moss, morass, wet and dry pasture, calculated separately, and the quantity of ground contained in each inserted on the plan by some of the methods, as will be particularly described in the next Section.*

* There are many land-measurers who prefer taking angles or bearings with a theodolite, and measuring round the boundary of a farm, although it should be three or four miles in circuit, and are very particular in marking, either in a field-book or on an eye-sketch, every thing remarkable near to the boundary as they measure along, leaving marks on the ground where they cross fences, brooks, roads, and also where they ascend or descend a hill, to make an allowance to bring the acclivity or declivity to the horizontal distance for plotting, as described in page 88; and they are also very particular in digging a hole with a spade, or driving in a stake, at each station the instrument was planted at, till such time as they have gone all round and made a close. When this is done, they protract the bearings or angles, and lay off the dis-

SURVEYING BY TRAVERSING.

In a former part of this work, the method of surveying with the circumferenter has been explained, and its use, as generally employed, fully illustrated. Of late years, the prismatic compass has been much employed in filling up the interior of surveys of moderate dimensions. It generally reads to single degrees, and fractions may be estimated to 30′ or 20′. When made of a somewhat large size, the card may be divided to 30′ or 20′, and, by estimation, to smaller quantities; but these are sufficient for any purpose to which such an instrument can be applied.

The bearings being taken, and distances measured round a moderate space, affords the means of plotting it in the usual manner, from which, by the plotting scale employed, the area may be readily obtained with sufficient accuracy for most purposes, especially in the colonies and in uncleared countries.

Instead of plotting it by the protractor and plane scale, as in general practice, Mr John Gale proposed an improved method of plotting, by employing a traverse-table which he had computed expressly for this purpose, and published as an appendix to Adams' *Geometrical and Graphical Essays*, in 1791. The same method is solely employed in a treatise by Mr Gummere, on surveying, of which the second edition was published at Philadelphia in America, in 1817.

Mr Gale's traverse-table extends from 1 to 100 of distance for every degree and 15 minutes of the quadrant, and the results are carried to three places of decimals. Mr Gummere's table is carried to two places, and is, in fact, the same as that in Robertson's *Navigation*.

A traverse-table may be found in all books of navigation, to every point and quarter-point of the compass, as well as to every degree—generally carried to 300 or 400 of distance, and one place of decimals. The most extensive table, however, hitherto pub-

tances from any scale they choose to adopt. If no error is made, the angles and distances will meet in a point, if great care and attention is taken. Afterwards they go to the field, and measure and finish all the other lines with the chain alone, in the same manner as described in the survey of Langlee, page 24. Those that practise this method of surveying are very partial to it, as it saves the inconvenience of carrying the theodolite to the field, unless they meet with particular parts of the farm where angles are required to be taken, which they can perform quicker than measuring with the chain alone, it being every surveyor's chief aim to be as expeditious and accurate as possible, which can only be attained by attention and practice.

lished is that of Mr William Garrard, which is carried to every *ten minutes* of the quadrant, with distances extending to 300, and the results carried to two places of decimals. The last which we have seen is that of Captain Boileau of the Bengal Engineers, expressly calculated for surveying on Mr Gale's plan, and published in 1839. The angles are carried to every *single minute* of the quadrant, but the distances to 10 only, which, by shifting the decimal point and successive additions, may be employed for all purposes where the angles are carried to a minute only, the maximum accuracy in ordinary surveying. Mr J. V. Massaloup's table in German, published at Leipsic in 1847, may also be used for this purpose. By the circumferenter and prismatic compass, the accuracy of the result depends entirely upon that of the needle. Captain Boileau, however, recommends both *exterior* and *interior* angles at each station to be measured, showing the accuracy at each step by the sum of both making 360°, if the measures be correct. This occupies much time, however, and cannot be followed by the ordinary surveyor, on account of both time and expense. As far as I know, it has not occurred to any that the same method of plotting and computation may be followed by the usual method of traversing, as explained in the survey of Bonnyton, page 102.

To introduce this method, I shall begin with Mr Gale's example, p. 290 of Adams' *Geometrical and Graphical Essays*, published by W. Jones in 1803,* but with bearings as read from the card of the prismatic compass reading up to 360°, as that instrument has nearly superseded the circumferenter.

FIELD-BOOK.

BEGINNING AT THE BOTTOM OF THE PAGE AND READING UPWARDS, AS IS NOW THE GENERAL PRACTICE.

	Offsets.	Bearings and Distances.	Offsets.	
		2100		
	65	1560		
	10	845		
	40	360		
H. Denman's lands .	0	N. 353° 0′ E.		
		⊙ 1		
		COMMENCEMENT.		

* This example has been republished in books of practical mathematics, and the method reproposed in some societies as new, very lately.

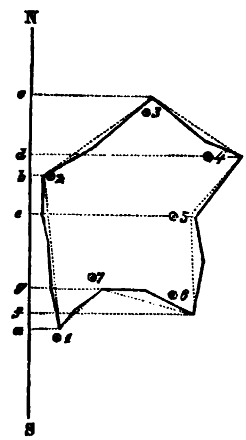
FIELD-BOOK—*concluded.*

	Offsets.	Bearings and Distances.	Offsets.	
		Closes at 1st Station, or, ⊙ 1		
W. Humphrey's lands .	0	**917** 410 N. 232° 0′ E. ⊙ 7	0 60 0	To boundary of the field
W. Humphrey's lands .	0	**1240** 750 320 N. 73° 45′ E. ⊙ 6	0 125 30 0	To boundary of the field
A corner . . . C. Ward's lands . . To the above corner .	0 125 68 0	**1400** 750 66 N. 175° 45′ E. ⊙ 5	0	
To a corner . .	95 40 0	**1100** 800 N. 220° 0′ E. ⊙ 4	0	
A corner . . .	0 0	**1440** 740 N. 117° 30′ E. ⊙ 3	0 90 0	To boundary of the field
A corner . . . W. Higgins' lands .	0 0	**1820** 610 N. 55° 15′ E. ⊙ 2	0 60 0	To boundary of the field
		CONTINUATION.		

Proper tables of northing, southing, easting, and westing, by navigators, called traverse-tables, being, from the enumeration formerly given, accessible to all, the plan, by their means, may be easily constructed, and the area readily determined with accuracy and expedition by the following—

THEOREM.

If the sum of each adjacent pair of distances, perpendicular to the meridian, true or magnetic, assumed without the survey, be multiplied by the meridian distance between them, in succession round the figure in the same order, the difference between the *north* products and the *south* products will be *double* the area of the survey.



The truth of this follows from the trapezoidal space ⊙ 1, a, b, ⊙ 2, being equal to half the product of (⊙ 1 a + ⊙ 2 b) × ab; that of ⊙ 2, b, c, ⊙ 3 being half the product of (⊙ 2 b + ⊙ 3 c) × bc; that of c, ⊙ 3, ⊙ 4, d, being equal to half the product of (c ⊙ 3 × d ⊙ 4) × cd, that of d, ⊙ 4, ⊙ 5, e, equal to half the product (d ⊙ 4 + e ⊙ 5) × de, that of e, ⊙ 5, ⊙ 6, f, equal to half the product of (e ⊙ 5 + f ⊙ 6) × ef, that of ⊙ 6, f, g, ⊙ 7, equal to half the product of (⊙ 6 f + ⊙ 7 g) × fg, and, finally, that of g, ⊙ 7, ⊙ 1, a is equal to half the product of (g ⊙ 7 + a ⊙ 1) × ag, from the principles of elementary geometry and mensuration. Hence, the difference of these is the area—that is, from the space ⊙ 1, a, c, ⊙ 3, ⊙ 4, ⊙ 5, ⊙ 6, ⊙ 7, ⊙ 1, subtract the space ⊙ 1, a, c, ⊙ 3, ⊙ 2, ⊙ 1, the remainder will be the area ⊙ 1, ⊙ 2, ⊙ 3, ⊙ 4, ⊙ 5, ⊙ 6, ⊙ 7, ⊙ 1, or that of the figure required. It is likewise more easy to put down the double areas, and then, once for all, taking half their difference for the area. The area of the offsets are computed in the usual manner, as previously shown. In addition to these, the offsets must be computed and applied in the usual manner.

In general, the card of the prismatic compass is divided into 360° from the north, eastward round the whole circle, and this is the case with the theodolite-compass, and the reading of the leading vernier of those instruments which have more than one.

Hence, to get the bearing from the meridian, the following *rule* must be observed:—

1. Between 0° and 90°, or in the N.E. quadrant, no alteration is required.
2. ... 90° and 180°, or in the S.E. quadrant, subtract the arc read from 180°.
3. ... 180° and 270°, or in the S.W. quadrant, substract 180° from the arc read.
4. ... 270° and 360°, or in the N.W. quadrant, subtract the arc read from 360°.

These results are to be accounted the courses to be found in the traverse-table, which, with the measured distances, will give the northings and southings, the eastings and westings, called by navigators the differences of latitude and departure, to be placed in a table formed for the purpose.

No.	Course.	Distance.	Difference of Latitude.		Departure.	
			N.	S.	E.	W.
		Links.	Links.	Links.	Links.	Links.
1	N. 7° 0′ W.	2100	2084			256
2	N. 55 15 E.	1820	1037		1495	
3	S. 62 30 E.	1440		665	1277	
4	S. 40 0 W.	1100		843		707
5	S. 4 15 E.	1400		1396	104	
6	N. 73 45 W.	1240	347			1190
7	S. 52 0 W.	917		564		723
	Sums, .	. . .	3468	3468	2876	2876

Results for Plotting.				Results for Area.		
No.	Meridian Distances.	Departure Distances.	Distances from the assumed meridian in survey. E. 300.	Sums of pairs of distances from the meridian.	North Products.	South Products.
	Links.	Links.		Links.	Square links.	Square links.
1	N. 2084	W. 256	E. 44	344	716,896	
2	N. 3121	E. 1239	1539	1583	1,641,571	
3	N. 2456	E. 2516	2816	4355		2,896,075
4	N. 1613	E. 1809	2109	4925		4,151,775
5	N. 217	E. 1913	2213	4322		6,033,512
6	N. 564	E. 723	1023	3236	1,122,892	
7	0	0	300	1323		746,172
Double areas, . . .				. . .	3,481,359	13,827,534
						3,481,359

No.		Offsets—Double Areas.	
		+	—
1		14,400	109,200
2		24,250	129,600
3		53,625	9,600
4		35,100	66,650
5		32,000	61,250
6		40,500	55,020
7		4,488	
8		132,012	
9		81,250	
	Sum, +	417,625	431,320
	Sum, —	431,320	
	Difference,	13,695	

Difference, . . .	10,346,175
Offsets, Subtract, .	13,695
Double area=Difference,	10,332,480
Area=Half, . .	5,166,240
	Acres.
Area, ...	51·66240
	A. R. P.
Or, ...	51 2 25·984

This example is sufficient to show the application of Schmalcalder's prismatic compass to this method of surveying. That of traversing by the theodolite can be finished in the same manner as will now be exemplified,—

EDINBURGH, *January* 20, 1844.

FIELD-BOOK OF THE WESTERN DIVISION OF THE MEADOWS,

SURVEYED WITH A THEODOLITE DIVIDED INTO TWICE 180°,

BY WILLIAM GALBRAITH.

	Offsets.	Bearings and Distances.		Offsets.	
		Theodolite.	Compass.		
Entry at Merchant Maiden Hospital (10, 30)	10	**1256**		25	
	64	920		4	
	22	900			
	21	80		31	
	20	107° 45′	N. 108° 20′ E.		
		⊙ 6			
		553			
Wall . Trees. 62 — 36 +	26	460		16	
	49	40		34	
		19° 38′	N. 19° 40′ E.		
		⊙ 5			
Corner of Wall . . .	83	**605**			
	34	512		8	
	12	260		26	
		129° 29	N. 309° 30′ E.		
		⊙ 4			
	2	**453**		36	
	34	150		4	
		138° 53′	N. 318° 45′ E.		
		⊙ 3			
		1894		27	To railing
To trees	12	1330		34	
End of Meadow Place wall	34	920		23	
Meadow Place . .	53	530		14	
Mr Sanderson's house .	45	300			
To wall	34	100		26	To railing
		S. 129° 21′ W.	N. 309° 20′ E.		
		⊙ 2			
East road . . .	62	**1576**			Breadth of plantation on the west side of Middle Walk, 55 links.
	8	1470		36	
	26	100		19	
East road		S. 27° 20′ W.	N. 207° 20′ E.		West road
		⊙ 1			
		COMMENCEMENT.			

FIELD-BOOK OF THE WESTERN DIVISION OF THE MEADOWS—*concluded.*

	Offsets.	Bearings and Distances.		Offsets.	
		Theodolite.	Compass.		
Return to station first		27° 20′ ⊙ 1	N. 217° 20′		Thus checking exactly
	33	528 450 122° 8′ ⊙ 8	N. 122° 15′ E.	19	
	26	1183 1150 108° 38′ ⊙ 7	N. 109° 15′ E.	11	
CONTINUATION.					

WESTERN DIVISION OF THE MEADOWS.

COMPUTATION OF THE AREA BY BOILEAU'S TABLE FOR TRAVERSING.

No.	Course.	Distance.	Difference of Latitude.		Departure.	
			N.	S.	E.	W.
		Links.	Links.	Links.	Links.	Links.
1	S. 27° 20′ W.	1576	1200·90	1400·03		723·63
2	N. 50 39 W.	1894	341·28			1464·61
3	N. 41 7 W.	453	384·69			297·89
4	N. 50 31 W.	605	520·85			466·95
5	N. 19 38 E.	553			185·81	
6	S. 72 15 E.	1256		382·90	1196·20	
7	S. 71 22 E.	1183		377·98	1120·99	
8	S. 57 52 E.	528		280·84	447·12	
			*2447·72 2441·75	2441·75	2950·12	2953·08 2950·12
	Errors in 8 sides, . . : ,, in 1 side, . : + S.,—N., 0·74 . :		5·97 S. + 0·744 N. —			2·96 E. + } W. — } 0·37
CORRECTED RESULTS.						
1	S. 27° 20′ W.	1576		1400·78		723·26
2	N. 50 39 W.	1894	1200·15			1464·24
3	N. 41 7 W.	453	340·54			297·52
4	N. 50 31 W.	605	383·94			466·58
5	N. 19 38 E.	553	520·11		186·18	
6	S. 72 15 E.	1256		383·64	1196·57	
7	S. 71 22 E.	1183		378·73	1121·36	
8	S. 57 52 E.	528		281·59	447·49	
			2444·74	2444·74	2951·60	2951·60

Preparatory Process for calculating the Area and protracting the Plan on Mr Gale's Method, the working Magnetic Meridian passing through the First Station.

1		1400·78	S.	. .		723·26	W.
2		1200·15	N.	. .		1464·24	W.
	Difference,	200·63	S.	. .	Sum,	2187·50	W.
3		340·54	N.	. .		297·52	W.
	Difference,	139·91	N.	. .	Sum,	2485·02	W.
4		383·94	N.	. .		466·58	W.
	Sum,	523·85	N.	. .	Sum,	2951·60	W.
5		520·11	N.	. .		186·18	E.
	Sum,	1043·96	N.	. .	Difference,	2765·42	W.
6		383·64	S.	. .		1196·57	E.
	Difference,	660·32	N.	. .	Difference,	1568·85	W.
7		378·73	S.	. .		1121·36	E.
	Difference,	281·59	N.	. .	Difference,	447·49	W.
8		281·59	S.	. .		447·49	E.

Meridian passing through ⊙ 1 — **Double Areas.**

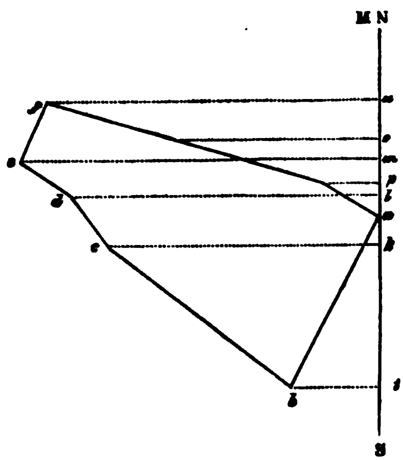
		Links.			Links.		Square Links.	Square Links.
		0·00			0·00			
1		1400·78	S.	. .	723·26	W.	+	—
	Difference,	1400·78	S.	× Sum,—	723·26	W.	. . . .	1013128·1428
2		200·63	S.	. .	2187·50	W.		
	Difference,	1200·15	S.	× Sum, +	2910·76	W.	3493348·6140	
3		200·63	S.	. .	2187·50	W.		
4		139·91	N.	. .	2485·02	W.		
	Sum,	340·54	N.	× Sum, +	4672·52	W.	1591179·9608	
4		139·91	N.	. .	2485·02	W.		
5		523·85	N.	. .	2951·60	W.		
	Differ nce,	383·94	N.	× Sum, +	5436·62	W.	2087335·8828	
5		523·85	N.	.	2951·60	W.		
6		1043·96	N.	. .	2765·42	W.		
	Difference,	520·11	N.	× Sum,	5717·02	W.	2973479·2722	
6		1043·96	N.	. .	2765·42	W.		
7		660·32	N.	. .	1568·85	W.		
	Difference,	383·64	N.	× Sum,—	4334·27	W.	. . . .	1662799·3428
7		660·32	N.	. .	1568·85	W.		
8		281·59	N.	. .	447·49	W.		
	Difference,	378·73	N.	× Sum,—	2016·34	W.	. . . .	763648·4482
8		281·59	N.	. .	447·49	W.		
1		0·00		. .	0·00			
	Difference,	281·59	N.	× Sum, —	447·49		. . . .	126008·7061
							10145343·7298	3565584·6429

Sum of additive double areas in square links,	10145344
Sum of subtractive areas,	3565585
Double area within the lines,	6579759
Double area of offsets,	461363
Double area within the boundary,	6118396
Area of the field in square links,	3059198
Dividing by 100000, and the area will be	30·59198 acres.

Or 30 acres 2 roods and 15 poles nearly, in imperial measure, because the chain used was an imperial chain.

In actual practice, the sum of the northings and southings will seldom agree exactly, nor will the sum of the eastings and westings. Here the discordance has been merely divided by the number of courses, but it would have been more accurate if the discordance had been divided by the sum of the measured distances, then multiplying the quotient, carried to four or five places of decimals by each distance, to obtain its correction. With these corrected distances, the difference of latitude and departure must be again determined, when, as above, the results will agree, and in general be more accurate. It is, however, hardly necessary to take such great trouble, since much depends upon the accuracy of the measured *angles* as well as the distances. This consideration is an inducement, not ill founded, to adopt the more simple method, especially since traversing by the theodolite is much superior to any circumferentor, where the sole dependence is on the needle.

For proof of this, see the discordances between the needle and the limb at our stations, 6 and 7 in the preceding field-book.



This survey may be plotted by the protractor in the usual manner, as was done in the farm of Bonnytoun, when surveying by what is technically called the back angle. This, for the sake of example, will be plotted by means of the traverse-table.

First draw the line M. N. S., signifying the magnetic meridian. 1. Let *a* be the first station upon the meridian in this case. From *a* lay off the first southing, 1400·78 to *i*, at right angles to which draw *i b*, equal to 723·26, the first westing, and join *a b*, the first side of the field.

2. Again lay off *a k* equal to 200·63 S. and *k c* 2187·50 W.
... *a l* equal to 139·91 N. and *l d* 2485·02 W.
... *a m* equal to 523·85 N. and *m e* 2951·60 W.
... *a n* equal to 1043·96 N. and *n f* 2765·42 W.
... *a o* equal to 660·32 N. and *o g* 1568·85 W.
... *a p* equal to 281·59 N. and *p h* 447·49 W.

Now join *a b*, *b c*, *c d*, *d e*, *e f*, *f g*, *g h*, *h a*, and the course of the chain will lead down from the traverse-table with all desirable accuracy. For practice, and a comparison of the two methods, let this be protracted by the angles, and the interior space obtained by dividing it into triangles, by the method usually employed by land-surveyors. *See* Plate X.

The latter method is the more easy, but less accurate, though sufficient, with due care, for most practical purposes.

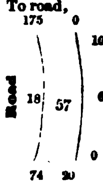
EDINBURGH, *December* 2, 1843.

SURVEY OF BRUNTSFIELD LINKS.

	Offsets.	Bearings and Distances.		Offsets.
		Theodolite.	Compass.	
	13	**1524**		
	128	1110		
	63	1014		
	36	827		
	29	600		
		N. 277° 8′ E.	N. 277⅛° E.	
		⊙ 3		
	38	**1118**		
	10	800		Elevation of ground, 1° 30′
	7	700		
	4	373		
	56	246		
	52	N. 290° 32′ E.	N. 290½° E.	
		⊙ 2		
	13	**206**		
To wall . . .	9	N. 220° E.	N. 220° E.	
		⊙ 1		Meadow Place wall

COMMENCEMENT.

BRUNTSFIELD LINKS—*continued.*

	Offsets.	Bearings and Distances. Theodolite.	Bearings and Distances. Compass.	Offsets.
		Remeasurement. N. 220° 4′ E. Return to ⊙ 1	Check. N. 220° E.	
	0	**474** N. 120° 45′ E. ⊙ 10	N. 121° E.	
	73 55 123 73	**1518** 840 575 30 N. 131° 32′ E. ⊙ 9	N. 132° E.	
To wall, ... To house, ... To road, ...	60 320 79	**735** 100 N. 88° 53′ E. ⊙ 8	N. 89° E.	
To house, ... To road, ...  To wall, . To road, .	26 190 0 18 0 20	**2067** 1580 1500 1000 680 200 N. 60° 31′ E. ⊙ 7	N. 60½° E.	{ 57 links breadth of road to Bruntsfield Links
	31 130 193 200 158 30	**1458** 1200 900 700 400 48 N. 313° 8′ E. ⊙ 6	N. 313½° E.	
	15 18 33	**1028** 700 274 N. 183° 46′ E. ⊙ 5	N. 184½° E.	
	8	**358** N. 234° 16′ E. ⊙ 4	N. 234¼° E.	

CONTINUATION.

BRUNTSFIELD LINKS—*continued.*

CALCULATION FOR PLOTTING AND AREA.

No.	Course.	Distance in Imperial links.	Difference of Latitude.		Departure.	
			N.	S.	E.	W.
1	S. 40° 0′ W.	206		157·8		132·4
2	N. 69 23 W.	1118	389·3			1039·5
3	N. 82 52 W.	1524	189·3			1512·2
4	S. 54 16 W.	358		209·1		290·6
5	S. 3 46 W.	1028		1025·8		67·5
6	N. 46 52 W.	1458	996·8			1063·9
7	N. 60 31 E.	2067	1017·3		1799·3	
8	N. 88 53 E.	735	15·2		734·9	
9	S. 48 28 E.	1518		1001·2	1130·3	
10	S. 59 15 E.	474		242·4	407·4	
			2607·9	2636·3	4071·9	4106·1
				2607·9		4071·9
				28·4		34·2
			$\frac{1}{10}$ error	2·84	. .	3·42
1				155·0		129·0
2			392·1			1036·1
3			192·2			1508·8
4				206·2		287·2
5				1023·0		64·1
6			999·6			1060·4
7			1020·2		1802·7	
8			18·0		738·3	
9				998·4	1133·8	
10				239·5	410·8	
			2622·1	2622·1	4085·6	4085·6

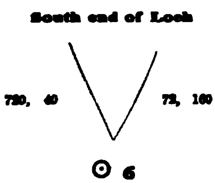
			Area in Square Links.			
				—		+
1	155·0	× 129·0		1995		
2	392·1	× 1294·1	. .	. . . .	. .	507417
3	192·2	× 3839·0	. .	. . . .	. .	737856
4	206·2	× 5635·0		1161937		
5	1023·0	× 5986·3		6123985		
6	999·6	× 7110·8	. .	. . . .	. .	7107956
7	1020·2	× 6368·5	. .	. . . .	. .	6497145
8	18·0	× 3827·5	. .	. . . .	. .	68895
9	998·4	× 1955·4		1952271		
10	410·8	× 239·5		98387		
				9356575		14919269
	Offsets, . .	. .	. .	45600	. .	1337196
				9402175		+16256465
						— 9402175
						2 | 6854290
						3427145 S.L.
						Acres 34.27145
						34A. 1R. 3·432 P.

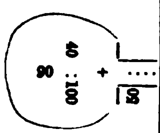
January 21, 1846.

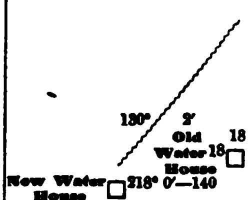
LOCHEND FIELD-BOOK.

SURVEY OF THE LOCH AS ITS WATERS STOOD ON THAT DAY.

	Offsets.	Bearings and Distances.	Offsets.	
	8	**444**		
	12	400		
	32	300		
	24	150		
		N. 35° 17′ E.		
	5	⊙ 7		
		1154		
		1000	. . . .	Farm House
	0	646	. . . .	Hedge to House
	8	400		
	24	240		
	72	160		
		N. 16° 12′ E.		
		⊙ 6		
		900		
	40	720		
	20	500		
	0	300		
		N. 153° 24′ E.		
		⊙ 5		
	80	**498**		
	88	400		
	60	350		
	50	N. 164° 38′ E.		
		⊙ 4		
	50	**664**		Burn
	10	515		
	52	450		
	58	320		
	9	80		
		N. 220° 28′ E.		
		⊙ 3		
	12	**560**		
	5	400		
		N. 243° 27′ E.		
		⊙ 2		
	20	**367**		
		220	22	□ Dovecot
		60		
		N. 276° 21′ E.		
		⊙ 1		
		COMMENCEMENT.		







LOCHEND FIELD-BOOK—*continued.*

	Offsets.	Bearings and Distances. / Check Angle.	Offsets.	
		N. 276° 25′ Close at ⊙ 1		
		130 N. 13° 24′ E. ⊙ 12		
		90 N. 344° 0′ E. ⊙ 11		
		218 N. 332° 13′ E. ⊙ 10	....	40 Corner of Walls
		140 N. 80° 50′ E. ⊙ 9		
		168 N. 51° 24′ E. ⊙ 8		

CONTINUATION.

CALCULATION FOR PLOTTING AND AREA.

No.	Course.	Distance.	Difference of Latitude. N.	Difference of Latitude. S.	Departure. E.	Departure. W.
		Links.	Links.	Links.	Links.	Links.
1	N. 83° 39′ W.	367	40·59			364·75
2	S. 63 27 W.	560		250·31		500·94
3	S. 40 28 W.	664		505·16		430·94
4	S. 15 22 E.	498		480·19	131·97	
5	S. 26 36 E.	900		804·74	402·98	
6	N. 16 12 E.	1154	1108·17		321·96	
7	N. 35 17 E.	444	362·44		256·46	
8	N. 51 24 E.	168	104·81		131·29	
9	N. 8 50 E.	140	22·30		138·21	
10	N. 27 47 W.	218	192·87			101·61
11	N. 16 0 W.	90	86·51			24·81
12	N. 13 24 E.	130	126·46		30·12	
			2044·15 2040·90	2040·40	1413·00	1423·05 1413·01
			12)3·25			12)10·04
			0·27			0·84

LOCHEND—*continued.*

CALCULATION FOR PLOTTING AND AREA.

No.	N.	S.	E.	W.	Meridian.	Perpendicular.	Sums.
				Assumed Meridian Distance, from ⊙ 1 100 E			
1	40			364	4 N.	464 W.	564
2		251		500	211 S.	964 W.	1428
3		506		430	717 S.	1394 W.	2358
4		480	133		1197 S.	1261 W.	2655
5		805	404		2002 S.	857 W.	2118
6	1108		323		894 S.	534 W.	1391
7	362		257		532 S.	277 W.	811
8	105		132		427 S.	145 W.	422
9	22		139		405 S.	6 W.	151
10	193			101	212 S.	107 W.	113
11	86			24	126 S.	131 W.	238
12	126		31		126 N.	31 W.	162
	2042	2042	1419	1419			
				COMPUTATION OF AREA.			
					—		+
1	N.	× 40	564		22560		
2	S.	× 251	1428	. .	. . . . .	. . . . .	358428
3	S.	× 506	2358	. .	. . . . .	. . . . .	1193148
4	S.	× 480	2655	. .	. . . . .	. . . . .	1274400
5	S.	× 805	2118	. .	. . . . .	. . . . .	1704990
6	N.	× 1108	1391		1541228		
7	N.	× 362	811		293582		
8	N.	× 105	422		44310		
9	N.	× 22	151		3322		
10	N.	× 193	113		21809		
11	N.	× 86	238		20468		
12	N.	× 126	162		20412		
					1967691	+	4530966
						—	1967691
							2563275
	Area within the Measured Distances,						12·81637
	Offsets to Water that day,						1·26496
	Area covered by Water in Imperial Acres,						11·55141
	Area in different denominations, 11 A. 2 R. 8½ P.						

See PLATE X.

[The following Field-Book commences at the lower end of St Mary's Loch, whence the Yarrow issues. The first Bearing is reckoned from the south towards the west.]

August 1846.

THE FIELD-BOOK OF ST MARY'S LOCH, AND THE LOCH OF THE LOWES, SELKIRKSHIRE—See PLATE XI.

By JOHN LEARMONT, TRAQUAIR.

	Offsets.	Bearings and Distances.	Offsets.	
	95	1600	0	
Trees	80	236		Road
		85° 30′		
		⊙ 9		
	300	1057	0	Bridge
Stream		950		
	260	657	0	
		87° 30′		
		⊙ 8		
	80	640	0	
		96° 0′		
		⊙ 7		
	20	1763	0	Road
	7	1500	20	
	20	1100	0	
		1000	20	
	90	600	0	
		85° 40′		
		⊙ 6		
	10	756	0	Road
	15	500	0	
	20	400	20	
	30	100	0	
		77° 20′		
		⊙ 5		
	50	731		
		600	10	
		87° 30′		Road
		⊙ 4		
	30	457	0	
	40	300	0	
	30	200	10	
	40	100	10	
		76° 20′		
		⊙ 3		
	60	1600	0	Road
		1400		
	0	1200		
	120	700		
	104	400		
		129° 50′		
		⊙ 2		
	30	1087		
	0	800		
	42	400		
	40	300	20	To a Bush
	0	S. 110° 35′ W.		
		⊙ 1		

COMMENCEMENT.

THE FIELD-BOOK OF ST MARY'S LOCH, AND THE LOCH OF THE LOWES—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
		770	10	Road enters
		7° 20′		
		⊙ 18		
		685	5	Road issues
Road enters		38° 0′		
		⊙ 17		
	0	1080		
	10	600		
		41° 50′	┌	
		⊙ 16		
Trees	50	830		
		56° 30′		
		⊙ 15		
	0	4700		Road
Trees		3180	......	Meggat Water
		3070		
	800	2730	......	
	800	1950		
	30	1100		
	20	800		
	0	600		
	100	100		
		45° 0′		
		⊙ 14		
		460		
	40	300	0	
		112° 15′		
		⊙ 13		
	5	697	0	
	10	500	0	
	50	200		Road
		101° 30′		
		⊙ 12		
	10	1538	0	
	30	1050		
	0	800		
	20	660	0	
Trees	5	550	35	
	20	500	35	
	30	300		
	10	200	0	
	30	50	5	
		99° 45′		
		⊙ 11		Road
		730	0	
	30	600	10	
		120° 30′		
		⊙ 10		

CONTINUATION.

THE FIELD-BOOK OF ST MARY'S LOCH, AND THE LOCH OF THE LOWES—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
Road issues	20 100 20 200	2298 1850 1700 200 48° 50′ ⊙ 28	0 5 0	Road enters
	20 100	412 300 14° 0′ ⊙ 27	0 0	
	0 50 200	645 300 100 45° 0′ ⊙ 26	0 40 0	50 □ Summerhope House
Road enters	170 120 30 130	550 400 240 100 38° 0′ ⊙ 25		Road issues
	130 0	677 600 47° 5′ ⊙ 24		Return to ⊙ 16
House □ Road issues Cappercleugh □ House	0 50 30	446 300 200 Toll-bar 85° 0′ ⊙ 23		Closes at ⊙ 14 Road enters
Church □		553 400 98° 0′ ⊙ 22	0 15	Road issues
Road enters	0 100	946 600 121° 0′ ⊙ 21	Γ	
		2160 70 41° 0′ ⊙ 20		Meggat Water
Road issues	0 10 20	730 300 200 145° 35′ ⊙ 19		

CONTINUATION.

THE FIELD-BOOK OF ST MARY'S LOCH, AND THE LOCH OF THE LOWES—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
	5 0	1850 1700 1000 51° 30′ ⊙ 38		
Road issues	0 5 0	416 300 200 35° 30′ ⊙ 37		
		500 46° 20′ ⊙ 36	0	Road enters
		333 300 29° 0′ ⊙ 35	0 0	Road issues
Road enters Road issues	0 0	1078 800 700 500 38° 40′ ⊙ 34	0	Road enters
		860 200 43° 0′ ⊙ 33	0 10	
	5 0 200	1112 1000 600 69° 15′ ⊙ 32	0 10	Loch of Lowes
	10 30 150 30	1070 600 300 200 177° 30′ ⊙ 31	350 70 20 10	
	40 60	611 400 25° 30′ ⊙ 30	0 0	
Trees Road enters	10 576	2242 1500 600 200 7° 30′ ⊙ 29	0	Road issues

CONTINUATION.

THE FIELD-BOOK OF ST MARY'S LOCH, AND THE LOCH OF THE LOWES—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
Boat □ House	30	420		
	200	0		
		103° 0′		
		⊙ 46		
	3	241	0	
	30	100	10	Trees
		4° 0′		
		⊙ 45		
		274		
	....	.... 200	....	Bridge
		148° 0′		
		⊙ 44		
		1878	0	Tibby □ Shiels
	100	1740		
	20	1500		
	250	1100		
		800		
	200	700		
	.. 50 ..	.... 150	....	Trees
		6° 0′		
		⊙ 43		
Road	20	5100		
	170	4000		
	200	3800		
	120	3100		
	50	2400		
	20	1000		
	50	300		
		100		
	⌝	45° 30′		
		⊙ 42		
		850		
	30	745		
	....	.... 600	50	Trees
	50	200		
		122° 0′		
		⊙ 41		
	58	1500		
	50	1300		
	20	1240		Yarrow
		1193		
	....	.... 1040	....	
	100	900		
	80	700		
		118° 10′		
		⊙ 40		
Road enters	10....3	1200	70	Road issues
		200		
		70° 15′		
		⊙ 39		

CONTINUATION.

THE FIELD-BOOK OF ST MARY'S LOCH, AND THE LOCH OF THE LOWES—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
Trees	50	2600		
	10	2100		
	30	2000		
	0	1950		
	70	1700		
	10	1050		
	30	500		
		95° 20′		
		⊙ 57		
	20	1600		
	200	1100		
		82° 45′		
		⊙ 56		
	20	1200		
	120	500		
		57° 40′		
		⊙ 55		
	30	2800		
	150	2000		
	350	1200		
	300	1000		
		46° 30′		
		⊙ 54		
	30	892		
	0	100		
		57° 55′		
		⊙ 53		
	10	280		
		53° 0′		
		⊙ 52		
		142		
		123° 30′		
		⊙ 51		
	10	1000		Trees
	80	270		
		17° 30′		
		⊙ 50		
	50	1850		
	30	1500		
	100	100		
		25° 0′		
		⊙ 49		
	0	310		
		300		
		74° 45′		
		⊙ 48		
	30	1210		Trees
	170	700		
	150	470		
	0	300		
		83° 0′		
		⊙ 47		

CONTINUATION.

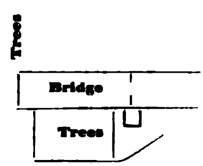
THE FIELD-BOOK OF ST MARY'S LOCH, AND THE LOCH OF THE LOWES—*concluded.*

	Offsets.	Bearings and Distances.	Offsets.	
	0	800 700		Closes at ⊙ 1 Yarrow
		300 164° 0′ ⊙ 67		
To Sluice ☐	150 100 20 0	466 400 300 200 35° 0′ ⊙ 66		
	20 0 10	1300 1100 800 68° 25′ ⊙ 65		
	10 60	408 100 104° 30′ ⊙ 64		
	30 250 70	1400 1000 300 69° 0′ ⊙ 63		
	200 200	2000 1500 250 80° 0′ ⊙ 62		
	0 0 20 0 60	2000 1150 500 350 100 81° 30′ ⊙ 61		
	40 0 50	760 600 200 117° ⊙ 60		
	0 30	270 150 98° 0′ ⊙ 59		Bowerhope ☐
	0 80 30	900 460 200 56° 0′ ⊙ 58		

CONTINUATION.

THE FIELD-BOOK OF INNERLEITHEN FARM, PEEBLESSHIRE.

PLATE XII.

	Offsets.	Bearings and Distances.	Offsets.	
		965	80	
		800	15	
		751	0	
		700	0	
		500	30	
		400	40	
		300	40	
		200	30	
		30	0	
		60° 0′		
		⊙ 8		
		186		
		34		
		130° 0′		
		⊙ 7		
	0	467		
		100	34	
		17° 20′		
		⊙ 6		
To Dyke on	~~~~	~ 1277 ~	~~~~	Cowford Road
		1100	15	
		1060	0	
		1000	2	
		800	24	
		750	20	
		600	0	
		500	10	
		350	40	
		200	60	
		100	60	
		67° 45′		
		⊙ 5		
		1030	10	
		900	15	
		160	10	
		100		To arable ground 100 links back
Continued this line and took ⊙ 10 here		137° 30′		
		⊙ 4		
		518		
		400	0	
		206	25	
		190	4	
	10	170	0	
		150	24	
	24	0	0	
		70° 30′		
		⊙ 3		
		220		
		159° 0′		
		⊙ 2		
		170		Toll ☐ house
		70		
		125° 30′		
		⊙ 1		

COMMENCEMENT.

THE FIELD-BOOK OF INNERLEITHEN FARM, PEEBLES-SHIRE.—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
		365	10	
	133	0		
		168° 30′		
		⊙ 14		
		791	10	Arable ground
Tweed	0	700		
168	30	480		
148	33	230		
Return to ⊙ 13 Arable ground		156° 45′		
		⊙ 13		
		2070		
		1210		
		30° 0′		119° 45′ Length 1325.
Return to ⊙ 13		⊙ 13		
		750		Side of pond
		74° 15		To pond
		⊙ 13		
	88	1561	10	
	80	1100		
	60	700		
	50	300		
		0	10	
		137° 0′		
		⊙ 12		
	46	1045		To Cowford
		1000	30	
		800	10	
	77	400	0	
		250	0	
	70	200	20	
River Tweed		100	60	
		134° 40′		Arable ground
		⊙ 11		
	7	1246		
	33	1160	65	
	57	900	26	
	72	600	40	
	50	300	70	
	12	100	87	
Return to ⊙ 4		65° 30′		
		⊙ 10		
		1080		Closes at ⊙ 1
		1000	120	Hedge
		800	185	
		500	60	
		400	25	
		330	0	
		200	0	
		100	40	
		119° 30′		
		⊙ 9		

CONTINUATION.

FIELD-BOOK OF INNERLEITHEN FARM, PEEBLES-SHIRE—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
		553		
		500	0	
	25	400	23	
	7	270	45	
	0	200	55	
	6	100	54	
Return to ⊙ 20		86° 0′		
		⊙ 20		
		937	30	
	0	20	30	**Road to Quarry**
Elevation 5°		157° 0′		
		⊙ 20		
	15	1018	35	
	23	900	25	
	30	800	19	
	25	700	22	
	15	600	33	
	10	500	39	
	10	400	35	
	20	200	28	
	33	100	15	
		74° 45′		
		⊙ 19		
	40	905	0	
	20	800		
	7	742	30	**To end of dyke**
	10	700		
	7	630		
	4	400		
	9	300		
	12	200		
	17	100		
		91° 30′		
		⊙ 18		
	36	1023	4	
	30	990		
	30	900		
	35	800		
	30	700		
	15	600	32	
	7	400	40	
	0	100	48	
	25	0	22	
		121° 45′		
		⊙ 17		
		80		**On Peebles road**
	9	0		
Elevation 18°		43° 30′		
		⊙ 16		
		1238		
	80	1200		
	60	740		
	78	500		
	110	0		
		171° 0′		
		⊙ 15		

CONTINUATION.

FIELD-BOOK OF INNERLEITHEN FARM, PEEBLES-SHIRE—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
		763		
		743		
Trees		700		Trees
Return to ⊙ 29		117° 30′		Elevation 9° 30′
		⊙ 30		
Length to hedge		590	0	
		250	60	
		140	60	
		4° 0′		
		⊙ 29		
		1056	5	
		790	77	
		54° 45′		
		⊙ 28		
		754	5	
		114° 30′		
		⊙ 27		
		1128		
		930	210	
		68° 20′		
		⊙ 26		
		530		Corner of Garden
		270	230	Corner of House
	......	......60.....	......	
Return to ⊙ 21		28° 0′		
		⊙ 25		
		700		Went off Bridge
		667		Edge of Water
		300		Came on to Bridge
		173° 0′		
		⊙ 24		
Toll-house		2800		
10 Links till it joins ⊙ 1		2700		
81° 30′ Length of Hedge 1544		1900		
82° 30′ Length of Hedge 1211		695		
	.. 0..	105	......	
Return to 334 on		168° 0′		⊙ 21
		⊙ 23		
		483		Closes at ⊙
		24° 0′		
		⊙ 22		
		3647		Dyke joins
		2027		
End of Dyke 34		460		
Took ⊙ 23 here		334		
		40	0	
		30° 45′		
		⊙ 21		

CONTINUATION.

FIELD-BOOK OF INNERLEITHEN FARM, PEEBLES-SHIRE—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
	......	405	......	
	14	332	130	Corner of Garden
	54	300		
	......	105	......	
Return to ⊙ 38		86° 15′		
		⊙ 39		
Corner		480		of Farm-House
	24	460		
Corner of Garden	16	370		
	6	240		
	12	140		
	40	100		
		122° 20′		
		⊙ 38		
Corner of Garden	5	657		
Corner of House	271	500		
	50	100		
	......	64	......	
Return to ⊙ 21		118° 30′		
		⊙ 37		
		230	0	To road-side dyke
		100	20	
Declivity 5°		3° 0′		
		⊙ 36		
		296	4	
		200	36	
		100	72	
		50	42	
Declivity 15° 30′		155° 30′		
		⊙ 35		
		197	0	
		100	0	
		71° 30′		
		⊙ 34		
		464	10	
		56° 55′		
		⊙ 33		
		457	5	
		140	0	
		5	15	
		50° 45′		
		⊙ 32		
		96	3	
		179° 0′		Declivity, 14° 30′
		⊙ 31		

CONTINUATION.

FIELD-BOOK OF INNERLEITHEN FARM, PEEBLES-SHIRE—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
	0	455		
Leithen	35	400		
	80	174		
	4	150		
	25	60	10	
Return to ⊙ 42		11° 45′		
		⊙ 47		
		312		
		160	250	Corner of Feu.
		110° 0′		
		⊙ 46		
		1316	10	
		1110	127	
		1060	110	
		690	169	
		510	136	
		400	130	
		29° 30′		
		⊙ 45		
		415	5	
		390	20	
		300	36	
		225	36	
	……	4	……	
		113° 0′		
		⊙ 44		
	3	725		
	10	50		
		39° 50′		
		⊙ 43		
Head of Boyd's Feu	5	1250		
Foot of Boyd's Feu	0	670		
	0	406		
		23° 0′		
		⊙ 42		
		522	0	
	30	510		
Dam	38	300		
	45	200		
	25	100		
		50		
Dam		0		Dam
		42° 25′		
		⊙ 41		
		149		
	0	100		
		50° 0′		
		⊙ 40		

CONTINUATION.

FIELD-BOOK OF INNERLEITHEN FARM, PEEBLES-SHIRE—*continued.*

	Offsets.	Bearings and Distances.	Offsets.	
	30	373		
	0	100		
		95° 30′		
		⊙ 54		
	0	720		
	20	600		
		500	0	
		450	20	
	0	400		
	30	200		
Tweed		113° 50′		
		⊙ 53		
	0	Lth. 425		Foot of Dam
	40	200		
		94° 20′		
		⊙ 52		
Foot of Leithen	0	840		
	0	200		
	20	100		
		154° 10′		
		⊙ 51		
		1200		
	70	900		
	70	700		
	60	600		
	40	500		
	10	400		
	0	300		
	0	200		
		0	200	Back fall
		160° 45′		
		⊙ 50		
Run enters		Lth. 332		71° 30′ 200 Foot of Fen
		250		Run issues
		166° 30′		
		⊙ 49		
	30	1075		Run Back fall
Leithen				Factory
		800	100	Run
		780		Dobson's fen
				Run
				Gaswork
		660	70	250
	0	400		
		250		260 to Dam 350 Barn
		200		
50	0	100		270 to Dam and Corner of Steading.
		54		
	0	0		
		160° 0′		
		⊙ 48		

CONTINUATION.

FIELD-BOOK OF INNERLEITHEN FARM, PEEBLES-SHIRE—*concluded.*

	Offsets.	Bearings and Distances.	Offsets.	
		260 235 100 175° 45′ ⊙ 60	 10 25	Closes at Farm-House at end of ⊙ 38 line. Head of Stack-Yard
Return to ⊙ 58		429 340 200 80 125° 45′ ⊙ 59	10 100 40 50	Corner of Stack-Yard Hedge joins
	0 5	273 220 .... 75 66° 30′ ⊙ 58		Joins on line ⊙ 48 at 730 on line Corner of Gas Work Corner of Gas Work
		928 792 294 200 160′ 15° ⊙ 57	 20 10 10	Factory Corner of Feu
Hedge Joins Return to ⊙ 53		1580 162° 45′ ⊙ 56	10	
	 7 0	540 400 200 77° 45′ ⊙ 55		Closes at ⊙ 2
CONTINUATION.				

NOTE.—The survey commenced at the east side of the toll in a direction S.E., that is, at station 1, page 159, the bearing 125° 30 is N.E., thus bringing it to a south-easterly direction.

SECTION THIRD.

OF AREAS.

ART. I.—TO FIND THE AREA OF REGULAR AND IRREGULAR FIGURES.

THE *Area* or *Content* of land is in general named in England and Ireland, *acres*, *roods*, and *perches;* in many parts of England the *perches* are named *poles;* in Scotland, the perches are named *falls*. Now, there being 100 links in a chain, and ten chains long and one chain wide being one acre, 1000 links, which is the number of links contained in ten chains, if multiplied by 100 links, the product is 100,000 square links, which is equivalent to one acre, and is known by cutting off the five cyphers on the right hand, thus 1.00000.

Also five chains in length and two chains in breadth is one acre, and four chains long and two and a half wide is also an acre. Land-surveyors seldom write down the words chains and links when in the field; but thus, 500 for 5 chains, 200 for 2 chains.* Now if 500 be multiplied by 200, the product is 100,000; and when the 5 right-hand figures are cut off, there remains 1.00000 as before; which 1 is an acre: but if a distance is measured of 11 chains and 50 links, it is marked 1150; or, suppose you have measured a line 19 chains and 75 links in length, it is expressed 1975. Now if 1975 is multiplied by 1150, the product is 2271250; by cutting off 5 figures on the right hand, there remains 22.71250, or 22 acres on the left; the decimals .71250 are square links, which when multiplied by 4 (the number of roods in an acre,) brings the decimals into roods. After cutting off 5 figures on the right, the product is 2.85000 roods; then .85000 multiplied by 40 (the number of perches, poles, or falls

* It is in general preferable to write down all distances in links only.

in a rood,) the product is 34.00000, and is 34 perches; which gives in the whole 22 acres, 2 roods, and 34 perches or falls.

The calculation as under will appear plain.

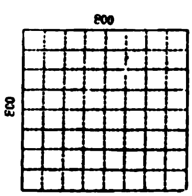
```
           1975
           1150 multiplied by
          ------
          98750
         1975
        1975
       --------
acres 22.71250 product
             4 multiplied by
       --------
roods  2.85000
            40 multiplied by
       --------
perches 34.00000
```

A practical surveyor is seldom at the trouble of casting up the contents of the decimals of an acre, but cuts off the 5 figures on the right hand, and to what is above 5, adds the letter A, which represents acres: the rest are decimals.

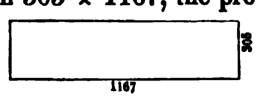
In the following examples I shall use a few algebraic signs, merely for shortening the calculations, viz. + for the sign of addition; — for the sign of subtraction; × for the sign of multiplication; ÷ for the sign of division; and = as signifying equal to.

The area or superficial content of a square is easily found.

1. Suppose the square represented by this figure to have each side 800 links, what is the area? The product of 800 × 800 is 6.40000 acres, which × by 4, to bring the decimals into roods, and the remaining decimals × by 40, to bring them into perches as before directed, is = 6 acres, 1 rood, 24 perches.

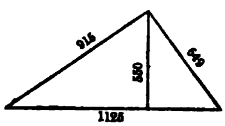


2. Suppose the parallelogram, or oblong, represented by this figure to be 1167 long and 305 wide; then 305 × 1167, the product is 3.55935; which × 4, to bring the decimals .55935 into roods, is = 2.23740 roods; then .23740 × 40 is = 9.49600 perches, which is = 3 acres, 2 roods, 9 perches and a half nearly.



It is presumed, from what has been stated above, that a pupil will be at no loss in understanding how the amount in square links is turned into acres, roods, and perches.

3. This figure is a triangle, the base of which is 1125, and the perpendicular 550: the half is 275; which × 1125, the product is 3.09375 = 3 acres 15 perches. It will turn out the same measure if half the base had been multiplied by the whole of the perpendicular; half the base 562 × 550, the product is 3.09375 = 3 acres 15 perches.



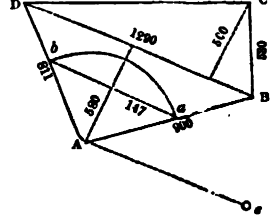
All triangles are measured in the same manner, either by taking half the length of the perpendicular, and multiplying it by the length of the base, or by taking half the length of the base, and multiplying it by the perpendicular, or taking half the product of the base and perpendicular, when both are odd numbers.

4. The right-angled triangle I G H, (*fig. p.* 3,) whose sides are each 740; the square of which is 5.47600 acres, the half is 2.73800 acres, = 2 acres, 2 roods, 38 perches.

5. The trapezium, (*fig.* 3, *p.* 3,) whose diagonal from A to B is 1200, and the sum of the perpendiculars are 1080; the half of which is 540 × 1200, the product is 648000 = 6 acres, 1 rood, 37 perches.

In place of measuring the two perpendiculars separately, and adding them together, and then halving the sum, a practical surveyor would lay a parallel ruler upon the diagonal A B, and move it parallel to the angle D, and draw the line D *e*, and take off the distance with his compasses from the angle C, by putting one point in C, and measuring to the nearest part of the dotted line D *e*; which will give the same distance as the sum of the two perpendiculars, by applying that extent on a scale equal to double the extent of the scale the figure is laid down by. This operation saves the trouble of adding and dividing.

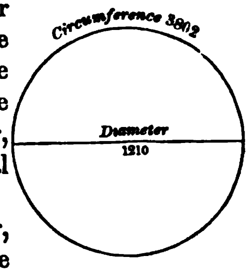
This figure is another trapezium, whose diagonal from D to B is 1290, and the sum of the perpendiculars is 1080, the half is 540 × 1290 = 6.96600. In this figure an angle A is supposed to be taken with the chain, which saves the trouble of measuring the diagonal across the enclosure: it also shows, by drawing a line parallel to the diagonal D B from A to *e*, and by applying the compasses to the angle C, and extending them to the nearest part of the line A *e* is the sum of the two perpendiculars; which distance, if



applied to a double scale, saves the trouble of adding and halving the same.

The area or contents of irregular fields, of whatever number of sides, are generally determined by dividing them into triangles, trapeziums, &c., and measuring them separately; and the products, being added together, will be the sum total or area. Regular circles, polygons, or ellipses, seldom occurring in practice, for the sake of the curious, I shall merely give a few examples, showing how the areas are commonly calculated.

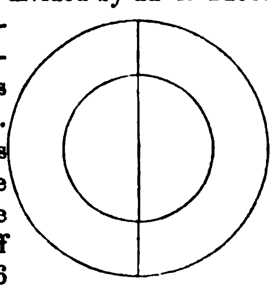
This figure is a circle, the contents of which is wanted. Various methods have been tried to get the exact contents; the common method is by multiplying half the diameter by half the circumference, which gives the area. For example, half the diameter of the above figure is 605, half the circumference is 1901; when multiplied into one another, gives 1150105 square links, which is equal to 11 acres 2 roods.



Another method is by squaring the diameter, and multiplying the amount by .7854: the square of the diameter is 1464100, multiplied by .7854 is 1149904.04; omit the decimals, and there remains 1149904; cut off 5 figures to the right hand, multiply the remainder by 4, and cut off 5 figures on the right; then multiply by 40, and cut off 5 figures, and there remains 84772 decimals of a pole or perch; the amount is 11 acres, 1 rood, 39$\frac{8}{10}$, very near the first calculation.

It frequently happens that the diameter of a circle cannot be measured, particularly in a round plantation, a round building, hay-cock, or fish-pond: in that case, if you can get the circumference measured, the diameter is found by multiplying it by 0.31831; or thus roughly.

The circumference of the outmost circle in this figure is 4400, which, if multiplied by 7, is 30800, and divided by 22 is 1400. For the length of the diameter, the content is found in the same way as described above in last figure, which amounts to 15.40000, or 15 acres, 1 rood, 24 perches. The circumference of the inner circle is 2515.2, and the diameter is found as above to be 800: square the diameter as before directed, and multiply by .7854, and cut off the four decimals, and there remains 5.02656



for the amount of the inner circle, which subtract from the outermost, which is 15.40000, which also gives the amount of the space between the circles, which is 10 acres, 1 rood, 2 perches.

Another method for finding the contents of the space betwixt the outermost and innermost circle. The diameter of the large circle is 1400, and the diameter of the little or innermost circle is 800 links; when they are added together is 2200. This sum, multiplied by 600, the difference of the diameters, is 1320000, and again multiplied by .7854, gives 1026728, or 10 acres, 1 rood, 2 perches, the same as before.

1. To find the area of a circle when the circumference is given: multiply the square of the circumference by 0.0795775, or 0.08 nearly correct, when the circle is small. Thus $\frac{4400 \times 4400 \times 0.0795775}{100000} = 15.406204$ acres = 15 acres, 1 rood, 25 perches.

2. The areas of circles are as the squares of their diameters, or as the squares of their circumferences. But the difference of the squares of any two numbers is equal to the produce of their sum and difference. Whence, the product of the sum and difference multiplied by the constant 0.7854 for the diameter, or 0.07958 for the circumference, will give the area between two circles of different diameters with the same centre.

Contents of the triangles and trapeziums of the farm of Broad Meadows, surveyed with the circumferentor.* Fig. page 86.

					A. Dec.
Triangle	I.	1930 ×	515	half the perpendicular	9.93950
Do.	II.	2200 ×	200	do. do. . .	4.40000
Do.	III.	2810 ×	900	do. do. . .	25.29000
Do.	IV.	2410 ×	160	do. do. . .	3.85600
Do.	V.	4130 ×	525	do. do. . .	21.68250
Trapez.	VI.	2600 ×	1015	half sum of the perpendiculars. .	26.39000
Do.	VII.	2690 ×	900	do. do. . .	24.21000
Do.	VIII.	5100 ×	2119	do. do. . .	108.06900
Triangle	IX.	3940 ×	380	half of the perpendicular	14.97200
Do.	X.	2280 ×	150	do. do. . .	3.42000
Do.	XI.	3112 ×	554	do. do. . .	17.24048
Trapez.	XII.	2280 ×	450	half sum of the perpendiculars . .	10.26000

A. R. P.
269·72948 = 269 2 36 $\frac{7}{10}$

Plate VI. (*Fig.* 4.)—Is a plan of the same *Common field*, as *Fig.*

* It is a saving of labour to put down the *double areas*, dividing the sum at last by 2, once instead of several times, and also thus avoiding fractions.

3, each division of which is divided into equal distances of 250 links wide, which is introduced merely to point out the best method of calculating the contents of such an irregular field, which has not a straight line; the boundaries being very crooked and curved, would require much trouble in dividing it in the common way into trapeziums and triangles, to ascertain the areas. This plan being protracted from a scale of eight chains in an inch, four chains or half an inch will then make a scale of acres—that is to say, when each division is 250 links, each half inch upon a scale of equal parts is one acre. The making scales of acres will be particularly taken notice of in an after part of this work. In the figure, there are only inserted the amount in each of the columns of George Peacock's property, merely to show how they ought to have been inserted in the same way in all the other properties; and by simply adding the amount of all the columns together, we have 5·972 for the area of George Peacock's property; and if the amount of the others had been inserted in each column as in George Peacock's, the sum of them would be the area. If the breadth of the columns is short, as in Joseph Dice's property, the amount may be ascertained by applying the compasses but once to the scale of equal parts or acres. Thus, put one foot of the compasses in the centre as near as you can guess, in the left hand column at *a*, on the boundary betwixt Joseph Dice and Robert Thomas, and extend the other foot to the centre of John Wilson's boundary; then with that extent put one foot of the compasses in the middle of the next column between Joseph Dice and Robert Thomas, and let the other foot next you be fixed upon the paper, which will reach into George Peacock's property in the second column; then extend the compasses from thence, over John Dice's property, to John Wilson's; then put in one foot of the compasses between Robert Thomas and Joseph Dice, and let the other foot, which is towards yourself, be fixed somewhere about the middle of the third column of George Peacock's property, and extend the other foot to John Wilson's boundary; go over the other columns in the same way. When done, apply the compasses to the scale of acres, which is 1.736; then add to it the amount of the small square, which is .132 square links, and the sum is 1.868 acres, equal to 1 acre, 3 roods, 19 perches. This small plan is merely introduced to give a pupil an idea of the quickest method of finding out the content of an irregular boundary like what the figure is, which should be laid down upon a larger scale, and the area of each property will be correctly ascertained. If the learner chooses to keep a field-book in preference to the sketches *Fig.* 1 and 2, there is one in pages 121 and 122.

Fig. 3. page 10, represents a hexagon, each side of which is 520. First calculate the contents as described in fig. 1. page 169, for finding the content of a triangle, by multiplying half the perpendicular by the base, and multiplying the amount of one triangle by 6, the number of sides, which gives the area. For example, the side B C is 520, the perpendicular is 450, the half is 225, × 520, is 1.17000 × 6, the number of sides, is 7.02000, = 7 acres, 3 perches.

When one side only is given, the area may be found by the multiplier in the annexed table.

No. of sides.		Multiplier.
3.	Equilateral Triangle	0.433013
4.	Square	1.000000
5.	Pentagon	1.720477
6.	Hexagon	2.598076
7.	Heptagon	3.633912
8.	Octagon	4.828427
9.	Nonagon	6.181824
10.	Decagon	7.694209
11.	Undecagon	9.365640
12.	Duodecagon	11.916153

Rule. Square the side of any regular polygon, and multiply the square by the multiplier in the table.

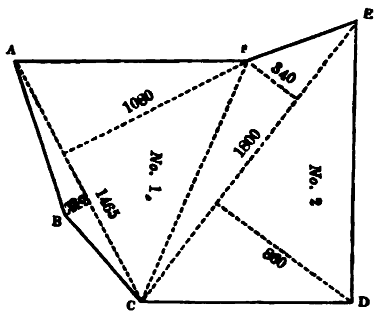
Example. One side of the above Hexagon is 520 × 520 × 2·598076 = 270400 × 2·598076, is 7·02519,75040 = 7 acres, 4 perches.

Fig. 1. page 12, is an octagon. One side is 340, and a perpendicular let fall from one of its sides to the centre is 410, the half is 205 and 205 × 340 × 8 is 5·57600 = 5 acres, 2 roods, 13 perches.

Or thus, by the multiplier in the table; the square of 340, one of the sides, is 115600 and 115600 × 4·828427 is 558166·1612 square links = 5 acres, 2 roods, 13 perches.

Fig. 2. page 12, is an Oval or Ellipse. 1st, Let the longest or transverse diameter be 1740, and its shortest or conjugate diameter 1270. *Rule.* Multiply the 2 diameters into one another, and the product by ·7854. Thus 1740 × 1270 × 0·7854 is 2209800 × ·7854 is 1735576·9200 square links = 17 acres, 1 rood, 17 perches.

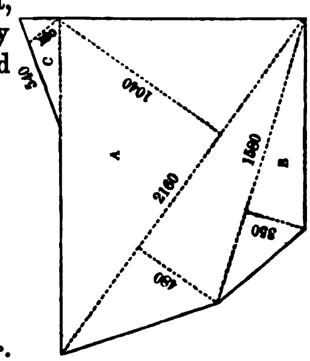
A B C D E F is a field of six sides divided into two trapeziums, represented by dotted lines, which should be drawn with a sharp pointed black lead pencil on your plan. In finding the contents, some surveyors draw the lines with red ink, and write in the length of the diagonals and perpendiculars in the same manner as I have done, but, in general, most surveyors use a black lead pencil, which is rubbed out after the work is calculated.



The calculation turns out as follows:—

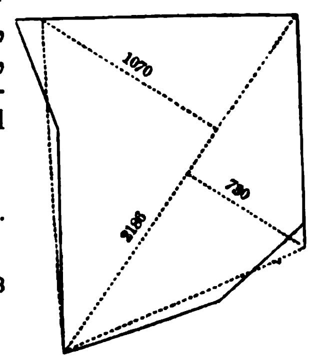
The Trapezium No. 1, 1465 × 605, half sum of the perpendiculars, ...	8·86325			
Trapezium No. 2, 1800 × 600, half do. do.	10·80000			
		A.	R.	P.
Total,	19·66325 =	19	2	26

This figure is the same field as last, but calculated in a different manner, by dividing it into one large trapezium and two triangles.



The diagonal of trapezium A, 2160 × 760, half sum of the perpendiculars, is	16·41600			
The base of the triangle B, 1580 × 175, half the perpendicular, is	2·76500			
The base of the triangle C, 540 × 90, half do.	48·600			
		A.	R.	P.
	19·66700=	19	2	27

This is the same field as the two last, and is calculated by equalising the sides, as particularly described in the following pages, and the contents ascertained by one calculation, viz. :—



	A. Dec.	A.	R.	P.
The diagonal 2186 × 900, half sum of the perpendiculars, is . .	19·67400 =	19	2	28

This figure (fig. 1,) is a very irregular field of ten sides, divided into one trapezium and five triangles.

The trapezium A,	Diagonal	1438 × 615, half sum of the perpendiculars, is	8·84370
Triangle B,	Base	980 × 250, half the perpendicular, ...	2·45000
Do. C,	do.	920 × 150, half do.	1·38000
Do. D,	do.	1680 × 290, half do.	4·77200
Do. E,	do.	1200 × 228, half do.	2·73600
Do. F,	do.	860 × 140, half do.	1·20400

A. R. P.
21·38570 = 21 1 21

Fig. 1.

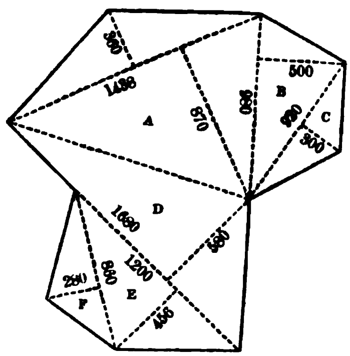


Fig. 2.

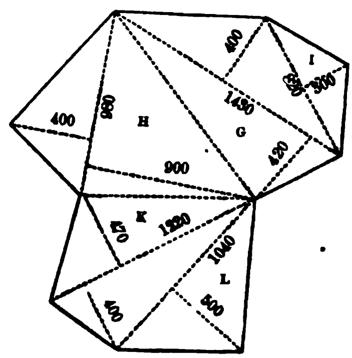
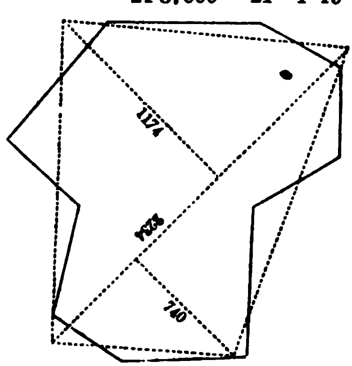


Fig. 2. is the same field as last, divided in a different manner, viz., into three trapeziums and two triangles.

The trapezium G,	Diagonal	1430 × 410, half sum of the perpendiculars, ...	5·86300
Do. H,	do.	980 × 650, half do.	6·37000
Do. K,	do.	1220 × 435, half do.	5·30700
The triangle I,	Base	820 × 150, half the perpendicular, ...	1·23000
Do. L,	do.	1040 × 250, half do.	2·60000

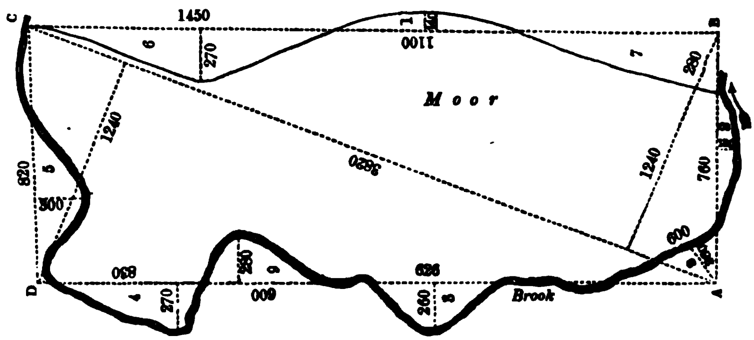
A. R. P.
21·37000 = 21 1 19

This figure represents the same field as the two last, and the contents ascertained by one calculation, by equalising the different sides into a square.



	A. Dec.	A. R. P.
The diagonal 2234 × 957, half sum of the perpendiculars, is	21·37938 =	21 1 20

This figure is a very irregular moor, having a brook or burn running round the greatest part of it.



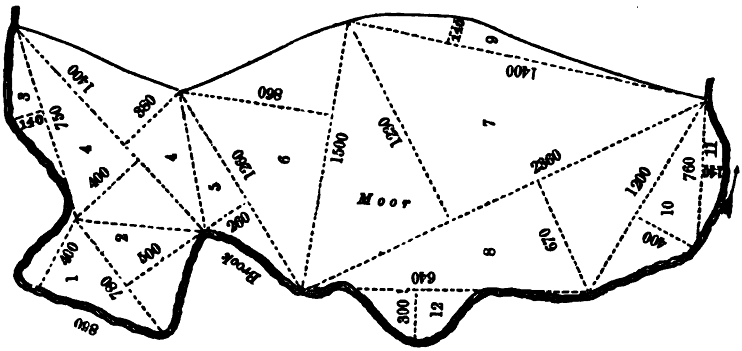
The contents are cast up or calculated by dividing it upon the plan into a rectangular figure, as A B C D, and casting up all the different triangles and corners separately, and that part of the land falling without the square to be added, and the vacant part within the square to be deducted.

The calculation will be thus :—

Amount of the square A B C D, Diagonal 3820 × 1240, half sum of the perpendiculars,				48·36800
Amount of triangle No. 1, Base	1100 × 70	half the perpendicular,	·77000	
Do. No. 2, do.	760 × 60	half do.	·45600	
Do. No. 3, do.	620 × 130	half do.	·80600	
Do. No. 4, do.	830 × 135	half do.	1·12050	
				3·15250
				51·52050
Deduct,				
Amount of triangle No. 5, Base	820 × 150	half the perpendicular,	1·23000	
Do. triangle No. 6, do.	1450 × 135	half do.	1·95750	
Do. No. 7, do.	1000 × 140	half do.	1·40000	
Do. No. 8, do.	600 × 130	half do.	·78000	
Do. No. 9, do.	830 + 135	half do.	1·12050	
		Deduct,		6·48800

		A.	R.	P.
Total amount of the moor,	45·03250 =	45	0	5

This figure is the same moor as represented in last, divided into eleven triangles and one trapezium.



Triangle	No. 1, Base	860 × 200 half the perpendicular is . .	1·72000	
Do.	No. 2, do.	780 × 250 half do.	1·95000	
Do.	No. 3, do.	750 × 70 half do.	·52500	
Trapezium	No. 4, Diagonal	1400 × 390 half sum of the perpendiculars, .	5·46000	
Triangle	No. 5, Base	1260 × 130 half the perpendicular, .	1·63800	
Do.	No. 6, do.	1500 × 430 half do.	6·45000	
Do.	No. 7, do.	2360 × 615 half do.	14·51400	
Do.	No. 8, do.	2360 × 335 half do.	7·90600	
Do.	No. 9, do.	1400 × 70 half do.	·98000	
Do.	No. 10, do.	1200 × 200 half do.	2·40000	
Do.	No. 11, do.	760 × 70 half do.	·53200	
Do.	No. 12, do.	640 × 150 half do.	·96000	
				A. R. P.
			45·03500	= 45 0 5·6

The figure on next page is the same moor as last two, with the contents calculated in a different way from any of the above-mentioned methods, viz., by drawing lines round it, which forms a quadrilateral E F G H, and calculated as other trapeziums by multiplying the diagonal by the half sum of the two perpendiculars; but there falls to be deducted all the land that lies betwixt the dotted lines E F and G H and the moor.

The calculation will be as under:—

Diagonal 4300		× 1540 half the perpendiculars, .		66·22800
Triangle	*a* Base	820 × 150 half its perpendicular, . .	1·23000	
Do.	*c* do.	1860 × 200 half do.	3·72000	
Do.	*b* do.	1860 × 300 half do.	5·58000	
		Carry over,	10·53000	66·22800

		Brought forward,	10·53000	66.22800
Triangle	*d* Base	300 × 40 half its perpendicular, . .	·12000	
Do.	*e* do.	250 × 55 half do.	·13750	
Do.	*f* do.	300 × 50 half do.	·15000	
Do.	*g* do.	770 × 160 half do.	1·23200	
Trapezium	*h* Diagonal	1550 × 190 half sum of the perpendiculars, .	2·94500	
Triangle	*i* Base	200 × 35 half the perpendicular, . .	·07000	
Do.	*k* do.	540 × 90 half do.	·48600	
Do.	*l* do.	1280 × 325 half do.	4·16000	
Do.	*m* do.	740 × 185 half do.	1·35900	
		Subtract		21·18950

		A.	R.	P.
Total,	45·03850 =	45	0	6

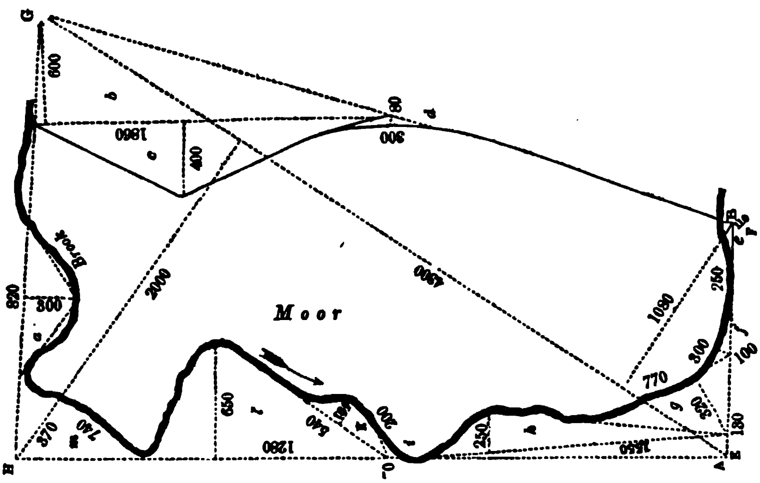


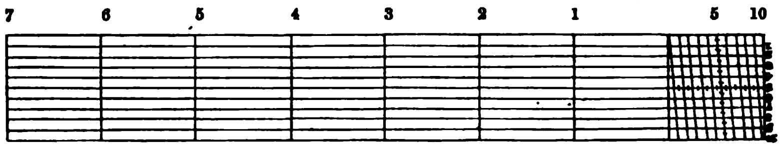
Plate IV. is a reduced plan of the farm of *Hardacres*, which is calculated as under. This will give the learner an idea how to arrange his rough draught before he inserts the contents into a finished plan.

			Diag. and Bases. × Perpendiculars.				A.	R.	P.
a	House & Yard		440 × 190	length and breadth . . .		·83600 =	0	3	14
b	Trapezium	D.	760 × 345	half sum of perpend.	2·62200				
	Do.	D.	1150 × 525	half do.	6·04750	8·96950 =	8	3	35
	Triangle	B.	500 × 60	half the perpend.	·30000				
c	Trapezium	D.	400 × 615	half sum of perpend.	8·61000	10·50000 =	10	2	0
	Triangle	B.	900 × 210	half the perpend.	1·89000				
d	Trapezium	D.	1610 × 545	half sum of perpend.	8·77450	10·96280 =	10	3	34
	Triangle	B.	1385 × 158	half the perpend.	2·18830				
e	Trapezium	D.	1800 × 710	half sum of perpend.	12·78000				
	Triangle	B.	1130 × 150	half the perpend.	1·65000	14·87000 =	14	3	19
	Do.	B.	440 × 100	half do.	44000				
				Carry forward,		46.13830	46	0	22

		Diag. and Bases.	Perpendiculars.						
				Brought forward,		46·13830 =	46	0	22
	Trapezium	D. 1510	× 540	half sum of perpend.		8·22950 =	8	0	37
g	Do.	D. 1600	× 490	half do.	7·84000	8·54000 =	8	2	6
g	Do.	D. 700	× 100	half do.	·70000				
h	Do.	D. 1590	× 716	half do.	11·24120	13·43120 =	13	1	29
h	Triangle	B. 1460	× 150	half the perpend.	2·19000				
	Trapezium	D. 1505	× 545	half sum of perpend.		8·20225 =	8	0	32
						84·54125 =	84	2	6

Various methods have been invented to facilitate and lessen the labour attending the calculation of the contents of land by different scales; but few deserve notice except the following:—It must be admitted that as few figures as possible ought to be used in the calculations, because, when the figures are complex, there is a greater liability to error; and no one can be certain his calculation is right without recalculating, which ought to be done in some other way from the former. If the two do not agree, a third must be resorted to, and sometimes a fourth. It has been observed before, that ten chains in length and one chain in breadth is one acre; when multiplied into one another is 100,000 square links, the square root of which is 316·22777 links, for the sides of a square acre to be delineated on a plan; and *Euclid*, Book 1, Prop. 47, demonstrates that the hypotenuse is equal to the sum of the squares of the two other sides; therefore the square of 316·22777 links is 100000 links; when doubled, is 200000 links; the square root of which is 447·2136 links for the diagonal of a square acre; that distance, taken from the scale the plan is protracted and plotted by, and made one of the large divisions of another scale, and very finely divided into a diagonal scale in 100 equal parts, forms a new scale for the diagonal of a square acre. The divisions, being larger than the plotting scale, in proportion as 447 exceeds 1000, this will give the area in fewer figures than the plotting scale; and in proportion as a square or parallelogram is to a triangle of the same base and altitude; consequently, if the diagonal and perpendiculars of any trapezium are measured upon a plan, or the base and perpendiculars of any triangle by this diagonal scale, and multiplied into one another, the product is the whole of the area, and saves the trouble of halving the perpendiculars. The perpendiculars have only to be added together, and their product multiplied by the whole length of the diagonal, which shows that a fewer number of figures are used to multiply into one another (to ascertain the area) than by the plotting scale.

This figure is a diagonal scale, constructed for calculating a plan plotted by a scale of seven chains to an inch. Suppose the meadow close in the farm of Tipperty, fig. page 94, which is plotted from



the scale just mentioned, and the calculation of enclosure No. 4. cast up by the new scale.

The Trapezium M B C L, Diagonal 409 × 288, the sum of the perpendiculars is	11·7792	
Triangle L M *a*, Base 328 × 80, the whole of the perpendicular, ...	2·6240	
		A. R. P.
	14·4032 =	14 1 24

In using this new scale, there requires only four decimals to be cut off upon the right hand, which multiply by 4 and then by 40, which converts the decimals into roods and perches in the same manner as formerly described. It will be proper to observe, that a scale of this kind is very useful to take off the diagonals and perpendiculars, to prove the calculations; but by no means will I recommend it as superior to the plotting scale the work has been laid down from, notwithstanding some land-measurers give it the preference on account of its expedition in casting up the contents in fewer figures than the scale the enclosure is plotted by.*

The first figure represents a meadow, which is bounded on one side by a brook or rivulet, and on the other side by a steep bank,

Fig. 1.

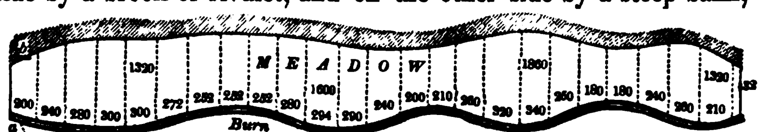


Fig. 2.

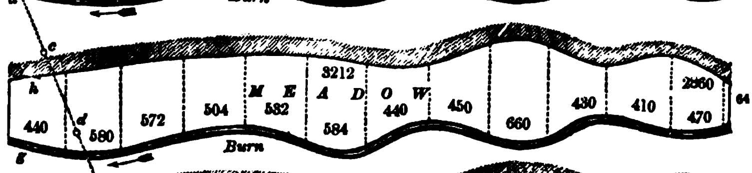
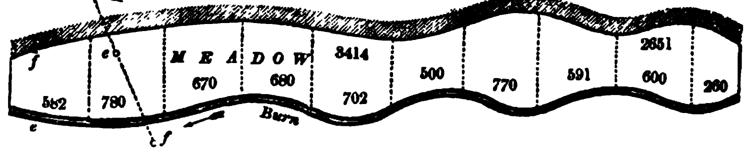


Fig. 3.



* There are several scales contrived for this purpose, as those used by the Tithe Commission, &c.

the lower part of which is very winding. To divide it into trapeziums and triangles, and calculate them all separately, would require more time than a practical surveyor might conveniently spare. The best and most expeditious method is to divide it into a number of equal distances, suppose each division a chain in breadth; then with a pair of compasses take the length of each division as near the middle as you can guess, and apply that extent to the same scale the meadow is plotted by, and note how far it measures upon the scale; insert the distance, which is 200, as you will perceive marked on the figure in the first column on the left hand; do the same with every division, and set down the distance as above in each respective column; then by a simple sum of addition the area is pointed out by cutting off three figures on the right hand; the figures on the left are acres. If the amount of the figures in the columns had been multiplied by the breadth, which is 100 links, then five figures must have been cut off. The total sum of all the lengths, when added, is 6 acres 102 decimals; but there is a small piece of ·032 decimals, which is cast up by multiplying the length by the breadth; when added to 6 acres 102 decimals, it makes the whole amount 6 acres 134 decimals = 6 acres 0 roods 21 perches. After multiplying the decimals ·134 by 4, and then by 40, there remains the odd 21 perches.

The second figure, last page, represents the same meadow as fig. 1, with the contents cast up by a method more expeditious, viz., by dividing it into equal distances of two chains each, and instead of ten chains being an acre, as fig. 2, five chains is a scale of acres, as may be seen by the annexed scale, fig. 2, p. 186, (this as well as the other is plotted from a scale of ¼th of an inch to a chain.) 1st, From the plotting scale take half an inch or two chains between the compasses, and with that extent divide the figure into equal distances; and if the last division is short, the length and breadth must be multiplied into one another, and the amount inserted in that column; then use a scale half the size of the plotting scale, and ten chains of it will be one acre; apply the compasses between the brook at *g* and *h* as near the middle of each column as you can guess, which upon the scale of acres of five chains is 440; place that number in its respective column; take off all the other squares or columns in the same way, and insert each distance in its respective column, as you will see marked on the figure; then copy all the distances, and add them up, which amounts to 6 acres 136 decimals = 6 acres 0 roods 21 perches.—*Note,* If all the distances had been taken from the plotting scale, and the sum multiplied by 2, it would have given the amount, except the small column, which is added after the multiplication by 2.

The third figure represents the same meadow as fig. 1 and fig. 2, in which the area is ascertained by a method still more expeditious than the former, and equally correct, by dividing it into 2½ chains, or 250 links each column; then every four chains or one inch is a scale of acres. This is a more ready scale than that of five chains. Apply the compasses, suppose from *e* to *f*, and note how much it measures upon the inch scale, which is 582; insert that distance in its respective column or square; do the same with all the other columns, and insert the distance of each in its respective column; add up all those distances, and the sum is 6 acres 125 decimals; multiply the decimals as before directed by 4, and then by 40, which is equal to 6 acres 0 roods 22 perches.

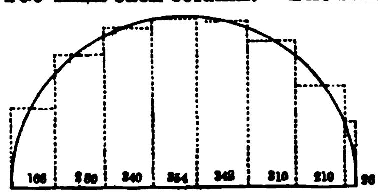
There is another method still shorter than any yet mentioned, particularly to a surveyor who can use the compasses expeditiously, by taking the extent of four, five, or six columns all at once, and applying that extent to the scale. For example, put one foot of the compasses at the side of the rivulet at *a*, fig. 1, p. 180, and extend the other foot up to *b*, and with that extent put one foot of the compasses at the side of the brook in the next column under 240, and the other foot next you will reach to *c*; keep that foot in *c*, and extend the other foot over the meadow to the bank in the second column; then with the same extent put one foot of the compasses at the edge of the rivulet in the third column under 280, and the point of the compasses that is next you will reach to *d*; keep that point of the compasses in *d*, and extend the other to the bank in the third square or column; then put one foot of the compasses at the edge of the brook under 300 in the fourth column, and the foot next you will reach to *e*; keep that point in *e*, and extend the other foot to the bank in the fourth column; then put one foot of the compasses at the edge of the brook in the fifth square or column, and the foot next you will reach to *f*; keep that foot in *f*, and extend the other foot over the meadow to the bank; then apply the compasses to the scale of acres, which is 1320, which insert in the fifth column; begin again at column sixth, under 272, and proceed in the same manner till you take in the length of six columns, which is 1600, which insert in the eleventh column; begin again at the edge of the rivulet in the twelfth column under 290, and proceed on to the eighteenth column, and see how far the compasses extend upon the scale, which is 1860, which insert in the eighteenth square; begin again in the nineteenth column under 250, and proceed in the same way as directed in the first five columns to the last, which extends to 1320 on the scale of acres; add up all the numbers, including the small piece of 32 at the end, and the sum will be 6·132 = 6 acres 0 roods 21 perches.

The calculations following are all considerably shorter than if the meadow had been divided into triangles and trapeziums, (which is the common method of finding out the contents. Figs. 2 and 3 might have been done in the same way as fig. 1.

The calculation of the meadow by four different methods, as under :—

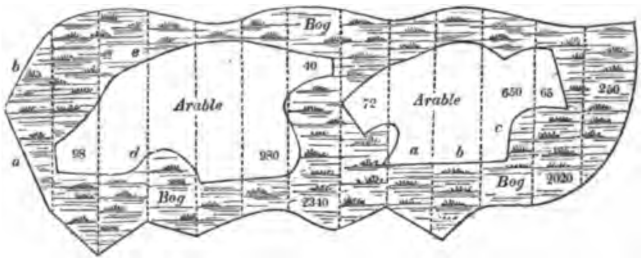
First Method refers to Fig. 1.	First Method continued.	Second Method refers to Fig. 2.	Third Method refers to Fig 3.	Fourth Method refers to Fig. 1.
1 Column 200	Brought up ......3·652	1 Column 440	1 Column 582	5 Column 1320
2 ... 240	15 Column 210	2 ... 580	2 ... 780	6 ... 1600
3 ... 280	16 ... 260	3 .. 572	3 ... 670	6 ... 1860
4 ... 300	17 ... 320	4 ... 504	4 ... 680	6 ... 1320
5 ... 300	18 ... 340	5 ... 532	5 ... 702	little piece 32
6 ... 272	19 ... 250	6 ... 584	6 ... 500	——
7 ... 252	20 ... 180	7 ... 440	7 ... 770	Total, 6·132
8 ... 252	21 ... 180	8 ... 450	8 ... 591	
9 ... 252	22 ... 240	9 ... 660	9 ... 600	
10 ... 280	23 ... 260	10 ... 430	little piece 260	
11 ... 294	24 ... 210	11 ... 410	——	
12 ... 290	little piece 32	12 ... 470	Total, 6·135	
13 ... 240	——	little piece 64		
14 ... 200	Total, 6·134	——		
——		Total, 6·136		
3·652				

This figure is a semicircle, the diameter of which is 720, and it is divided into equal distances of 100 links each column. The sum of the divisions when added, including a small segment of the circle of ·026 decimals or part of an acre, amounts to 2·034. Land-surveyors seldom meet with regular curved lines in practice; it is introduced here merely to show that the dividing of curved lines into equal distances may sometimes be made use of with success for ascertaining the area. Many surveyors seldom think of calculating the contents, but by dividing the arc as near as they can guess with their eye, either into a triangle, parallelogram, or trapezium, by including a part, and excluding another, so as to compensate. By this method, a person with a correct eye can attain exactness even beyond conception. Square the diameter 720 × 720, which is 51840 and



51840 × ·7854, as described in page 170, which amounts to 4·07151 for the area of a circle, the half of it is 2·03575 for the content of the semicircle. It would have turned out the same thing, in place of squaring the diameter, to have multiplied the half of it, which is 360, into the whole diameter: thus 720 × 360 is 259000, which again × ·7854 = 203575, or 2 acres 0 roods 5 falls.

This figure represents a bog meadow, with two pieces of dry arable land surrounded by the bog. The only correct and expe-



ditious method of ascertaining the area is by dividing it into distances of 100 links each, as in fig. 1, page 180, formerly explained. The parallel lines should be drawn in upon the rough plan, either with red ink or a black-lead pencil through the whole figure. It is presumed, from what has already been said, this method will appear very evident. It is done by taking a number of lengths of the different columns, and applying the whole extent to the plotting scale, 1000 then becomes an acre; you insert the number of acres and decimals of an acre in the columns, and add them up into one sum; then go over the two pieces of arable land in the same way, and if there are any corners or small pieces that do not occupy the whole breadth of a column, take the length and breadth of them, and multiply the length by the breadth, and insert the decimals in their proper places on the rough plan: when all is gone over, add each spot of the arable land together, and subtract the sum from the total.

Suppose you begin at *a*, and extend the foot of the compasses to *b*; then remove the compasses to the low end of the second column, and make a mark with the point next you; keep one foot of the compasses in that mark, and extend the other to the upper part of the column over both the bog and arable ground; do the same in every column (or square) till you take in the seventh square, and apply the extent between the compasses to the scale of acres, which is 2·340 acres, which insert in the column left off at; begin again in the next column, and take in five more of the divisions, and apply the extent to the scale of acres, which is 2·020 acres, which also insert in the column; lastly, take the length of the next column, which is 250 parts of an acre; add them all up into one sum, which is 4·610 acres. The next thing to be done is to take the length and breadth of the small square on the left in the arable land, which, when multiplied, the product is ·098, which insert; then take the length and breadth of the small piece on the right hand, and multiply them, and the product is ·040; add those distances to the amount of the other columns, which is ·980, and that piece of arable land is 1·118 acres; begin again on the other piece of arable land in the middle of the bog, and take the length and breadth of the left hand small piece, and multiply the length by the breadth, which is ·072 parts or decimals of an acre, which insert in that column; begin again at *a*, and extend the compasses over the arable land; then put one foot in *b*, and make a mark towards yourself, and extend the compasses from that mark to the far side of the arable land; then put one foot of the compasses in *c*, and let the other foot extend towards yourself, and make a mark; then extend the other foot over the arable land, and apply the compasses to the scale of acres, which is ·650 parts of an acre, which insert on your rough plan; then take the length and breadth of the small piece on the right hand and multiply them together, and the product is ·065 decimals of an acre; add all the three distances together, and the sum ·787 decimals of an acre; then add the amount of this piece of arable land to the other piece of arable land, and the sum is 1·905, which subtract out of the total sum, which is 4·610 acres, and there remains 2·715 acres of bog meadow, = 2 acres 2 roods 34 perches; and the sum of the two pieces of arable land is 1 acre 3 roods 25 perches.

From what has been stated, and the variety of methods pointed out of calculating the contents or areas of land, although some of them may at first view appear a little complicated to a learner, yet I entertain little doubt that, after a minute and strict examination of them, with a little practice and a careful inspection of the figures in this work, he will be enabled, in a short time, to cast up the contents with expedition and accuracy of regular as well as the most irregular fields to be met with in practice. Numerous valuable treatises have been published, but have seldom gone farther than illustrating straight lines, triangles, trapeziums, and figures of five, six, seven, or eight sides, and are in general deficient for giving a beginner a just idea of all the varieties he may expect to meet with in practice. It is well known that many estates are bounded by very crooked lines, curved boundaries, and many serpentine turns and windings are frequently met with in the interior parts of grounds. This has induced me to give rather a detailed account of one of the most useful parts of surveying. The equalising of irregular boundaries, to facilitate the calculations, and which has not been as yet thoroughly explained, will be found not only expeditious, but correct, even to a nicety, in reducing irregular boundaries to straight lines—which land-surveyors ought to make themselves masters of, particularly in dividing a crooked march or boundary betwixt the proprietors of two adjacent farms, who wish to have a straight boundary in preference to a zigzag one; and this a surveyor may sometimes have occasion to do.

This figure represents a scale of acres, where each division is 100 links or a chain wide. Ten of those divisions make 1 acre, twenty make 2 acres, thirty make 3 acres, and so on.

1

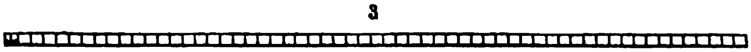
This figure represents a scale of acres, where each division is 200 links or 2 chains wide. Ten of the small divisions on the scale

2

then become an acre, twenty make 2 acres, thirty make 3 acres, and so on.

This figure represents a small scale of acres, where each division is 250 links wide; ten of the small divisions on the scale make 1 acre, twenty make 2 acres, and so on.

3

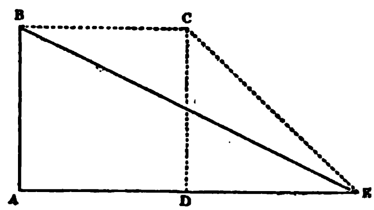


For example, take any length you please, suppose an inch and a half, between your compasses, and apply that extent to any of the scales of acres, suppose to the scale of four chains in an inch, which will reach to 600 decimals of an acre: or, if you apply the same extent to the scale of acres of 250 links wide, it will extend to 1500, which is equal to 1 acre and 500 decimals of an acre.

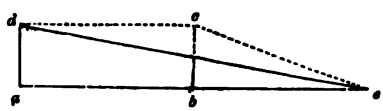
Again, if you apply the same extent to the scale of acres of 200 links wide, it will reach to 1225, or 1 acre and 225 decimals of an acre. These scales have been reduced to two-thirds of their original size.

ART. II.—EQUALISING DIFFERENT FIGURES TO REDUCE THEM TO TRIANGLES, &c.

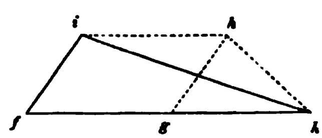
Suppose the four-sided square A B C D is to be reduced to a triangle whose area will be the same. 1*st*, Produce or extend one of the sides, suppose A D; then lay a parallel ruler or the T square with its companion upon the points B and D, and move it parallel to C; and where the edge of the ruler crosses the produced line, make a mark as at E; then draw the line E B, and it is done. It would be the same thing if you pricked off upon the produced line the length of one of the sides, suppose A D, by putting one foot of the compasses in D, the other will extend to E; draw the line E B, and it will be the same as the former.



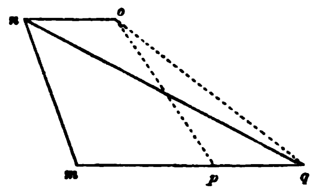
Suppose the four-sided figure *a b c d* is to be reduced to a triangle whose area will be equal to the parallelogram. 1*st*, Produce the line *a b*, lay the T square upon the points *d* and *b*, and move it parallel to *c*, and make a mark upon the produced line *a b* at *e*; lastly, draw the line *e d*, and it is done. Or you may take the length of *a b*, and put one foot of the compasses in *b*, and the other will reach to *e*, on the produced line; then draw the hypotenuse as before.



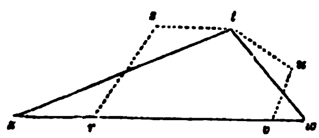
Suppose the four-sided figure (called a rhomboid) is to be reduced to a triangle, whose area will be equal to the figure *f g h i*; extend the line *fg*, and lay the T square upon the points *g* and *i*; move it parallel to *h*, and make a mark where the edge of the T or parallel ruler cuts the produced line *fg*, and make a mark upon the produced line at *k*; then draw the line *k i*, which will be the same area as the rhomboid.



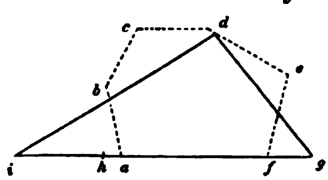
Let the figure *m n o p* be reduced to a triangle whose area will be equal to the trapezium. 1*st*, Extend the line *m p;* then lay the ruler upon the points *p* and *n*, and move the T or a parallel ruler parallel to *o*, and make a mark where the edge cuts the produced line at *q;* then draw the hypotenuse from *q* to *n*, and it is finished.



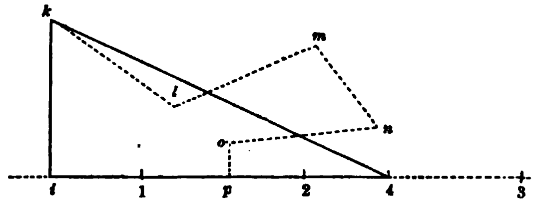
Let the figure *r s t u v*, of five sides, be reduced to a triangle whose area will be the same as the figure. 1*st*, Produce the line *v r* both ways; that is to say, past *r* on the left and past *v* on the right; then lay the T upon *r* and *t*, and move it parallel to *s*, ard make a mark upon the produced line on the left at *x;* then lay the T upon the points *v* and *t*, move it parallel to *u*, and make a mark on the produced line on the right at *w;* then draw the hypotenuse line *x t*, and also the line *t w;* and the triangle will be equal to the five-sided figure *r s t u v*.



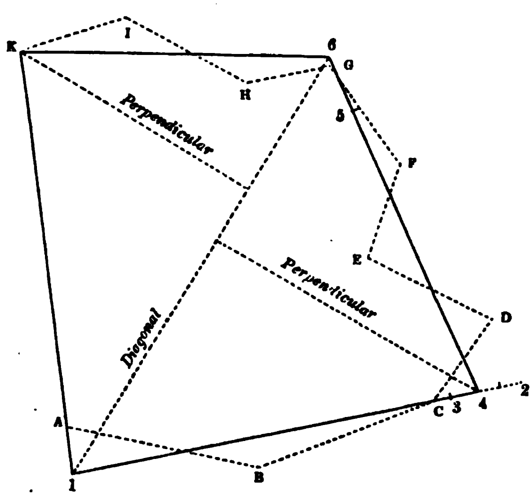
Let the six-sided figure *a b c d e f* be reduced to a triangle of the same dimensions or area. 1*st*, Produce or extend the base *a f* to the left of *a* and to the right of *f*, then lay the parallel ruler or T upon the angles *a* and *c*, and move it parallel to *b*, and make a mark upon the produced line on the left of *a* at *h;* then lay the ruler upon the mark at *h* and the point *d*, and move it forward to the point *c*, and where the edge of the ruler cuts the produced line make a mark, at *i;* then lay the parallel ruler or T upon the point *f* and *d*, and move it parallel to *e;* and where the edge of the ruler cuts the extended line on the right of *f* make a mark at *g;* lastly, draw in the hypotenuse from *i* to *d* and from *d* to *g*, and extend the base from *f* to *g* and from *a* to *i*, and it is finished.



This is a seven-sided figure, *i k l m n o p*. First extend the base *i p* a considerable way on the right past *p*, and apply the ruler or the T to *p* and *n;* move it parallel to *o*, and where the edge of the ruler cuts the base line at 1 make a mark; then lay the parallel ruler upon the mark 1 and the angle at *m* and move it parallel to *n*; and make a mark where the edge of the ruler crosses the extended line at 2; then lay the ruler upon the mark 2 and the angle *l*, and move it parallel to *m*, and where the edge cuts the produced line, make a mark at 3: then lay the ruler upon the mark 3 and the angle *k*, and move it parallel to *l*; make a mark upon the extended line at 4; then draw the hypotenuse from *k* to the mark at 4, and the triangle is formed; which reduces the seven-sided figure *iklmnop* to a three-sided one whose content is the same, and may be calculated, as all other triangles are done, by multiplying the hypotenuse by half the perpendicular let fall thereon from the opposite angle *l*.

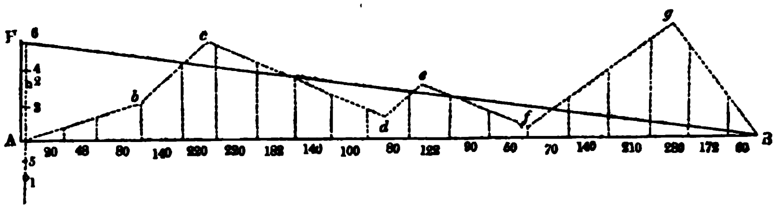


This figure is an irregular field of ten sides to be reduced to a four-sided figure. First produce the line K A past A; then lay the parallel ruler or the T square upon the points A and C, and move it parallel to B; where the edge of the T crosses the produced line, make a mark as at 1; then draw a line from the mark 1 through the angle C, which extend past C; then lay the T upon C and E, and move it parallel to D; make a mark at 2, on the extended line 1 C; then lay the T upon the mark 2 and the angle F, and move it parallel to E, and where the edge cuts the extended line make a mark at 3,



on the produced line 1 C ; then lay the parallel ruler or T upon the mark 3 and the angle G, and move it parallel to the angle F, and where the edge cuts the extended line 1 C make a mark at 4 ; then draw the line from the mark 4 through the angle G, which produce past G ; this will reduce the four sides, CD, DE, EF, and FG, to one side ; then apply the T to the angles G and I, and move it parallel to H, and where the edge cuts the line 4 G make a mark as at 5 ; then lay the ruler upon the mark 5 and the angle K, and move it parallel to the angle I, and where the edge cuts the produced line 4 G make a mark at 6 ; then draw the line from 6 to K and the short line from A to 1, which reduces the ten-sided figure to one of four sides, which is calculated by dividing it into 2 triangles, or one trapezium, as is drawn in on the figure by a diagonal and perpendiculars let fall thereon.

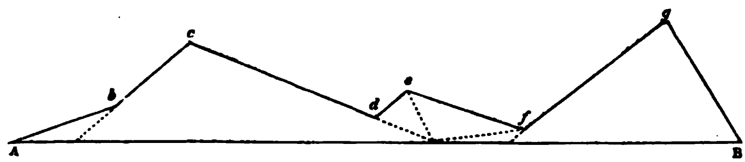
This figure represents a very irregular boundary, which it is proposed to reduce to one mean line by equalising the different sides.



1st, Extend the line H A below A ; then lay the parallel ruler upon the points A and *c*, and move it parallel to *b* ; where the edge of the ruler cuts the produced line below A make a mark as 1 ; then lay the ruler upon the mark at 1 and the angle *d*, move it parallel to *c*, and make a mark at 2 on the dotted line A H ; then lay the ruler upon the mark 2 and the angle *e*, move it parallel to *d*, and make a mark upon the dotted line at 3 ; then lay the ruler upon the mark 3 and the angle *f*, move it parallel to *e*, and make a mark upon the dotted line at 4 ; then lay the ruler upon the mark 4 and the angle at *g*, and move it parallel to *f*, making a mark at 5 on the produced line below A ; then lay the ruler upon the point 5 and B, and move it parallel to the angle *g*, and make a mark upon the dotted line at 6 ; then draw the line through the mark 6 to B, which equalises the boundary line of 7 sides to one line ; and the triangle may be cast up by one calculation, which will amount to 2 acres 1 rood 29 perches.

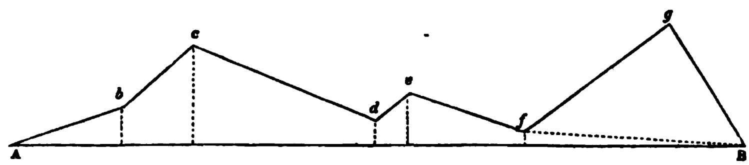
Another method may be used for ascertaining the contents of the above figure, which is, by dividing it into equal divisions of 100 links wide. In that case, 10 chains in length is one acre; the length of each column is set down under it in decimals of an acre; when they are added together, the sum amounts to 2.424, which is equal to 2 acres 1 rood 28 perches. The method of dividing irregular fields, like the above, by equal distances, is particularly described in Areas; which will be found the most expeditious method of finding the contents of such irregular boundaries as this figure.

This is the same irregular figure of 8 sides divided into six triangles, which are all calculated separately, and which, when



summed up, amount to 2 acres 1 rood 28 perches, the same as the other, which is calculated by equal divisions of 100 links each.

This figure represents another method of calculating the same figure, namely, by dividing into three triangles and four trapeziums.



The content turns out much the same as the others when the different calculations are added up in one sum.

There are other methods than those that have already been defined for the calculations of triangles, trapeziums, squares, and irregular figures, to find out the contents; but none of them deserve notice except the following:

Many surveyors draw lines with red ink all over their rough plans, squares of equal distances, each of which they make 316$\frac{2}{10}$ links each side; then each square becomes an acre, which they number; and from these calculate all the small pieces, which they either add or deduct according as they are either within or without the squares. If the amount of the whole corresponds nearly to a former calculation, they rest satisfied that their calculation is right. If they do not agree, the surveyor examines the different fields by the number of squares in each field till he finds out his error, and then recalculates that field over again where a wrong calculation has been committed.

A good method of casting up the contents and proving the calculations of triangles, trapeziums, squares, or irregular figures, is by a table of logarithms, which is done in the following manner :

Suppose the square, fig. page 168, each side of which is 800 links,—

Look for that number in the left hand column in a book of logarithms, and opposite it is	2.90309
Look again for the same number, 800, ...	2.90309
The sum is, ...	5.80618

The index being five denotes that you have six figures in the sum, and if the index had been six, there would have been seven figures in the sum, and so on; the index always reckoning one figure less than the sum.

Then look for the above logarithm 5.80618, paying no regard to the index 5, and opposite .80618 in the left hand column under *numb.* you will find 6400; to which add 2 cyphers, the index being 5 gives 6.40000 * = 6 acres 1 rood 24 perches.

Again, suppose the parallelogram Plate II. fig. 7, which is 1167 links long and 305 wide;

Look what logarithm is opposite 1167, and you will find .	3.06707
Look what logarithm is opposite 305, and you will find .	2.48430
	5.55137

Then look what number is opposite the logarithm of .55137, paying no regard to the index 5, which denotes that there are 6 figures in the sum; the nearest number to the logarithm of 5.55137 is 3559; to which add 2 cyphers, and cut off 5 figures on the right hand, and there remains 3.55900 = 3 acres 2 roods 9 perches.

* Since the *logarithm* of 100000 square links, the content of an acre, is 5.0000000 it has been the practice of the editor in such calculations to reject 5 from the index; that is, dividing by 100000, the remaining logarithm will give the area in acres and decimals. Lalande's small collection of tables, bound up with a tuck, is the most convenient for this purpose from its portability.

Again, suppose the diagonal of a trapezium to be 2200 links, and the half sum of the perpendiculars 840:

Look what logarithm is opposite 2200, and you will find .	3.34242
Again look what logarithm is opposite 840, and you will find .	2.92428
	6.26670

Then look what number is opposite .26670, and you will find 1848; to which add 3 cyphers. The index being 6 denotes that 7 figures are in the sum; and when 5 figures are cut off on the right, there remain 18 acres, 48000 decimals = 18 acres 1 rood 37 perches nearly.

It sometimes occurs that the logarithm of a number is not exactly found in the tables. In that case, look for the next less logarithm, and subtract it from the given logarithm; then look for the next greater logarithm, and subtract the next less one from it; then divide it by the difference of the greater and less logarithm found in the table. Thus, suppose the diagonal of a trapezium to be 8748, and the half sum of the perpendicular 4644;

Look for the logarithm opposite 8748 links, and opposite it is	3.94191
Then look for the logarithm opposite 4644 links, and opposite it is	3.66689
Given logarithm, . . .	7.60880

Then look what number is opposite the logarithm of .60880, which is 4062, being 4 of the figures sought; the index being 7, shows that 8 figures are in the sum.

The logarithm being not exactly found in the table, you must subtract the next less logarithm from the given logarithm; thus—

The given logarithm is		60880
The next less is		60874
The difference is 6 for the dividend, ...		6
To which add 4 cyphers, ...		6.0000
Then the next larger logarithm is .	60885	
Subtract the next less logarithm, .	60874	

Divide by the remainder 11)6.0000(5454, annex this sum to 4062 and it will be 406.25454 = 406 acres 1 rood 1 perch nearly.

It is unnecessary to carry the decimals so far in practice.

When the side of a square, the sides of a rectangle, or the base and perpendicular of a triangle, are given to find the area, logarithms are almost unnecessary. Their chief advantage arises when the three sides, two sides and contained angle, or two angles and adjacent side of triangles, are given to find the area.

1. To find the area of a triangle when the three sides are given.

Rule.—Add the three sides, and take half the sum. From the half sum subtract each side successively, then multiply the half sum and the three remainders together; the square root of the product will be the area of the triangle.

In using logarithms, add the logarithm of the half sum and those of the three remainders together; half the sum of these four logarithms will be the logarithm of the area.

2. To find the area of a triangle when two sides and the contained angle are given.

Rule.—Multiply the natural sine of the contained angle by each of the sides successively; half the product will be the area.

By logarithms. To the constant logarithm of ½ or 0·5, add the logarithmic sine of the contained angle and the logarithms of the sides; the sum will be the logarithm of the area.

3. To find the area of a triangle when the two angles and interjacent side are given.

Rule.—Multiply the sines of the two angles together, the cosecant of the sum and the product by the square of the interjacent side; half the continued product will be the area.

By logarithms. To the constant logarithm of ½ or 0·5 add the logarithmic sines of the given angles, the logarithm cosecant of the sum and twice logarithm of the interjacent side; the sum will be the logarithm of the area in square measure, of the same name as that of the lineal in which the sides were taken.

Example 1. Given the three sides of a triangle, A B C, namely, A B=1900 links, B C=1230, and A C=1620; required the area?

A B = 1900	Half sum	2375	2375	2375
A C = 1620	A B =	1900	A C = 1620	B C = 1230
B C = 1230		——	——	——
——	1st diff.	475	2d = 755	3d = 1145
Sum = 4750				
Half = 2375				

2375 × 475 × 755 × 1145 = 975,235,859,375, of which the square root is 987,540 square links, nearly the area required.

Dividing 987,540 by 100,000, the number of square links in an acre, which is done by pointing off five figures from the right as decimals; those at the left are integers.

The decimals may be converted into roods by multiplying by 4, and the remaining decimal reduced to poles by multiplying by 40; and, finally, if thought necessary, the last decimal may be reduced to square feet by multiplying by 30¼. Thus—

```
9.87540
      4
-------
3.50160
     40
-------
20.06400
     30¼
--------
  192000
    1600
--------
 1.93600
```

Hence the area is 9 A. 3 R. 20 P. and 1.936 square yards.

BY LOGARITHMS—

AB = 1900 = c
AC = 1620 = b
BC = 1230 = a

$2s$ = 4750

s = 2375 log.	. . . . . .	3.375664
$s-c$ = 475 —	. . . . . .	2.676694
$s-b$ = 755 —	. . . . . .	2.877947
$s-a$ = 1145 —	. . . . . .	3.058805
	Sum, rejecting 10 =	1.989110
Area, 9.8754 acres,	. . . . . . log.	0.994555 half.

Example 2.—Given the side A B=667 links, the side A C=866 links, and the contained angle B A C = 85° 43′ 30″, required the area in acres.

Here the result is obtained by dividing by 100,000 and by 2, or by 200,000 at once, as is easily inferred from the rule. But, instead of subtracting the log. of 200,000, its arithmetical complement may be added from the nature of logarithms. The log. of 2,000,000 is 5·3010300, which, subtracted from 10, leaves 4.6989700, the arithmetical complement.

Constant logarithm,		4.6989700
Contained angle B A C = 85° 48′ 30″	log. sine.	9.9987900
A B = 667 links,	log.	2.8241258
A C = 866 links,	log.	2.9375179
Area = 2.880074 in acres (sum rejecting 20) =		0.4594037

Converting, if necessary, the decimal into roods and poles, the whole will be 2 A. 3 R. 20.8 poles.

Example 3.—Let the two angles of a triangle be A = 74° 58′ 18″, the angle B = 59° 5′ 54″, and the interjacent side A B = 2574 links; required the area in acres.

Constant logarithm as before,	4.6989700
A = 74° 58′ 18″ log. sine,	9.9848862
B = 59 5 54 log. sine,	9.9335126
A + B = 134 4 12 cosecant,	0.1435789
A B = 2574 log. × 2 =	6.8212170
Area = 38.209 acres, log.,	1.5821647
= 38 A. 0 R. 33.44 P.	

The following tables for turning perches into acres, roods, and perches, poles, or falls, by inspection, are frequently made use of for finding out where a mistake may have been made in the first calculation. It is done by making a scale of perches answering to the plotting scale by which the plan has been protracted and laid down, which will come very near the truth. For example, a perch-pole or fall is 25 links: take 2500 links from your plotting scale, and divide that length into 10 equal divisions; each of those divisions will be 10 perches; then divide either the right or left hand division into 10 equal parts, and each of those divisions will be one perch in length or breadth. With a pair of compasses take off the length of the base of a triangle, and apply it to the above scale of perches, which will show how many perches it is in length: take half the length of the perpendicular and do the same. Suppose the length of the base is 50 perches, and the half of the perpendicular is 9 perches or falls, look for 50 in the left-hand column, and for 9 at the top; then with your eye trace the line of 50 till you are immediately under 9 at the top, and in that column you will find 2 acres 3 roods and 10 perches.

Again, suppose you have a trapezium, the diagonal of which is

220 perches in length, and the half sum of the perpendicular is 80 perches—

	A.	R.	P.
First look out for 200 in the left-hand column, and opposite it, under 80, is	100	0	0
Then look out for 20 in the left-hand column, and under 80 you will find	10	0	0
The sum is the amount, . .	110	0	0

Again, suppose you have a triangle the base of which is 19 perches and a half, and the half of the perpendicular is 8 perches—

	A.	R.	P.
Look out for 19 perches in the left-hand column, and under 8 is	0	3	32
Then for the half perch look for 1 at the top, and take the half of 8, which is 4, in the left-hand column, and you will find	0	0	4
	0	3	36

All others are done in the same manner, and need no farther illustration.

The following tables and scale will be found useful to gentlemen that have plans of their property, if they wish to know the quantity of ground in any divisions they have made or intend to make on their grounds.

FROM 1 TO 8 PERCHES, POLES, OR FALLS WIDE.

Perches, Poles, or Falls long.	1 Do. wide.	2 Do. wide.	3 Do. wide.	4 Do. wide.	5 Do. wide.	6 Do. wide.	7 Do. wide.	8 Do. wide.
	A R P	A R P.	A R P	A R P	A R P	A R P	A R P	A R P
1	0 0 1	0 0 2	0 0 3	0 0 4	0 0 5	0 0 6	0 0 7	0 0 8
2	0 0 2	0 0 4	0 0 6	0 0 8	0 0 10	0 0 12	0 0 14	0 0 16
3	0 0 3	0 0 6	0 0 9	0 0 12	0 0 15	0 0 18	0 0 21	0 0 24
4	0 0 4	0 0 8	0 0 12	0 0 16	0 0 20	0 0 24	0 0 28	0 0 32
5	0 0 5	0 0 10	0 0 15	0 0 20	0 0 25	0 0 30	0 0 35	0 1 0
6	0 0 6	0 0 12	0 0 18	0 0 24	0 0 30	0 0 36	0 1 2	0 1 8
7	0 0 7	0 0 14	0 0 21	0 0 28	0 0 35	0 1 2	0 1 9	0 1 16
8	0 0 8	0 0 16	0 0 24	0 0 32	0 1 0	0 1 8	0 1 16	0 1 24
9	0 0 9	0 0 18	0 0 27	0 0 36	0 1 5	0 1 14	0 1 23	0 1 32
10	0 0 10	0 0 20	0 0 30	0 1 0	0 1 10	0 1 20	0 1 30	0 2 0
11	0 0 11	0 0 22	0 0 33	0 1 4	0 1 15	0 1 26	0 1 37	0 2 8
12	0 0 12	0 0 24	0 0 36	0 1 8	0 1 20	0 1 32	0 2 4	0 2 16
13	0 0 13	0 0 26	0 0 39	0 1 12	0 1 25	0 1 38	0 2 11	0 2 24
14	0 0 14	0 0 28	0 1 2	0 1 16	0 1 30	0 2 4	0 2 18	0 2 32
15	0 0 15	0 0 30	0 1 5	0 1 20	0 1 35	0 2 10	0 2 25	0 3 0
16	0 0 16	0 0 32	0 1 8	0 1 24	0 2 0	0 2 16	0 2 32	0 3 8
17	0 0 17	0 0 34	0 1 11	0 1 28	0 2 5	0 2 22	0 2 39	0 3 16
18	0 0 18	0 0 36	0 1 14	0 1 32	0 2 10	0 2 28	0 3 6	0 3 24
19	0 0 19	0 0 38	0 1 17	0 1 36	0 2 15	0 2 34	0 3 13	0 3 32
20	0 0 20	0 1 0	0 1 20	0 2 0	0 2 20	0 3 0	0 3 20	1 0 0
30	0 0 30	0 1 20	0 2 10	0 3 0	0 3 30	1 0 20	1 1 10	1 2 0
40	0 1 0	0 2 0	0 3 0	1 0 0	1 1 0	1 2 0	1 3 0	2 0 0
50	0 1 10	0 2 20	0 3 30	1 1 0	1 2 10	1 3 20	2 0 30	2 2 0
60	0 1 20	0 3 0	1 0 20	1 2 0	1 3 20	2 1 0	2 2 20	3 0 0
70	0 1 30	0 3 20	1 1 10	1 3 0	2 0 30	2 2 20	3 0 10	3 2 0
80	0 2 0	1 0 0	1 2 0	2 0 0	2 2 0	3 0 0	3 2 0	4 0 0
90	0 2 10	1 0 20	1 2 30	2 1 0	2 3 10	3 1 20	3 3 30	4 2 0
100	0 2 20	1 1 0	1 3 20	2 2 0	3 0 20	3 3 0	4 1 20	5 0 0
200	1 1 0	2 2 0	3 3 0	5 0 0	6 1 0	7 2 0	8 3 0	10 0 0
300	1 3 20	3 3 0	5 2 20	7 2 0	9 1 20	11 1 0	13 0 20	15 0 0
400	2 2 0	5 0 0	7 2 0	10 0 0	12 2 0	15 0 0	17 2 0	20 0 0
500	3 0 20	6 1 0	9 1 20	11 2 0	15 2 20	18 3 0	21 3 20	25 0 0
600	3 3 0	7 2 0	11 1 0	15 0 0	18 3 0	22 2 0	26 1 0	30 0 0

FROM 9 TO 16 PERCHES, POLES, OR FALLS WIDE.

Perches long.	9 Do. wide.	10 Do. wide.	11 Do. wide.	12 Do. wide.	13 Do. wide.	14 Do. wide.	15 Do. wide.	16 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 0 9	0 0 10	0 0 11	0 0 12	0 0 13	0 0 14	0 0 15	0 0 16
2	0 0 18	0 0 20	0 0 22	0 0 24	0 0 26	0 0 28	0 0 30	0 0 32
3	0 0 27	0 0 30	0 0 33	0 0 36	0 0 39	0 1 2	0 1 5	0 1 8
4	0 0 36	0 1 0	0 1 4	0 1 8	0 1 12	0 1 16	0 1 20	0 1 24
5	0 1 5	0 1 10	0 1 15	0 1 20	0 1 25	0 1 30	0 1 35	0 2 0
6	0 1 14	0 1 20	0 1 26	0 1 32	0 1 38	0 2 4	0 2 10	0 2 16
7	0 1 23	0 1 30	0 1 37	0 2 4	0 2 11	0 2 18	0 2 25	0 2 32
8	0 1 32	0 2 0	0 2 8	0 2 16	0 2 24	0 2 32	0 3 0	0 3 8
9	0 2 1	0 2 10	0 2 19	0 2 28	0 2 37	0 3 6	0 3 15	0 3 24
10	0 2 10	0 2 20	0 2 30	0 3 0	0 3 10	0 3 20	0 3 30	1 0 0
11	0 2 19	0 2 30	0 3 1	0 3 12	0 3 23	0 3 34	1 0 5	1 0 16
12	0 2 28	0 3 0	0 3 12	0 3 24	0 3 36	1 0 8	1 0 20	1 0 32
13	0 2 37	0 3 10	0 3 23	0 3 36	1 0 9	1 0 22	1 0 35	1 1 8
14	0 3 6	0 3 20	0 3 34	1 0 8	1 0 22	1 0 36	1 1 10	1 1 24
15	0 3 15	0 3 30	1 0 5	1 0 20	1 0 35	1 1 10	1 1 25	1 2 .0
16	0 3 24	1 0 0	1 0 16	1 0 32	1 1 8	1 1 24	1 2 0	1 2 16
17	0 3 33	1 0 10	1 0 27	1 1 4	1 1 21	1 1 38	1 2 15	1 2 32
18	1 0 2	1 0 20	1 0 38	1 1 16	1 1 34	1 2 12	1 2 30	1 3 8
19	1 0 11	1 0 30	1 1 9	1 1 28	1 2 7	1 2 26	1 3 5	1 3 24
20	1 0 20	1 1 0	1 1 20	1 2 0	1 2 20	1 3 0	1 3 20	2 0 0
30	1 2 30	1 3 20	2 0 10	2 1 0	2 1 30	2 2 20	2 3 10	3 0 0
40	2 1 0	2 2 0	2 3 0	3 0 0	3 1 0	3 2 0	3 3 0	4 0 0
50	2 3 10	3 0 20	3 1 30	3 3 0	4 0 10	4 1 20	4 2 30	5 0 0
60	3 1 20	3 3 0	4 0 20	4 2 0	4 3 20	5 1 0	5 2 20	6 0 0
70	3 3 30	4 1 20	4 3 10	5 1 0	5 2 30	6 0 20	6 2 10	7 0 0
80	4 2 0	5 0 0	5 2 0	6 0 0	6 2 0	7 0 0	7 2 0	8 0 0
90	5 0 10	5 2 20	6 0 30	6 3 0	7 1 10	7 3 20	8 1 30	9 0 0
100	5 2 20	6 1 0	6 3 20	7 2 0	8 0 20	8 3 0	9 1 20	10 0 0
200	11 1 0	12 2 0	13 3 0	15 0 0	16 1 0	17 2 0	18 3 0	20 0 0
300	16 3 20	18 3 0	20 2 20	22 2 0	24 1 20	26 1 0	28 0 20	30 0 0
400	22 2 0	25 0 0	27 2 0	30 0 0	32 2 0	35 0 0	37 2 0	40 0 0
500	28 0 20	31 1 0	34 1 20	37 2 0	40 2 20	43 3 0	46 3 20	50 0 0
600	33 3 0	37 2 0	41 1 0	45 0 0	48 3 0	52 2 0	56 1 0	60 0 0

FROM 17 TO 24 PERCHES, POLES, OR FALLS WIDE.

Perches long.	17 Do. wide.	18 Do. wide.	19 Do. wide.	20 Do. wide.	21 Do. wide.	22 Do. wide.	23 Do. wide.	24 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 0 17	0 0 18	0 0 19	0 0 20	0 0 21	0 0 22	0 0 23	0 0 24
2	0 0 34	0 0 36	0 0 38	0 1 0	0 1 2	0 1 4	0 1 6	0 1 8
3	0 1 11	0 1 14	0 1 17	0 1 20	0 1 23	0 1 26	0 1 29	0 1 32
4	0 1 28	0 1 32	0 1 36	0 2 0	0 2 4	0 2 8	0 2 12	0 2 16
5	0 2 5	0 2 10	0 2 15	0 2 20	0 2 25	0 2 30	0 2 35	0 3 0
6	0 2 22	0 2 28	0 2 34	0 3 0	0 3 6	0 3 12	0 3 18	0 3 24
7	0 2 39	0 3 6	0 3 13	0 3 20	0 3 27	0 3 34	1 0 1	1 0 8
8	0 3 16	0 3 24	0 3 32	1 0 0	1 0 8	1 0 16	1 0 24	1 0 32
9	0 3 33	1 0 2	1 0 11	1 0 20	1 0 29	1 0 38	1 1 7	1 1 16
10	1 0 10	1 0 20	1 0 30	1 1 0	1 1 10	1 1 20	1 1 30	1 2 0
11	1 0 27	1 0 38	1 1 9	1 1 20	1 1 31	1 2 2	1 2 13	1 2 24
12	1 1 4	1 1 16	1 1 28	1 2 0	1 2 12	1 2 24	1 2 36	1 3 8
13	1 2 21	1 2 34	1 3 7	1 3 20	1 2 33	1 3 6	1 3 19	1 3 32
14	1 1 38	1 2 12	1 2 26	1 3 0	1 3 14	1 3 28	2 0 2	2 0 16
15	1 2 15	1 2 30	1 3 5	1 3 20	1 3 35	2 0 10	2 0 25	2 1 0
16	1 2 32	1 3 8	1 3 24	2 0 0	2 0 16	2 0 32	2 1 8	2 1 24
17	1 3 9	1 3 26	2 0 3	2 0 20	2 0 37	2 1 14	2 1 13	2 2 8
18	1 3 26	2 0 4	2 0 22	2 1 0	2 1 18	2 1 36	2 2 14	2 2 32
19	2 0 3	2 0 22	2 1 1	2 1 20	2 1 39	2 2 18	2 2 37	2 3 16
20	2 0 20	2 1 0	2 1 20	2 2 0	2 2 20	2 3 0	2 3 20	3 0 0
30	3 0 30	3 1 20	3 2 10	3 3 0	3 3 30	4 0 20	4 1 10	4 2 0
40	4 1 0	4 2 0	4 3 0	5 0 0	5 1 0	5 2 0	5 3 0	6 0 0
50	5 1 10	5 2 20	5 3 30	6 1 0	6 2 10	6 3 20	7 0 30	7 2 0
60	6 1 20	6 3 0	7 0 20	7 2 0	7 3 20	8 1 0	8 2 20	9 0 0
70	7 1 30	7 3 20	8 1 10	8 3 0	9 0 30	9 2 20	10 0 10	10 2 0
80	8 2 0	9 0 0	9 2 0	10 0 0	10 2 0	11 0 0	11 2 0	12 0 0
90	9 2 10	10 0 20	10 2 30	11 1 0	11 3 10	12 1 20	12 3 30	13 2 0
100	10 2 20	11 1 0	11 3 20	12 2 0	13 0 20	13 3 0	14 1 20	15 0 0
200	21 1 0	22 2 0	23 3 0	25 0 0	26 1 0	27 2 0	28 3 0	30 0 0
300	31 3 20	33 3 0	35 2 20	37 2 0	39 1 20	41 1 0	43 0 20	45 0 0
400	42 2 0	45 0 0	47 2 0	50.0 0	52 2 0	55 0 0	57 2 0	60 0 0
500	53 0 20	56 1 0	59 1 20	62 2 0	65 2 20	68 3 0	71 3 20	75 0 0
600	63 3 0	67 2 0	71 1 0	75 0 0	78 3 0	82 2 0	86 1 0	90 0 0

FROM 25 TO 32 PERCHES, POLES, OR FALLS WIDE.

Perches long.	25 Do. wide.	26 Do. wide.	27 Do. wide.	28 Do. wide.	29 Do. wide.	30 Do. wide.	31 Do. wide.	32 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 0 25	0 0 26	0 0 27	0 0 28	0 0 29	0 0 30	0 0 31	0 0 32
2	0 1 10	0 1 12	0 1 14	0 1 16	0 1 18	0 1 20	0 1 22	0 1 24
3	0 1 35	0 1 38	0 2 1	0 2 4	0 2 7	0 2 10	0 2 13	0 2 16
4	0 2 20	0 2 24	0 2 28	0 2 32	0 2 36	0 3 0	0 3 4	0 3 8
5	0 3 5	0 3 10	0 3 15	0 3 20	0 3 35	0 3 30	0 3 35	1 0 0
6	0 3 30	0 3 36	1 0 2	1 0 8	1 0 14	1 0 20	1 0 26	1 0 32
7	1 0 15	1 0 22	1 0 29	1 0 36	1 1 3	1 1 10	1 1 17	1 1 24
8	1 1 0	1 1 8	1 1 16	1 1 24	1 1 32	1 2 0	1 2 8	1 2 16
9	1 1 25	1 1 34	1 2 3	1 2 12	1 2 21	1 2 30	1 2 39	1 3 8
10	1 2 10	1 2 20	1 2 30	1 3 0	1 3 10	1 3 20	1 3 30	2 0 0
11	1 2 35	1 3 6	1 3 17	1 3 28	1 3 39	2 0 10	2 0 21	2 0 32
12	1 3 20	1 3 32	2 0 4	2 0 16	2 0 28	2 1 0	2 1 12	2 1 24
13	2 0 5	2 0 18	2 0 31	2 1 4	2 1 17	2 1 30	2 2 3	2 2 16
14	2 0 30	2 1 4	2 1 18	2 1 32	2 2 6	2 2 20	2 2 34	2 3 8
15	2 1 15	2 1 30	2 2 5	2 2 20	2 2 35	2 3 10	2 3 25	3 0 0
16	2 2 0	2 2 16	2 2 32	2 3 8	2 3 24	3 0 0	3 0 16	3 0 32
17	2 2 25	2 3 2	2 3 19	2 3 36	3 0 13	3 0 30	3 1 7	3 1 24
18	2 3 10	2 3 28	3 0 6	3 0 24	3 1 2	3 1 20	3 1 38	3 2 16
19	2 3 35	3 0 14	3 0 33	3 1 12	3 1 31	3 2 10	3 2 29	3 3 8
20	3 0 20	3 1 0	3 1 20	3 2 0	3 2 20	3 3 0	3 3 20	4 0 0
30	4 2 30	4 3 20	5 0 10	5 1 0	5 1 30	5 2 20	5 3 10	6 0 0
40	6 1 0	6 2 0	6 3 0	7 0 0	7 1 0	7 2 0	7 3 0	8 0 0
50	7 3 10	8 0 20	8 1 30	8 3 0	9 0 10	9 1 20	9 2 30	10 0 0
60	9 1 20	9 3 0	10 0 20	10 2 0	10 3 20	11 1 0	11 2 20	12 0 0
70	10 3 30	11 1 20	11 3 30	12 1 0	12 2 30	13 0 20	13 2 10	14 0 0
80	12 2 0	13 0 0	13 2 0	14 0 0	14 2 0	15 0 0	15 2 0	16 0 0
90	14 0 10	14 2 20	15 0 30	15 3 0	16 1 10	16 3 20	17 1 30	18 0 0
100	15 2 20	16 1 0	16 3 20	17 2 0	18 0 20	18 3 0	19 1 20	20 0 0
200	31 1 0	32 2 0	33 3 0	35 0 0	36 1 0	37 2 0	38 3 0	40 0 0
300	46 3 20	48 3 0	50 2 20	52 2 0	55 0 20	56 1 0	58 0 20	60 0 0
400	62 2 0	65 0 0	67 2 0	70 0 0	72 2 0	75 0 0	77 2 0	80 0 0
500	78 0 20	81 1 0	84 1 20	87 2 0	90 2 20	93 3 0	96 3 20	100 0 0
600	93 3 0	97 2 0	101 1 0	105 0 0	108 3 0	112 2 0	116 1 0	120 0 0

FROM 33 TO 40 PERCHES, POLES, OR FALLS WIDE.

Perches long.	33 Do. wide.	34 Do. wide.	35 Do. wide.	36 Do. wide.	37 Do. wide.	38 Do. wide.	39 Do. wide.	40 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 0 33	0 0 34	0 0 35	0 0 36	0 0 37	0 0 38	0 0 39	0 1 0
2	0 1 26	0 1 28	0 1 30	0 1 32	0 1 34	0 1 36	0 1 38	0 2 0
3	0 2 19	0 2 22	0 2 25	0 2 28	0 2 31	0 2 34	0 2 37	0 3 0
4	0 3 12	0 3 16	0 3 20	0 3 24	0 3 28	0 3 32	0 3 36	1 0 0
5	1 0 5	1 0 10	1 0 15	1 0 20	1 0 25	1 0 30	1 0 35	1 1 0
6	1 0 38	1 1 4	1 1 10	1 1 16	1 1 22	1 1 28	1 1 34	1 2 0
7	1 1 31	1 1 38	1 2 5	1 2 12	1 2 19	1 2 26	1 2 33	1 3 0
8	1 2 24	1 2 32	1 3 0	1 3 8	1 3 16	1 3 24	1 3 32	2 0 0
9	1 3 17	1 3 26	1 3 35	2 0 4	2 0 13	2 0 22	2 0 31	2 1 0
10	2 0 10	2 0 20	2 0 30	2 1 0	2 1 10	2 1 20	2 1 30	2 2 0
11	2 1 3	2 1 14	2 1 25	2 1 36	2 2 7	2 2 18	2 2 29	2 3 0
12	2 1 36	2 2 8	2 2 20	2 2 32	2 3 4	2 3 16	2 3 28	3 0 0
13	2 2 29	2 3 2	2 3 15	2 3 28	3 0 1	3 0 14	3 0 27	3 1 0
14	2 3 32	2 3 36	3 0 10	3 0 24	3 0 38	3 1 12	3 1 26	3 2 0
15	3 0 15	3 0 30	3 1 5	3 1 20	3 1 35	3 2 10	3 2 25	3 3 0
16	2 1 8	3 1 24	3 2 0	3 2 16	3 2 32	3 3 8	3 3 14	4 0 0
17	3 2 1	3 2 18	3 2 35	3 3 12	3 3 29	4 0 6	4 0 13	4 1 0
18	3 2 34	3 3 12	3 3 30	4 0 8	4 0 26	4 1 4	4 1 12	4 2 0
19	3 3 27	4 0 6	4 0 25	4 1 4	4 1 23	4 2 2	4 2 11	4 3 0
20	4 0 20	4 1 0	4 1 20	4 2 0	4 2 20	4 3 0	4 3 20	5 0 0
30	6 0 30	6 1 20	6 2 10	6 3 0	6 3 30	7 0 20	7 1 10	7 2 0
40	8 1 0	8 2 0	8 3 0	9 0 0	9 1 0	9 2 0	9 3 0	10 0 0
50	10 1 10	10 2 20	10 3 30	11 1 0	11 2 10	11 3 20	12 0 30	12 2 2
60	12 1 20	12 3 0	13 0 20	13 2 0	13 3 20	14 1 0	14 2 20	15 0 0
70	14 1 30	14 3 20	15 0 10	15 3 0	16 0 30	16 2 20	17 0 10	17 2 0
80	16 2 0	17 0 0	17 1 0	18 0 0	18 2 0	19 0 0	19 2 0	20 0 0
90	18 2 10	19 0 20	19 1 30	20 1 0	20 3 10	21 1 20	21 3 30	22 2 0
100	20 2 20	21 1 0	21 3 20	22 2 0	23 0 20	23 3 0	24 1 20	25 0 0
200	41 1 0	42 2 0	43 1 0	45 0 0	46 1 0	47 2 0	48 3 0	50 0 0
300	61 3 20	63 3 0	64 3 20	67 2 0	69 1 20	71 1 0	73 0 20	75 0 0
400	82 2 0	85 0 0	87 2 0	90 0 0	92 2 0	95 0 0	97 2 0	100 0 0
500	103 0 20	106 1 0	109 1 20	112 2 0	115 2 20	118 3 0	121 3 20	125 0 0
600	123 3 0	127 2 0	131 1 0	135 0 0	138 3 0	142 2 0	146 1 0	150 0 0

FROM 41 TO 48 PERCHES, POLES, OR FALLS WIDE.

Perches long.	41 Do. wide.	42 Do. wide.	43 Do. wide.	44 Do. wide.	45 Do. wide.	46 Do. wide.	47 Do. wide.	48 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 1 1	0 1 2	0 1 3	0 1 4	0 1 5	0 1 6	0 1 7	0 1 8
2	0 2 2	0 2 4	0 2 6	0 2 8	0 2 10	0 2 12	0 2 14	0 2 16
3	0 3 3	0 3 6	0 3 9	0 3 12	0 3 15	0 3 18	0 3 21	0 3 24
4	1 0 4	1 0 8	1 0 12	1 0 16	1 0 20	1 0 24	1 0 28	1 0 32
5	1 1 5	1 1 10	1 1 15	1 1 20	1 1 25	1 1 30	1 1 35	1 2 0
6	1 2 6	1 2 12	1 2 18	1 2 24	1 2 30	1 2 36	1 3 2	1 3 8
7	1 3 7	1 3 14	1 3 21	1 3 28	1 3 35	2 0 2	2 0 9	2 0 16
8	2 0 8	2 0 16	2 0 24	2 0 32	2 1 0	2 1 8	2 1 16	2 1 24
9	2 1 9	2 1 18	2 1 27	2 1 36	2 2 5	2 2 14	2 2 23	2 2 32
10	2 2 10	2 2 20	2 2 30	2 3 0	2 3 10	2 3 20	2 3 30	3 0 0
11	2 3 11	2 3 22	2 3 33	3 0 4	3 0 15	3 0 26	3 0 37	3 1 8
12	3 0 12	3 0 24	3 0 36	3 1 8	3 1 20	3 1 32	3 2 4	3 2 16
13	3 1 13	3 1 26	3 1 39	3 2 12	3 2 25	3 2 38	3 3 11	3 3 24
14	3 2 14	3 2 28	3 3 2	3 3 16	3 3 30	4 0 4	4 0 18	4 0 32
15	3 3 15	3 3 30	4 0 5	4 0 20	4 0 35	4 1 10	4 1 25	4 2 0
16	4 0 16	4 0 32	4 1 8	4 1 24	4 2 0	4 2 16	4 2 32	4 3 8
17	4 1 17	4 1 34	4 2 11	4 2 28	4 3 5	4 3 22	4 3 39	5 0 16
18	4 2 18	4 2 36	4 3 14	4 3 32	5 0 10	5 0 28	5 1 6	5 1 24
19	4 3 19	4 3 38	5 0 17	5 0 36	5 1 15	5 1 34	5 2 13	5 2 32
20	5 0 20	5 1 0	5 1 20	5 2 0	5 2 20	5 3 0	5 3 20	6 0 0
30	7 2 30	7 3 20	8 0 10	8 1 0	8 1 30	8 2 20	8 3 10	9 0 0
40	10 1 0	10 2 0	10 3 0	11 0 0	11 1 0	11 2 0	11 3 0	12 0 0
50	12 3 10	13 0 20	13 1 30	13 3 0	14 0 10	14 1 20	14 2 30	15 0 0
60	15 1 20	15 3 0	16 0 20	16 2 0	16 3 20	17 1 0	17 2 20	18 0 0
70	17 3 30	18 1 20	18 3 10	19 1 0	19 2 30	20 0 20	20 2 10	21 0 0
80	20 2 0	21 0 0	21 2 0	22 0 0	25 1 10	23 0 0	23 2 0	24 0 0
90	23 0 10	23 2 10	24 0 30	24 3 0	25 2 10	25 3 20	26 1 30	27 0 0
100	25 2 20	26 1 0	26 3 20	27 2 0	28 0 20	28 3 0	29 1 20	30 0 0
200	51 1 0	52 2 0	53 3 0	55 0 0	56 1 0	57 2 0	58 3 0	60 0 0
300	76 3 20	78 3 0	80 1 20	82 2 0	84 1 20	86 1 0	88 0 20	90 0 0
400	102 2 0	105 0 0	107 2 0	110 0 0	112 2 0	115 0 0	117 2 0	120 0 0
500	128 0 20	131 1 0	134 1 20	137 2 0	140 2 20	143 3 0	146 3 20	150 0 0
600	153 3 0	157 2 0	161 1 0	165 0 0	168 3 0	172 2 0	176 1 0	180 0 0

FROM 49 TO 56 PERCHES, POLES, OR FALLS WIDE.

Perches long.	49 Do. wide.	50 Do. wide.	51 Do. wide.	52 Do. wide.	53 Do. wide.	54 Do. wide.	55 Do. wide.	56 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 1 9	0 1 10	0 1 11	0 1 12	0 1 13	0 1 14	0 1 15	0 1 16
2	0 2 18	0 2 20	0 2 22	0 2 24	0 2 26	0 2 28	0 2 30	0 2 32
3	0 3 27	0 3 30	0 3 33	0 3 36	0 3 39	1 0 2	1 0 5	1 0 8
4	1 0 36	1 1 0	1 1 4	1 1 8	1 1 12	1 1 16	1 1 20	1 1 24
5	1 2 5	1 2 10	1 2 15	1 2 20	1 2 25	1 2 30	1 2 35	1 3 0
6	1 3 14	1 3 20	1 3 26	1 3 32	1 3 38	2 0 4	2 0 10	2 0 16
7	2 0 23	2 0 30	2 0 37	2 1 4	2 1 11	2 1 18	2 1 25	2 1 32
8	2 1 32	2 2 0	2 2 8	2 2 16	2 2 24	2 2 32	2 3 0	2 3 8
9	2 3 1	2 3 10	2 3 19	2 3 28	2 3 37	3 0 6	3 0 15	3 0 24
10	3 0 10	3 0 20	3 0 30	3 1 0	3 1 10	3 1 20	3 1 30	3 2 0
11	3 1 19	3 1 30	3 2 1	3 2 12	3 2 23	3 2 34	3 3 5	3 3 16
12	3 2 28	3 3 0	3 3 12	3 3 24	3 3 36	4 0 8	4 0 20	4 0 32
13	3 3 37	4 0 10	4 0 23	4 0 36	4 1 9	4 1 22	4 1 35	4 2 8
14	4 1 6	4 1 20	4 1 34	4 2 8	4 2 22	4 2 36	4 3 10	4 3 24
15	4 2 15	4 2 30	4 3 5	4 3 20	4 3 35	5 0 10	5 0 25	5 1 0
16	4 3 24	5 0 0	5 0 16	5 0 32	5 1 8	5 1 24	5 2 0	5 2 16
17	5 0 33	5 1 10	5 1 27	5 2 4	5 2 21	5 2 38	5 3 15	5 3 32
18	5 2 2	5 2 20	5 2 38	5 3 16	5 3 34	6 0 12	6 0 30	6 1 8
19	5 3 11	5 3 30	6 0 9	6 0 28	6 1 7	6 1 26	6 2 5	6 2 24
20	6 0 20	6 1 0	6 1 20	6 2 0	6 2 20	6 3 0	6 3 20	7 0 0
30	9 0 30	9 1 20	9 2 10	9 3 0	9 3 30	10 0 20	10 1 10	10 2 0
40	12 1 0	12 2 0	12 3 0	13 0 0	13 1 0	13 2 0	13 3 0	14 0 0
50	15 1 10	15 2 20	15 3 30	16 1 0	16 2 10	16 3 20	17 0 30	17 2 0
60	18 1 20	18 3 0	19 0 20	19 2 0	19 3 20	20 1 0	20 2 20	21 0 0
70	21 1 30	21 3 20	22 1 10	22 3 0	23 0 30	23 2 20	24 0 10	24 2 0
80	24 2 0	25 0 0	25 2 0	26 0 0	26 2 0	27 0 0	27 2 0	28 0 0
90	27 2 10	28 0 20	28 2 30	29 1 0	29 3 10	30 1 20	30 3 3	31 2 0
100	30 2 20	31 1 0	31 3 20	32 2 0	33 0 20	33 3 0	34 1 20	35 0 0
200	61 1 0	62 2 0	63 3 0	65 0 0	66 1 0	67 2 0	68 3 0	70 0 0
300	91 3 20	93 3 0	95 2 20	97 2 0	99 1 20	101 1 0	103 0 20	105 0 0
400	122 2 0	125 0 0	127 2 0	130 0 0	132 2 0	135 0 0	137 2 0	140 0 0
500	153 0 20	156 1 0	159 1 20	162 2 0	165 2 20	168 3 0	171 3 20	175 0 0
600	183 3 0	187 2 0	191 1 0	195 0 0	198 3 0	202 2 0	206 1 0	210 0 0

FROM 57 TO 64 PERCHES, POLES, OR FALLS WIDE.

Perches long.	57 Do. wide.	58 Do. wide.	59 Do. wide.	60 Do. wide.	61 Do. wide.	62 Do. wide.	63 Do. wide.	64 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 1 17	0 1 18	0 1 19	0 1 20	0 1 21	0 1 22	0 1 23	0 1 24
2	0 2 34	0 2 36	0 2 38	0 3 0	0 3 2	0 3 4	0 3 6	0 3 8
3	1 0 11	1 0 14	1 0 17	1 0 20	1 0 23	1 0 26	1 0 29	1 0 32
4	1 1 28	1 1 32	1 1 36	1 2 0	1 2 4	1 2 8	1 2 12	1 2 16
5	1 3 5	1 3 10	1 3 15	1 3 20	1 3 25	1 3 30	1 3 35	2 0 0
6	2 0 22	2 0 28	2 0 34	2 1 0	2 1 6	2 1 12	2 1 18	2 1 24
7	2 1 39	2 2 6	2 2 13	2 2 20	2 2 27	2 2 34	2 3 1	2 3 8
8	2 3 16	2 3 24	2 3 32	3 0 0	3 0 8	3 0 16	3 0 24	3 0 32
9	3 0 33	3 1 2	2 3 11	3 1 20	3 1 29	3 1 38	3 2 7	3 2 16
10	3 2 10	3 2 20	3 2 30	3 3 0	3 3 10	3 3 20	3 3 30	4 0 0
11	3 3 27	3 3 38	4 0 9	4 0 20	4 0 31	4 1 2	4 1 13	4 1 24
12	4 1 4	4 1 16	4 1 28	4 2 0	4 2 12	4 2 24	4 2 36	4 3 8
13	4 2 21	4 2 34	4 3 7	4 3 20	4 3 33	5 0 6	5 0 19	5 0 32
14	4 3 38	5 0 12	5 0 26	5 1 0	5 1 14	5 1 28	5 2 2	5 2 16
15	5 1 15	5 1 30	5 2 5	5 2 20	5 2 35	5 3 10	5 3 25	6 0 0
16	5 2 32	5 3 8	5 3 24	6 0 0	6 0 16	6 0 32	6 0 32	6 1 24
17	6 0 9	6 0 26	6 1 3	6 1 20	6 1 37	6 2 14	6 2 14	6 3 8
18	6 1 26	6 2 4	6 2 22	6 3 0	6 3 18	6 3 36	6 3 36	7 0 32
19	6 3 3	6 3 22	7 0 1	7 0 20	7 0 39	7 1 18	7 1 18	7 2 16
20	7 0 20	7 1 0	7 1 20	7 2 0	7 2 20	7 3 0	7 3 20	8 0 0
30	10 2 30	10 3 20	11 0 10	11 1 0	11 1 30	11 2 20	11 3 10	12 0 0
40	14 1 0	14 2 0	14 3 0	15 0 0	15 1 0	15 2 0	15 3 0	16 0 0
50	17 3 10	18 0 20	18 1 30	18 3 0	19 1 10	19 1 20	19 2 30	20 0 0
60	21 1 20	21 3 0	22 0 20	22 2 0	22 3 20	23 1 0	23 2 20	24 0 0
70	24 3 30	25 1 20	25 3 10	26 1 0	26 2 30	27 0 20	27 2 10	28 0 0
80	28 2 0	29 0 0	29 2 0	30 0 0	30 2 20	31 0 0	31 2 0	32 0 0
90	32 0 10	32 2 20	33 0 30	33 3 0	34 1 10	34 3 20	35 1 30	36 0 0
100	35 2 20	36 1 0	36 3 20	37 2 0	38 0 20	38 3 0	39 1 20	40 0 0
200	71 1 0	72 2 0	73 3 0	75 0 0	76 1 0	77 2 0	78 3 0	80 0 0
300	106 3 20	108 3 0	110 2 20	112 2 0	114 1 20	116 1 0	118 0 20	120 0 0
400	142 2 0	145 0 0	147 2 0	150 0 0	152 2 0	155 0 0	157 2 0	160 0 0
500	178 0 20	181 1 0	184 1 30	187 2 0	190 2 20	193 3 0	196 3 20	200 0 0
600	213 3 0	217 2 0	221 1 0	225 0 0	228 3 0	232 2 0	236 1 0	240 0 0

FROM 65 TO 72 PERCHES, POLES, OR FALLS WIDE.

Perches long.	65 Do. wide.	66 Do. wide.	67 Do. wide.	68 Do. wide.	69 Do. wide.	70 Do. wide.	71 Do. wide.	72 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 1 25	0 1 26	0 1 27	0 1 28	0 1 29	0 1 30	0 1 31	0 1 32
2	0 3 10	0 3 12	0 3 14	0 3 16	0 3 18	0 3 20	0 3 22	0 3 24
3	1 0 35	1 0 38	1 1 1	1 1 4	1 1 7	1 1 10	1 1 13	1 1 16
4	1 2 20	1 2 24	1 2 28	1 2 32	1 2 36	1 3 0	1 3 4	1 3 8
5	2 0 5	2 0 10	2 0 15	2 0 20	2 0 25	2 0 30	2 0 35	2 1 0
6	2 1 30	2 1 36	2 2 2	2 2 8	2 2 14	2 2 20	2 2 26	2 2 32
7	2 3 15	2 3 22	2 3 29	2 3 36	3 0 3	3 0 10	3 0 17	3 0 24
8	3 1 0	3 1 8	3 1 16	3 1 24	3 1 32	3 2 0	3 2 8	3 2 16
9	3 2 25	3 2 34	3 3 3	3 3 12	3 3 21	3 3 30	3 3 39	4 0 8
10	4 0 10	4 0 20	4 0 30	4 1 0	4 1 10	4 1 20	4 1 30	4 2 0
11	4 1 35	4 2 6	4 2 17	4 2 28	4 2 39	4 3 10	4 3 21	4 3 32
12	4 3 20	4 3 32	5 0 4	5 0 16	5 0 28	5 1 0	5 1 12	5 1 24
13	5 1 5	5 1 18	5 1 31	5 2 4	5 2 17	5 2 30	5 3 3	5 3 16
14	5 2 30	5 3 4	5 3 18	5 3 32	6 0 6	6 0 20	6 0 34	6 1 8
15	6 0 15	6 0 30	6 1 5	6 1 20	6 1 35	6 2 10	6 2 25	6 3 0
16	6 2 0	6 2 16	6 2 32	6 3 8	6 3 24	7 0 0	7 0 16	7 0 32
17	6 3 25	7 0 2	7 0 19	7 0 36	7 1 13	7 1 30	7 2 7	7 2 24
18	7 1 10	7 1 28	7 2 6	7 2 24	7 3 2	7 3 20	7 3 38	8 0 16
19	7 2 35	7 3 14	7 3 33	8 0 12	8 0 31	8 1 10	8 1 29	8 2 8
20	8 0 20	8 1 0	8 1 20	8 2 0	8 2 20	8 3 0	8 3 20	9 0 0
30	12 0 30	12 1 20	12 2 10	12 3 0	12 3 30	13 0 20	13 1 10	13 2 0
40	16 1 0	16 2 0	16 3 0	17 0 0	17 1 0	17 2 0	17 3 0	18 0 0
50	20 1 10	20 2 20	20 3 30	21 1 0	21 2 10	21 3 20	22 0 80	22 2 0
60	24 1 20	24 3 0	25 2 20	25 2 0	25 3 20	26 1 0	26 2 20	27 0 0
70	28 1 30	28 3 20	29 1 10	29 3 0	30 0 30	30 2 20	31 0 10	31 2 0
80	32 2 0	33 0 0	33 2 0	34 0 0	34 2 0	35 0 0	35 2 0	36 0 0
90	36 2 10	37 0 20	37 2 30	38 1 0	38 3 10	39 1 20	39 3 30	40 2 0
100	40 2 20	41 1 0	41 3 20	42 2 0	43 0 20	43 3 0	41 1 20	45 0 0
200	81 1 0	82 2 0	83 3 0	85 0 0	86 1 0	87 2 0	88 3 0	90 0 0
300	121 3 20	123 3 0	125 2 20	127 2 0	130 1 20	131 1 0	133 0 20	135 0 0
400	162 2 0	165 0 0	167 2 0	170 0 0	172 2 0	175 0 0	177 2 0	180 0 0
500	203 0 20	206 1 0	209 1 20	212 2 0	215 2 20	218 3 0	221 3 20	225 0 0
600	243 3 0	247 2 0	151 1 0	255 0 0	258 3 0	262 2 0	266 1 0	270 0 0

FROM 73 TO 80 PERCHES, POLES, OR FALLS WIDE.

Perches long.	73 Do. wide.	74 Do. wide.	75 Do. wide.	76 Do. wide.	77 Do. wide.	78 Do. wide.	79 Do. wide.	80 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 1 33	0 1 34	0 1 35	0 1 36	0 1 37	0 1 38	0 1 39	0 2 0
2	0 3 36	0 3 28	0 3 30	0 3 32	0 3 34	0 3 36	0 3 38	1 0 0
3	1 1 19	1 1 22	1 1 25	1 1 28	1 1 31	1 1 34	1 1 37	1 2 0
4	1 3 12	1 3 16	1 3 20	1 3 24	1 3 28	1 3 32	1 3 36	2 0 0
5	2 1 5	2 1 10	2 1 15	2 1 20	2 1 25	2 1 30	2 1 35	2 2 0
6	2 2 38	2 3 4	2 3 10	2 3 16	2 3 22	2 3 28	2 3 34	3 0 0
7	3 0 31	3 0 38	3 1 5	3 1 12	3 1 19	3 1 26	3 1 33	3 2 0
8	3 2 24	3 2 32	3 3 0	3 3 8	3 3 16	3 3 24	3 3 32	4 0 0
9	4 0 17	4 0 26	4 0 35	4 1 4	4 1 13	4 1 22	4 1 31	4 2 0
10	4 2 10	4 2 20	4 2 30	4 3 0	4 3 10	4 3 20	4 3 30	5 0 0
11	5 0 3	5 0 14	5 0 25	5 0 36	5 1 7	5 1 18	5 1 29	5 2 0
12	5 1 36	5 2 8	5 2 20	5 2 32	5 3 4	5 3 16	5 3 28	6 0 0
13	5 3 29	6 0 2	6 0 15	6 0 28	6 1 1	6 1 4	6 1 27	6 2 0
14	6 1 22	6 1 36	6 2 10	6 2 24	6 2 38	6 3 12	6 3 26	7 0 0
15	6 3 15	6 3 30	7 0 5	7 0 20	7 0 35	7 1 10	7 1 25	7 2 0
16	7 1 8	7 1 24	7 2 0	7 2 16	7 2 32	7 3 8	7 3 24	8 0 0
17	7 3 1	7 3 18	7 3 35	8 0 12	8 0 29	8 1 0	8 1 23	8 2 0
18	8 0 34	8 1 12	8 1 30	8 2 8	8 2 26	8 3 4	8 3 22	9 0 0
19	8 2 27	8 3 6	8 3 25	9 0 4	9 0 23	9 1 2	9 1 21	9 2 0
20	9 0 20	9 1 0	9 1 20	9 2 0	9 2 20	9 3 0	9 3 20	10 0 0
30	13 2 30	13 3 20	14 0 10	14 1 0	14 1 30	14 2 20	14 3 10	15 0 0
40	18 1 0	18 2 20	18 3 0	19 0 0	19 1 10	19 2 0	19 3 0	20 0 0
50	22 3 10	23 0 20	23 1 30	23 3 0	24 0 10	24 1 20	24 2 30	25 0 0
60	27 1 20	27 3 0	28 0 20	28 2 0	28 3 20	29 1 0	29 2 20	30 0
70	31 3 10	32 1 20	32 3 10	33 1 0	33 2 30	34 0 20	34 2 10	35 0 0
80	36 2 0	37 0 0	37 2 0	38 0 0	38 2 0	39 0 0	39 2 0	40 0 0
90	41 0 10	41 2 20	42 0 30	42 3 0	43 1 10	43 3 20	44 1 30	45 0 0
100	45 2 20	46 1 0	46 3 20	47 2 0	48 0 20	48 3 0	49 1 20	50 0 0
200	91 1 0	92 2 0	93 3 0	95 0 0	96 1 0	97 2 0	98 3 0	100 0 0
300	136 3 20	138 3 0	140 2 20	142 2 0	144 1 20	146 1 0	148 0 20	150 0 0
400	182 2 0	185 0 0	187 2 0	190 0 0	192 2 0	195 0 0	197 2 0	200 0 0
500	228 0 20	231 1 0	234 1 30	237 2 0	240 2 20	243 3 0	246 3 20	250 0 0
600	273 3 0	277 2 0	281 1 0	285 0 0	288 3 0	292 2 0	296 1 0	300 0 0

FROM 81 TO 88 PERCHES, POLES, OR FALLS WIDE.

Perches long.	81 Do. wide.	82 Do. wide.	83 Do. wide.	84 Do. wide.	85 Do. wide.	86 Do. wide.	87 Do. wide.	88 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 2 1	0 2 2	0 2 3	0 2 4	0 2 5	0 2 6	0 2 7	0 2 8
2	1 0 2	1 0 4	1 0 6	1 0 8	1 0 10	1 0 12	1 0 14	1 0 16
3	1 2 3	1 2 6	1 2 9	1 2 12	1 2 15	1 2 18	1 2 21	1 2 24
4	2 0 4	2 0 8	2 0 12	2 0 16	2 0 20	2 0 24	2 0 28	2 0 32
5	2 2 25	2 2 10	2 2 15	2 2 20	2 2 25	2 2 30	2 2 35	2 3 0
6	3 0 6	3 0 12	3 0 18	3 0 24	3 0 30	3 0 36	3 1 2	3 1 8
7	3 2 7	3 2 14	3 2 21	3 2 28	3 2 35	3 3 2	3 3 9	3 3 16
8	4 0 8	4 0 16	4 0 24	4 0 32	4 1 0	4 1 8	4 1 16	4 1 24
9	4 2 9	4 2 8	4 2 27	4 2 36	4 3 5	4 3 14	4 3 23	4 3 32
10	5 0 10	5 0 20	5 0 30	5 1 0	5 1 10	5 1 20	5 1 30	5 2 0
11	5 2 11	5 2 22	5 2 33	5 3 4	5 3 15	5 3 26	5 3 37	6 0 8
12	6 0 12	6 0 24	6 0 36	6 1 8	6 1 20	6 1 32	6 2 4	6 2 16
13	6 2 13	6 2 26	6 2 39	6 3 12	6 3 25	6 3 38	7 0 11	7 0 24
14	7 0 14	7 0 28	7 1 2	7 1 16	7 1 30	7 2 4	7 2 18	7 2 32
15	7 2 15	7 2 30	7 3 5	7 3 20	7 3 35	8 0 10	8 0 25	8 1 0
16	8 0 16	8 0 32	8 1 8	8 1 24	8 2 0	8 2 16	8 2 32	8 3 8
17	8 2 17	8 2 34	8 3 11	8 3 28	9 0 5	9 0 22	9 0 39	9 1 16
18	9 0 18	9 0 36	9 1 14	9 1 32	9 2 10	9 2 28	9 3 6	9 3 24
19	9 2 19	9 2 38	9 3 17	9 3 36	10 0 15	10 0 34	10 1 13	10 1 32
20	10 0 20	10 1 0	10 1 20	10 2 0	10 2 20	10 3 0	10 3 20	11 0 0
30	15 0 30	15 1 20	15 2 10	15 3 0	15 3 30	16 0 20	16 1 10	16 2 0
40	20 1 0	20 2 0	20 3 0	21 0 0	21 1 0	21 2 0	21 3 0	22 0 0
50	25 1 10	25 2 20	25 3 30	26 1 0	26 2 10	26 3 20	27 0 30	27 2 0
60	30 1 20	30 3 0	31 0 20	31 2 0	31 3 20	32 1 0	32 2 20	33 0 0
70	35 1 30	35 3 20	36 1 10	36 3 0	37 0 30	37 2 20	38 0 10	38 2 0
80	40 2 0	41 0 0	41 2 0	42 0 0	42 2 0	43 0 0	43 2 0	44 0 0
90	45 2 10	46 0 20	46 2 30	47 1 0	47 3 10	48 1 20	48 3 30	49 2 0
100	50 2 20	51 1 0	51 3 20	52 2 0	53 0 20	53 3 0	54 1 20	55 0 0
200	101 1 0	102 2 0	103 3 0	105 0 0	106 1 0	107 2 0	108 3 0	110 0 0
300	151 3 20	153 3 0	155 2 20	157 2 0	159 1 20	161 1 0	163 0 20	165 0 0
400	202 2 0	205 0 0	207 2 0	210 0 0	212 2 0	215 0 0	217 2 0	220 0 0
500	253 0 20	256 1 0	259 1 20	262 2 0	265 2 20	268 3 0	271 3 20	275 0 0
600	303 3 0	307 2 0	311 1 0	315 0 0	318 3 0	322 2 0	326 1 0	330 0 0

FROM 89 TO 96 PERCHES, POLES, OR FALLS WIDE.

Perches long.	89 Do. wide.	90 Do. wide.	91 Do. wide.	92 Do. wide.	93 Do. wide.	94 Do. wide.	95 Do. wide.	96 Do. wide.
	A R P	A R P	A R P	A R P	A R P	A R P	A R P	A R P
1	0 2 9	0 2 10	0 2 11	0 2 12	0 2 13	0 2 14	0 2 15	0 2 16
2	1 0 18	1 0 20	1 0 22	1 0 24	1 0 26	1 0 28	1 0 30	1 0 32
3	1 2 27	1 2 30	1 2 33	1 2 36	1 2 39	1 3 2	1 3 5	1 3 8
4	2 0 36	2 1 0	2 1 4	2 1 8	2 1 12	2 1 16	2 1 20	2 1 24
5	2 3 5	2 3 10	2 3 15	2 3 20	2 3 25	2 3 30	2 3 35	3 0 0
6	3 1 14	3 1 20	3 1 26	3 1 32	3 1 18	3 2 4	3 2 10	3 2 16
7	3 3 23	3 3 30	3 3 37	4 0 4	4 0 11	4 0 18	4 0 25	4 0 32
8	4 1 32	4 2 0	4 2 8	4 2 16	4 2 24	4 2 32	4 3 0	4 3 8
9	5 0 1	5 0 10	5 0 19	5 0 28	5 0 37	5 1 6	5 1 15	5 1 24
10	5 2 10	5 2 20	5 2 30	5 3 0	5 3 10	5 3 20	5 3 30	6 0 0
11	6 0 19	6 0 30	6 1 1	6 1 12	6 1 23	6 1 34	6 2 5	6 2 16
12	6 2 28	6 3 0	6 3 12	6 3 24	6 3 36	7 0 8	7 0 20	7 0 32
13	7 0 37	7 1 10	7 1 23	7 1 35	7 2 8	7 2 21	7 2 35	7 3 8
14	7 3 6	7 3 20	7 3 34	8 0 8	8 0 22	8 0 36	8 1 10	8 1 24
15	8 1 15	8 1 30	8 2 5	8 2 20	8 2 35	8 3 10	8 3 25	9 0 0
16	8 3 24	9 0 0	9 0 16	9 0 32	9 1 8	9 1 24	9 2. 0	9 2 16
17	9 1 33	9 2 10	9 2 27	9 3 4	9 3 21	9 3 38	10 0 15	10 0 32
18	10 0 2	10 0 20	10 0 38	10 0 38	10 1 16	10 1 34	10 2 30	10 3 8
19	10 2 11	10 2 30	10 3 9	10 3 28	11 0 7	11 0 26	11 1 5	11 1 24
20	11 0 20	11 1 0	11 1 20	11 2 0	11 2 20	11 3 0	11 3 20	12 0 0
30	16 2 30	16 3 20	17 0 10	17 1 0	17 1 30	17 2 20	17 3 10	18 0 0
40	22 1 0	22 2 0	22 3 0	23 0 0	23 1 0	23 2 0	23 3 0	24 0 0
50	27 3 10	28 0 20	28 1 30	28 3 0	29 0 10	29 1 20	29 2 30	30 0 0
60	33 1 20	33 3 0	34 0 20	34 2 0	34 3 20	35 1 0	35 2 20	36 0 0
70	38 3 30	39 1 20	39 3 10	40 1 0	40 2 30	41 0 20	41 2 10	42 0 0
80	44 2 0	45 0 0	45 2 0	46 0 0	46 2 0	47 0 0	47 2 0	48 0 0
90	50 0 10	50 2 20	51 0 30	51 3 0	52 1 10	52 3 20	53 1 30	54 0 0
100	55 2 20	56 1 0	56 3 20	57 2 0	58 0 20	58 3 0	59 1 20	60 0 0
200	111 1 0	112 2 0	113 3 0	115 0 0	116 1 0	117 2 0	118 3 0	120 0 0
300	166 3 20	168 3 0	170 2 20	172 2 0	174 1 20	176 1 0	178 0 20	180 0 0
400	222 2 0	225 0 0	227 2 0	230 0 0	232 2 0	235 0 0	237 2 0	240 0 0
500	278 0 20	281 1 0	284 1 20	287 2 0	290 2 20	293 3 0	296 3 20	300 0 0
600	333 3 0	337 2 0	341 1 0	345 0 0	348 3 0	352 2 0	356 1 0	360 0 0

FROM 97 TO 102 PERCHES, POLES, OR FALLS WIDE.

Perches long.	97 Do. wide.			98 Do. wide.			99 Do. wide.			100 Do. wide.			101 Do. wide.			102 Do. wide.		
	A	R	P	A	R	P	A	R	P	A	R	P	A	R	P	A	R	P
1	0	2	17	0	2	18	0	2	19	0	2	20	0	2	21	0	2	22
2	1	0	34	1	0	36	1	0	38	1	1	0	1	1	2	1	1	4
3	1	3	11	1	3	14	1	3	17	1	3	20	1	3	23	1	3	26
4	2	1	28	2	1	32	2	1	36	2	2	0	2	2	4	2	2	8
5	3	0	5	3	0	10	3	0	15	3	0	20	3	0	25	3	0	30
6	3	2	22	3	2	28	3	2	34	3	3	0	3	3	6	3	3	12
7	4	0	39	4	1	6	4	1	13	4	1	20	4	1	27	4	1	34
8	4	3	16	4	3	24	4	3	32	5	0	0	5	0	8	5	0	16
9	5	1	33	5	2	2	5	2	11	5	2	20	5	2	29	5	2	38
10	6	0	10	6	0	20	6	0	30	6	1	0	6	1	10	6	1	20
11	6	2	27	6	2	38	6	3	9	6	3	20	6	3	31	7	0	2
12	7	1	4	7	1	16	7	1	28	7	2	0	7	2	12	7	2	24
13	7	3	21	7	3	34	8	0	7	8	0	20	8	0	33	8	1	6
14	8	1	38	8	2	12	8	2	26	8	3	0	8	3	14	8	3	28
15	9	0	15	9	0	30	9	1	5	9	1	20	9	1	35	9	2	10
16	9	2	32	9	3	8	9	3	24	10	0	0	10	0	16	10	0	32
17	10	1	9	10	1	26	10	2	3	10	2	20	10	2	33	10	3	14
18	10	3	26	11	0	4	11	0	22	11	1	0	11	1	18	11	1	36
19	11	2	3	11	2	22	11	3	1	11	3	20	11	3	39	12	0	18
20	12	0	20	12	1	0	12	1	20	12	2	0	12	2	20	12	3	0
30	18	0	30	18	1	20	18	2	10	18	3	0	18	3	30	19	0	20
40	24	1	0	24	2	0	24	3	0	25	0	0	25	1	0	25	2	0
50	30	1	10	30	2	20	30	3	30	31	1	0	31	2	10	31	3	20
60	36	1	20	36	3	0	37	0	20	37	2	0	37	3	20	38	1	0
70	42	1	30	42	3	20	43	1	10	43	3	0	44	0	30	44	2	20
80	48	2	0	49	0	0	49	2	0	50	0	0	50	2	0	51	0	0
90	54	2	10	55	0	20	55	2	30	56	1	0	56	3	10	57	1	20
100	60	2	20	61	1	0	61	3	20	62	2	0	63	0	20	63	3	0
200	121	1	0	122	2	0	123	3	0	125	0	0	126	1	0	127	2	0
300	181	3	20	183	3	0	185	2	20	187	2	0	189	1	20	191	1	0
400	242	2	0	245	0	0	247	2	0	250	0	0	252	2	0	255	0	0
500	303	0	20	306	1	0	309	1	20	312	2	0	315	2	20	318	3	0
600	363	3	0	367	2	0	371	1	0	375	0	0	378	3	0	382	2	0

ART. III.—CONTAINING USEFUL TABLES FOR THE REDUCTION OF MEASURES.

ENGLISH LONG MEASURE.

Inches 7.92	one link						
12	1.51	one foot					
36	4.54	3	one yard				
198	25	16	5.5	one pole or perch			
792	100	66	22	4	one chain		
7920	1000	660	220	40	10	one furlong	
63360	8000	5280	1760	320	80	8	one mile

ENGLISH SQUARE MEASURE.

Sq. inches 62.7264	one link					
144	Sq. links 2.29	one foot				
1296	20.66	9	one yard			
39204	625	272.25	30.25	one perch		
1568160	25000	10890	1210	40	one rood	
6272640	100000	43560	4840	160	4	one acre

SCOTS LONG MEASURE.

Inches 8.88	one link					
12	1.35	one foot				
37	4.16	3.08	one ell			
222	25	18.5	6	one rood		
880	100	74	24	4	one chain	
71040	8000	5920	1920	320	80	one mile

SCOTS SQUARE MEASURE.

Sq. inches 78.8544	one link					
144	Sq. links 1.82	one foot				
1369	17.36	9.50	one ell			
49284	625	342.25	36	one fall		
1971360	25000	13690	1440	40	one rood	
7885440	100000	54760	5760	160	4	one acre

IRISH LONG MEASURE.

Inches 9.53	one link					
12	1.19	one foot				
36	3.57	3	one yard			
252	25	21	7	one perch		
1008	100	84	28	4	one chain	
80640	8000	6720	2040	320	80	one mile

IRISH SQUARE MEASURE.

Inches 106.2144	one link					
144	Sq. links 1.42	one foot				
1296	12.78	9	one yard			
66384	.625	441•	49	one perch		
2655360	25000	17640	1960	40	one rood	
10681440	100000	70560	7840	160	4	one acre

Note.—These tables of Scots measure are still retained for the conversion of measures previous to 1826, when the new act was passed; because, up to that date, plans of estates were measured with such chains as stated by the surveyors on the face of them; and Ainslie himself always employed them.—W. G.

TABLE,

FOR ASCERTAINING, BY INSPECTION, THE NUMBER OF ROODS AND PERCHES CONTAINED IN THE TWO FIRST DECIMALS OF AN ACRE.

Decimals of an Acre. The last figure is only the decimal of a Perch or Fall.

Decimals.	Roods and Perches. R.	P.	Pts.	Decimals.	Roods and Perches. R.	P.	Pts.
01	0	01	.6	51	2	1	.6
02	0	03	.2	52	2	3	.2
03	0	04	.8	53	2	4	.8
04	0	06	.4	54	2	6	.4
05	0	08	.0	55	2	8	.0
06	0	09	.6	56	2	9	.6
07	0	11	.2	57	2	11	.2
08	0	13	.8	58	2	13	.8
09	0	14	.4	59	2	14	.4
10	0	16	.0	60	2	16	.0
11	0	17	.6	61	2	17	.6
12	0	19	.2	62	2	19	.2
13	0	20	.8	63	2	20	.8
14	0	22	.4	64	2	22	.4
15	0	24	.0	65	2	24	.0
16	0	25	.6	66	2	25	.4
17	0	27	.2	67	2	27	.2
18	0	28	.8	68	2	28	.8
19	0	30	.4	69	2	30	.4
20	0	32	.0	70	2	32	.0
21	0	33	.6	71	2	33	.6
22	0	35	.2	72	2	35	.2
23	0	36	.8	73	2	36	.8
24	0	38	.4	74	2	38	.4
25	1	0	.0	75	3	0	.0
26	1	1	.6	76	3	1	.6
27	1	3	.2	77	3	3	.2
28	1	4	.8	78	3	4	.8
29	1	6	.4	79	3	6	.4
30	1	8	.0	80	3	8	.0
31	1	9	.6	81	3	9	.6
32	1	11	.2	82	3	11	.2
33	1	12	.8	83	3	13	.8
34	1	14	.4	84	3	14	.4
35	1	16	.0	85	3	16	.0
36	1	17	.6	86	3	17	.6
37	1	19	.2	87	3	19	.2
38	1	20	.8	88	3	20	.8
39	1	22	.4	89	3	22	.4
40	1	24	.0	90	3	24	.0
41	1	25	.6	91	3	25	.6
42	1	27	.2	92	3	27	.2
43	1	28	.8	93	3	28	.8
44	1	30	.4	94	3	30	.4
45	1	32	.0	95	3	32	.0
46	1	33	.6	96	3	33	.6
47	1	35	.2	97	3	35	.2
48	1	36	.8	98	3	36	.8
49	1	38	.4	99	3	38	.4
50	2	0	.0				

This may be also performed by the Table for converting roods and poles into square links, by reversing the process.

TABLE I.

To convert Scottish Links into Imperial Links and Decimals.

Scots links.	Imperial links.	Scots links.	Imperial links.
100	112.302	5100	5727.424
200	224.605	5200	5839.726
300	336.907	5300	5952.028
400	449.210	5400	6064.331
500	561.512	5500	6176.633
600	673.815	5600	6288.936
700	686.117	5700	6401.238
800	898.419	5800	6513.541
900	1010.722	5900	6625.843
1000	1123.024	6000	6738.145
1100	1235.327	6100	6850.448
1200	1347.629	6200	6962.750
1300	1459.932	6300	7075.053
1400	1572.234	6400	7187.355
1500	1684.536	6500	7299.658
1600	1796.839	6600	7411.960
1700	1909.141	6700	7524.262
1800	2021.445	6800	7636.565
1900	2133.746	6900	7748.867
2000	2246.048	7000	7861.170
2100	2358.351	7100	7973.472
2200	2470.653	7200	8085.775
2300	2582.956	7300	8198.077
2400	2695.258	7400	8310.379
2500	2807.561	7500	8422.682
2600	2919.863	7600	8534.984
2700	3032.165	7700	8647.287
2800	3144.468	7800	8759.589
2900	3256.770	7900	8871.891
3000	3369.073	8000	8984.194
3100	3481.375	8100	9096.496
3200	3593.678	8200	9208.799
3300	3705.980	8300	9321.101
3400	3818.282	8400	9433.404
3500	3930.585	8500	9545.706
3600	4042.887	8600	9658.008
3700	4155.190	8700	9770.311
3800	4267.492	8800	9882.613
3900	4379.795	8900	9994.916
4000	4492.097	9000	10107.218
4100	4604.399	9100	10219.520
4200	4716.702	9200	10331.823
4300	4829.004	9300	10444.125
4400	4941.307	9400	10556.428
4500	5053.609	9500	10668.730
4600	5165.912	9600	10781.032
4700	5278.214	9700	10893.335
4800	5390.517	9800	11005.637
4900	5502.819	9900	11117.940
5000	5615.121	10000	11230.242

TABLE II.

To convert Scottish Acres into Imperial Acres and Decimals.

Scots acres.	Imperial acres.	Scots acres.	Imperial acres.
1	1.26118	51	64.32035
2	2.52237	52	65.58154
3	3.78355	53	66.84272
4	5.04473	54	68.10391
5	6.30592	55	69.36509
6	7.56710	56	70.62627
7	8.82828	57	71.88746
8	10.08947	58	73.14864
9	11.35065	59	74.40982
10	12.61183	60	75.67101
11	13.87302	61	76.93219
12	15.13420	62	78.19337
13	16.39528	63	79.45456
14	17.65657	64	80.71574
15	18.91775	65	81.97692
16	20.17893	66	83.23811
17	21.44012	67	84.49929
18	22.71030	68	85.76047
19	23.96248	69	87.02166
20	25.22367	70	88.28284
21	26.48485	71	89.54402
22	27.74603	72	90.80521
23	29.00722	73	92.06639
24	30.26840	74	93.32757
25	31.52959	75	94.58876
26	32.79077	76	95.84994
27	34.05195	77	97.11112
28	35.31314	78	98.37231
29	36.57432	79	99.63349
30	37.83550	80	100.89467
31	39.09669	81	102.15586
32	40.35787	82	103.41704
33	41.61905	83	104.67823
34	42.88024	84	105.93941
35	44.14142	85	107.20059
36	45.40260	86	108.46178
37	46.66379	87	109.72296
38	47.92497	88	110.98414
39	49.18615	89	112.24533
40	50.44734	90	113.50651
41	51.70852	91	114.76769
42	52.96970	92	116.02888
43	54.23089	93	117.29006
44	55.49207	94	118.55124
45	56.75325	95	119.81243
46	58.01444	96	121.07361
47	59.27562	97	122.33479
48	60.53680	98	123.59598
49	61.79799	99	124.85716
50	63.05917	100	126.11834

The preceding tables are founded on the determination of the Scottish standard ell, kept at Edinburgh, embodied in the act of Parliament of 1826, in reference to weights and measures.

It was determined to be 37.0598 imperial inches at the temperature of 62° Fahrenheit.

The Scottish chain contains 24 Scottish ells, and it is therefore equal to 74.1196 imperial feet. Hence the square chain will contain 610.412789 square yards.

This is the *new* Scottish acre, created by the act of 1826, while the old Scottish acre, derived from the chain formerly taken at 74 feet, gives the acre 6084$\frac{4}{9}$, thus introducing confusion instead of uniformity. Uniformity should have been enforced by legal enactments and penalties.

Hence the new Scottish acre is 6104$\frac{1}{8}$ square yards.
the old . . 6084$\frac{4}{9}$ square yards nearly.
and the ratio between these as 1.003235 to 1.

EXEMPLIFICATION OF FIRST TABLE.

1. In 7426 Scots links, how many imperial?

7400 Scots give	8310.379 imperial.
26 Scots, by shifting to the left the point two places, .	29.199
7426 Scots links are therefore equal to	8339.598 imperial.

EXEMPLIFICATION OF THE SECOND TABLE.

2. In 96.567 Scots acres, how many imperial?

96.00	Scots acres give	121.07861 imperial.
0.56	by shifting the point two places to the left, .	0.70626
0.007	by shifting three places to the left, . . .	0.00883
96.567	Scots acres are equal to	121.78870

TABLE, SHOWING THE LENGTH OF AN ACRE, IN CHAINS, LINKS, AND PARTS OF A LINK, TO A GIVEN BREADTH.

Chains.	Chains.	Links.	Pts.		Chains.	Chains.	Links.	Pts.	
1	10	00	.000	1 acre.	11	0	90	.909	1 acre.
2	5	00	.000	do.	12	0	83	.333	do.
3	3	33	.333	do.	13	0	76	.923	do.
4	2	50	.000	do.	14	0	71	.429	do.
5	2	00	.000	do.	15	0	66	.666	do.
6	1	66	.666	do.	16	0	62	.500	do.
7	1	42	.285	do.	17	0	58	.824	do.
8	1	25	.000	do.	18	0	55	.555	do.
9	1	11	.111	do.	19	0	53	.631	do.
10	1	00	.000	do.	20	0	50	.000	do.
Breadth.	Length.				Breadth.	Length.			

TABLE

TO CONVERT LINKS INTO FEET, AND CONVERSELY.

IMPERIAL LINKS.										
		100	200	300	400	500	600	700	800	900
Links.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
0	0.00	66.00	132.00	198.00	264.00	330.00	396.00	462.00	528.00	594.00
1	0.66	66.66	132.66	198.66	264.66	330.66	396.66	462.66	528.66	594.66
2	1.32	67.32	133.32	199.32	265.32	331.32	397.32	463.32	529.32	595.32
3	1.98	67.98	133.98	199.98	265.98	331.98	397.98	463.98	529.98	595.98
4	2.64	68.64	134.64	200.64	266.64	332.64	398.64	464.64	530.64	596.64
5	3.30	69.30	135.30	201.30	267.30	333.30	399.30	465.30	531.30	597.30
6	3.96	69.96	135.96	201.96	267.96	333.96	399.96	465.96	531.96	597.96
7	4.62	70.62	136.62	202.62	268.62	334.62	400.62	466.62	532.62	598.62
8	5.28	71.28	137.28	203.28	269.28	335.28	401.28	467.28	533.28	599.28
9	5.94	71.94	137.94	203.94	269.94	335.94	401.94	467.94	533.94	599.94
10	6.60	72.60	138.60	204.60	270.60	336.60	402.60	468.60	534.60	600.60
11	7.26	73.26	139.26	205.26	271.26	337.26	403.26	469.26	535.26	601.26
12	7.92	73.92	139.92	205.92	271.92	337.92	403.92	469.92	535.92	601.92
13	8.58	74.58	140.58	206.58	272.58	338.58	404.58	470.58	536.58	602.58
14	9.24	75.24	141.24	207.24	273.24	339.24	405.24	471.24	537.24	603.24
15	9.90	75.90	141.90	207.90	273.90	339.90	405.90	471.90	537.90	603.90
16	10.56	76.56	142.56	208.56	274.56	340.56	406.56	472.56	538.56	604.56
17	11.22	77.22	143.22	209.22	275.22	341.22	407.22	473.22	539.22	605.22
18	11.88	77.88	143.88	209.88	275.88	341.88	407.88	473.88	539.88	605.88
19	12.54	78.54	144.54	210.54	276.54	342.54	408.54	474.54	540.54	606.54
20	13.20	79.20	145.20	211.20	277.20	343.20	409.20	475.20	541.20	607.20
21	13.86	79.86	145.86	211.86	277.86	343.86	409.86	475.86	541.86	607.86
22	14.52	80.52	146.52	212.52	278.52	344.52	410.52	476.52	542.52	608.52
23	15.18	81.18	147.18	213.18	279.18	345.18	411.18	477.18	543.18	609.18
24	15.84	81.84	147.84	213.84	279.84	345.84	411.84	477.84	543.84	609.84
25	16.50	82.50	148.50	214.50	280.50	346.50	412.50	478.50	544.50	610.50
26	17.16	83.16	149.16	215.16	281.16	347.16	413.16	479.16	545.16	611.16
27	17.82	83.82	149.82	215.82	281.82	347.82	413.82	479.82	545.82	611.82
28	18.48	84.48	150.48	216.48	282.48	348.48	414.48	480.48	546.48	612.48
29	19.14	85.14	151.14	217.14	283.14	349.14	415.14	481.14	547.14	613.14
30	19.80	85.80	151.80	217.80	283.80	349.80	415.80	481.80	547.80	613.80
31	20.46	86.46	152.46	218.46	284.46	350.46	416.46	482.46	548.46	614.46
32	21.12	87.12	153.12	219.12	285.12	351.12	417.12	483.12	549.12	615.12
33	21.78	87.78	153.78	219.78	285.78	351.78	417.78	483.78	549.78	615.78
34	22.44	88.44	154.44	220.44	286.44	352.44	418.44	484.44	550.44	616.44
35	23.10	89.10	155.10	221.10	287.10	353.10	419.10	485.10	551.10	617.10
36	23.76	89.76	155.76	221.76	287.76	353.76	419.76	485.76	551.76	617.76
37	24.42	90.42	156.42	222.42	288.42	354.42	420.42	486.42	552.42	618.42
38	25.08	91.08	157.08	223.08	289.08	355.08	421.08	487.08	553.08	619.08
39	25.74	91.74	157.74	223.74	289.74	355.74	421.74	487.74	553.74	619.74
40	26.40	92.40	158.40	224.40	290.40	356.40	422.40	488.40	554.40	620.40
41	27.06	93.06	159.06	225.06	291.06	357.06	423.06	489.06	555.06	621.06
42	27.72	93.72	159.72	225.72	291.72	357.72	423.72	489.72	555.72	621.72
43	28.38	94.38	160.38	226.38	292.38	358.38	424.38	490.38	556.38	622.38
44	29.04	95.04	161.04	227.04	293.04	359.04	425.04	491.04	557.04	623.04
45	29.70	95.70	161.70	227.70	293.70	359.70	425.70	491.70	557.70	623.70
46	30.36	96.36	162.36	228.36	294.36	360.36	426.36	492.36	558.36	624.36
47	31.02	97.02	163.02	229.02	295.02	361.02	427.02	493.02	559.02	625.02
48	31.68	97.68	163.68	229.68	295.68	361.68	427.68	493.68	559.68	625.68
49	32.34	98.34	164.34	230.34	296.34	362.34	428.34	494.34	560.34	626.34

PROPORTIONAL PARTS TO TENTHS OF A LINK.

Link, .	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Foot, .	0.066	0.132	0.198	0.264	0.330	0.393	0.462	0.528	0.594

TABLE

TO CONVERT LINKS INTO FEET, AND CONVERSELY.

IMPERIAL LINKS.										
		100	200	300	400	500	600	700	800	900
Links.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
50	33.00	99.00	165.00	231.00	297.00	363.00	429.00	495.00	561.00	627.00
51	33.66	99.66	165.66	231.66	297.66	363.66	429.66	495.66	561.66	627.66
52	34.32	100.32	166.32	232.32	298.32	364.32	430.32	496.32	562.32	628.32
53	34.98	100.98	166.98	232.98	298.98	364.98	430.98	496.98	562.98	628.98
54	35.64	101.64	167.64	233.64	299.64	365.64	431.64	497.64	563.64	629.64
55	36.30	102.30	168.30	234.30	300.30	366.30	432.30	498.30	564.30	630.30
56	36.96	102.96	168.96	234.96	300.96	366.96	432.96	498 96	564.96	630 96
57	37.62	103.62	169.62	235.62	301.62	367.62	433.62	499 62	565.62	631.62
58	38.28	104.28	170.28	236.28	302.28	368.28	434.28	500.28	566.28	632.28
59	38.94	104.94	170.94	236.94	302.94	368.94	434.94	500.94	566.94	632.94
60	39.60	105.60	171.60	237.60	303.60	369.60	435.60	501.60	567.60	633.60
61	40.26	106.26	172.26	238.26	304.26	370.26	436.26	502.26	568.26	634.26
62	40.92	106.92	172.92	238.92	304.92	370.92	436.92	502.92	568.92	634.92
63	41.58	107.58	173.58	239.58	305.58	371.58	437.58	503.58	569.58	635.58
64	42 24	108.24	174.24	240.24	306.24	372.24	438.24	504.24	570.24	636.24
65	42.90	108.90	174.90	240.90	306.90	372.90	438.90	504.90	570.90	636.90
66	43.56	109.56	175.56	241.56	307.56	373.56	439.56	505.56	571.56	637.56
67	44.22	110.22	176.22	242.22	308.22	374.22	440.22	506.22	572.22	638.22
68	44.88	110.88	176.88	242.88	308.88	374.88	440.88	506.88	572.88	638.88
69	45.54	111.54	177 54	243.54	309.54	375.54	441.54	507.54	573.54	639.54
70	46.20	112.20	178.20	244.20	310.20	376.20	442.20	508.20	574.20	640.20
71	46.86	112.86	178.86	244.86	310.86	376.86	442.86	508.86	574.86	640.86
72	47.52	113.52	179 52	245.52	311.52	377.52	443.52	509.52	575.52	641.52
73	48.18	114.18	180.18	246.18	312 18	378 18	444.18	510.18	576.18	642.18
74	48.84	114.84	180.84	246.84	312.84	378.84	444.84	510.84	576.84	642.84
75	49.50	115.50	181.50	247.50	313.50	379.50	445.50	511.50	577.50	643.50
76	50.16	116.16	182.16	248.16	314.16	380.16	446.16	512.16	578.16	644.16
77	50.82	116.82	182.82	248.82	314.82	380.82	446.82	512.82	578.82	644.82
78	51.48	117.48	183.48	249.48	315.48	381.48	447.48	513.48	579.48	645.48
79	52.14	118.14	184.14	250.14	316.14	382.14	448.14	514.14	580.14	646.14
80	52.80	118.80	184.80	250.80	316.80	382.80	448.80	514.80	580.80	646.80
81	53.46	119.46	185.46	251.46	317.46	383.46	449.46	515.46	581.46	647.46
82	54.12	120.12	186.12	252.12	318.12	384.12	450.12	516.12	582.12	648.12
83	54.78	120.78	186.78	252.78	318.78	384.78	450.78	516.78	582.78	648.78
84	55.44	121.44	187.44	253.44	319.44	385.44	451.44	517.44	583.44	649.44
85	56.10	122.10	188.10	254.10	320.10	386.10	452.10	518.10	584.10	650.10
86	56.76	122.76	188.76	254.76	320.76	386.76	452.76	518.76	584.76	650.76
87	57.42	123.42	189.42	255.42	321.42	387.42	453.42	519.42	585.42	651.42
88	58.08	124.08	190.08	256.08	322.08	388.08	454.08	520.08	586.08	652.08
89	58.74	124.74	190.74	256.74	322.74	388.74	454.74	520.74	586.74	652.74
90	59.40	125.40	191.40	257.40	323.40	389.40	455.40	521.40	587.40	653.40
91	60.06	126.06	192.06	258.06	324.06	390.06	456.06	522.06	588.06	654.06
92	60.72	126.72	192.72	258.72	324.72	390.72	456.72	522.72	588.72	654.72
93	61.38	127.38	193.38	259.38	325.38	391.38	457.38	523.38	589.38	655.38
94	62.04	128.04	194.04	260.04	326.04	392.04	458.04	524.04	590.04	656.04
95	62.70	128.70	194.70	260.70	326.70	392.70	458.70	524.70	590.70	656.70
96	63.36	129.36	195.36	261.36	327.36	393.36	459.36	525.36	591.36	657.36
97	64 02	130.02	196.02	262.02	328.02	394.02	460.02	526.02	592.02	658.02
98	64.68	130.68	196.68	262.68	328.68	394.68	461.68	526.68	592.68	658.68
99	65.34	131.34	197.34	263.34	329.34	395.34	462.34	527.34	593.34	659.34

PROPORTIONAL PARTS TO TENTHS OF A LINK.

Links, .	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Feet, .	0.066	0.132	0.198	0.264	0.330	0.396	0.462	0.528	0.594

EXEMPLIFICATION OF THE USE OF THE TABLE.

1. In 636 links how many feet? . . . Ans. 419.76 feet.

2. In 758.5 links how many feet?

758 links give	500.28 feet.
0.5 links gives	0.33 feet.
758.5 links give	500.61 feet.

3. In 457.6 feet how many links?

457.38 feet give	693.00 links.
0.198 foot gives	0.30
0.022	0.03
457.600 feet give	693.33 links.

4. In 34568 links how many feet?

34500 links	22770.00 feet.
68 links	44.88 feet.
34568 links	22814.88 feet.

5. The length of a line of a survey is 4725.6 links; required the length in feet?

4720 links	3115.200 feet.
5.6 links	3.696 feet.
4725.6 links	3118.896 feet.

6. The length of a survey is 654.7 feet; required the length in links?

654.06 feet	991.00 links.
0.6402	0.97 links.
654.7002	991.97 links.

Or, since 654.72 feet = 992 links, this may be considered the equivalent very nearly.

TABLES FOR TURNING ROODS AND PERCHES INTO MONEY.

Roods	At 1s. per acre. L S D F P	At 2s. per acre. L S D F P	At 3s. per acre. L S D F P	At 4s. per acre. L S D F P	At 5s. per acre. L S D F P
1	0 0 3 0.0	0 0 6 0.0	0 0 9 0.0	0 1 0 0.0	0 1 3 0.0
2	0 0 6 0.0	0 1 0 0.0	0 1 6 0.0	0 2 0 0.0	0 2 0 0.0
3	0 0 9 0.0	0 1 6 0.0	0 2 3 0.0	0 3 0 0.0	0 3 9 0.0
Perches					
1	0 0 0 0.3	0 0 0 0.6	0 0 0 0.9	0 0 0 1.2	0 0 0 1.5
2	0 0 0 0.6	0 0 0 1.2	0 0 0 1.8	0 0 0 2.4	0 0 0 3.0
3	0 0 0 0.9	0 0 0 1.8	0 0 0 2.7	0 0 0 3.6	0 0 1 0.5
4	0 0 0 1.2	0 0 0 2.4	0 0 0 3.6	0 0 1 0.8	0 0 1 2.0
5	0 0 0 1.5	0 0 0 3.0	0 0 1 0.5	0 0 1 2.0	0 0 1 3.5
6	0 0 0 1.8	0 0 0 3.6	0 0 1 1.4	0 0 1 3.2	0 0 2 1.0
7	0 0 0 2.1	0 0 1 0.2	0 0 1 2.3	0 0 2 0.4	0 0 2 2.5
8	0 0 0 2.4	0 0 1 0.8	0 0 1 3.2	0 0 2 1.6	0 0 3 0.0
9	0 0 0 2.7	0 0 1 1.4	0 0 2 0.1	0 0 2 2.8	0 0 3 1.5
10	0 0 0 3.0	0 0 1 2.0	0 0 2 1.0	0 0 3 0.0	0 0 3 3.0
11	0 0 0 3.3	0 0 1 2.6	0 0 2 1.9	0 0 3 1.2	0 0 4 0.5
12	0 0 0 3.6	0 0 1 3.2	0 0 2 2.8	0 0 3 2.4	0 0 4 2.0
13	0 0 0 3.9	0 0 1 3.8	0 0 2 3.7	0 0 3 3.6	0 0 4 3.5
14	0 0 1 0.2	0 0 2 0.4	0 0 3 0.6	0 0 4 0.8	0 0 5 1.0
15	0 0 1 0.5	0 0 2 1.0	0 0 3 1.5	0 0 4 2.0	0 0 5 2.5
16	0 0 1 0.8	0 0 2 1.6	0 0 3 2.4	0 0 4 3.2	0 0 6 0.0
17	0 0 1 1.1	0 0 2 2.2	0 0 3 3.3	0 0 5 0.4	0 0 6 1.5
18	0 0 1 1.4	0 0 2 2.8	0 0 4 0.2	0 0 5 1.6	0 0 6 3.0
19	0 0 1 1.7	0 0 2 3.4	0 0 4 1.1	0 0 5 2.8	0 0 7 0 5
20	0 0 1 2.0	0 0 3 0.0	0 0 4 2.0	0 0 6 0.0	0 0 7 2.0
21	0 0 1 2.3	0 0 3 0.6	0 0 4 2.9	0 0 6 1.2	0 0 7 3.5
22	0 0 1 2.6	0 0 3 1.2	0 0 4 3.8	0 0 6 2.4	0 0 8 1.0
23	0 0 1 2.9	0 0 3 1.8	0 0 5 0.7	0 0 6 3.6	0 0 8 2.5
24	0 0 1 3.2	0 0 3 2.4	0 0 5 1.6	0 0 7 0.8	0 0 9 0.0
25	0 0 1 3.5	0 0 3 3.0	0 0 5 2.5	0 0 7 2.0	0 0 9 1.5
26	0 0 1 3.8	0 0 3 3.6	0 0 5 3.4	0 0 7 3.2	0 0 9 3.0
27	0 0 2 0.1	0 0 4 0.2	0 0 6 0.3	0 0 8 0.4	0 0 10 0.5
28	0 0 2 0.4	0 0 4 0.8	0 0 6 1.2	0 0 8 1.6	0 0 10 2.0
29	0 0 2 0.7	0 0 4 1.4	0 0 6 2.1	0 0 8 2.8	0 0 10 3.5
30	0 0 2 1.0	0 0 4 2.0	0 0 6 3.0	0 0 9 0.0	0 0 11 1.0
31	0 0 2 1.3	0 0 4 2.6	0 0 6 3.9	0 0 9 1.2	0 0 11 2.5
32	0 0 2 1.6	0 0 4 3.2	0 0 7 0.8	0 0 9 2.4	0 1 0 0.0
33	0 0 2 1.9	0 0 4 3.8	0 0 7 1.7	0 0 9 3.6	0 1 0 1.5
34	0 0 2 2.2	0 0 5 0.4	0 0 7 2.6	0 0 10 0.8	0 1 0 3.0
35	0 0 2 2.5	0 0 5 1.0	0 0 7 3.5	0 0 10 2.0	0 1 1 0.5
36	0 0 2 2.8	0 0 5 1.6	0 0 8 0.4	0 0 10 3.2	0 1 1 2.0
37	0 0 2 3.1	0 0 5 2.2	0 0 8 1.3	0 0 11 0.4	0 1 1 3.5
38	0 0 2 3.4	0 0 5 2.8	0 0 8 2.2	0 0 11 1.6	0 1 2 1.0
39	0 0 2 3.7	0 0 5 3.4	0 0 8 3.1	0 0 11 2.8	0 1 2 2.5

Roods	At 6s. per acre. L S D F P	At 7s. per acre. L S D F P	At 8s. per acre. L S D F P	At 9s. per acre. L S D F P	At 10s. per acre. L S D F P
1	0 1 6 0.0	0 1 9 0.0	0 2 0 0.0	0 2 3 0.0	0 2 6 0.0
2	0 3 0 0.0	0 3 6 0.0	0 4 0 0.0	0 4 6 0.0	0 5 0 0.0
3	0 4 6 00	0 5 3 0.0	0 6 0 0.0	0 6 9 0.0	0 7 6 0.0
Perches					
1	0 0 0 1.8	0 0 0 2.1	0 0 0 2.4	0 0 0 2.7	0 0 0 3.0
2	0 0 0 3.6	0 0 1 0.2	0 0 1 0.8	0 0 1 1.4	0 0 1 2.0
3	0 0 1 1.4	0 0 1 2.3	0 0 1 3.2	0 0 2 0.1	0 0 2 1.0
4	0 0 1 3.2	0 0 2 0.4	0 0 1 1.6	0 0 2 2.8	0 0 3 0.0
5	0 0 2 1.0	0 0 2 2.5	0 0 3 0.0	0 0 3 1.5	0 0 3 3.0
6	0 0 2 2.8	0 0 3 0.6	0 0 3 2.4	0 0 4 0.2	0 0 4 2.0
7	0 0 3 0.6	0 0 3 2.7	0 0 4 0 8	0 0 4 2.9	0 0 5 1.0
8	0 0 3 2.4	0 0 4 0.8	0 0 4 3.2	0 0 5 1.6	0 0 6 0.0
9	0 0 4 0.2	0 0 4 2.9	0 0 5 1.6	0 0 6 0.3	0 0 6 3.0
10	0 0 4 2.0	0 0 5 1.0	0 0 6 0.0	0 0 6 3.0	0 0 7 2.0
11	0 0 4 3.8	0 0 5 3.1	0 0 6 2.4	0 0 7 1.7	0 0 8 1.0
12	0 0 5 1.6	0 0 6 1.2	0 0 7 0.8	0 0 8 0.4	0 0 9 0.0
13	0 0 5 3.4	0 0 6 3.3	0 0 7 3.2	0 0 8 3.1	0 0 9 3.0
14	0 0 6 1.2	0 0 7 1.4	0 0 8 1.6	0 0 9 1.8	0 0 10 2.0
15	0 0 6 3.0	0 0 7 3.5	0 0 9 0.0	0 0 10 0.5	0 0 11 1.0
16	0 0 7 0.8	0 0 8 1.6	0 0 9 2.4	0 0 10 3.2	0 1 0 0.0
17	0 0 7 2.6	0 0 8 3.7	0 0 10 0.8	0 0 11 1.9	0 1 0 3.0
18	0 0 8 0.4	0 0 9 1.8	0 0 10 3.2	0 1 0 0.6	0 1 1 2.0
19	0 0 8 2.2	0 0 9 3.9	0 0 11 1.6	0 1 0 3.3	0 1 2 1.0
20	0 0 9 0.0	0 0 10 2.0	0 1 0 0.0	0 1 1 2.0	0 1 3 0.0
21	0 0 9 1.8	0 0 11 0.1	0 1 0 2.4	0 1 2 0.7	0 1 3 3.0
22	0 0 9 3.6	0 0 11 2.2	0 1 1 0.8	0 1 2 3.4	0 1 4 2.0
23	0 0 10 1.4	0 1 0 0.8	0 1 1 3.2	0 1 3 2.1	0 1 5 1.0
24	0 0 10 3.2	0 1 0 2.4	0 1 2 1.6	0 1 4 0.8	0 1 6 0.0
25	0 0 11 1.0	0 1 1 0.5	0 1 3 0.0	0 1 4 3.5	0 1 6 3.0
26	0 0 11 2.8	0 1 1 2.6	0 1 3 2.4	0 1 5 2.2	0 1 7 2.0
27	0 1 0 0.6	0 1 2 0.7	0 1 4 0.8	0 1 6 0.9	0 1 8 1.0
28	0 1 0 2.4	0 1 2 2.8	0 1 4 3.2	0 1 6 3.6	0 1 9 00
29	0 1 1 0.2	0 1 3 0.9	0 1 5 1.6	0 1 7 2.3	0 1 9 3.0
30	0 1 1 2.0	0 1 3 3.0	0 1 6 0.0	0 1 8 1.0	0 1 10 2.0
31	0 1 1 3.8	0 1 4 1.1	0 1 6 2.4	0 1 8 3.7	0 1 11 1.0
32	0 1 2 1.6	0 1 4 3.2	0 1 7 0.8	0 1 9 2.4	0 2 0 0.0
33	0 1 2 3.4	0 1 5 1.3	0 1 7 3.2	0 1 10 1.1	0 2 0 3.0
34	0 1 3 1.2	0 1 5 3.4	0 1 8 1.6	0 1 10 3.8	0 2 1 2.0
35	0 1 3 3.0	0 1 6 1.5	0 1 9 0.0	0 1 11 2.5	0 2 2 1.0
36	0 1 4 0.8	0 1 6 3.6	0 1 9 2.4	0 2 0 1.2	0 2 3 0.0
37	0 1 4 2.6	0 1 7 1.7	0 1 10 0.8	0 2 0 3.9	0 2 3 3.0
38	0 1 5 0.4	0 1 7 3.8	0 1 10 3.2	0 2 1 2.6	0 2 4 2.0
39	0 1 5 2.2	0 1 8 1.9	0 1 11 1.6	0 2 2 1.3	0 2 5 1.0

TABLES FOR TURNING ROODS AND PERCHES INTO MONEY—*continued.*

Roods	At 11s. per acre.	At 12s. per acre.	At 13s. per acre.	At 14s. per acre.	At 15s. per acre.	At 16s. per acre.	At 17s. per acre.	At 18s. per acre.	At 19s. per acre.	At 20s. per acre.
	L S D FP	L S D FP	L S D FP	L S D FP	L S D FP	L S D FP	L S D FP	L S D FP	L S D FP	L S D FP
1	0 2 9 0.0	0 3 0 0.0	0 3 3 0.0	0 3 6 0.0	0 3 9 0.0	0 4 0 0.0	0 4 3 0.0	0 4 6 0.0	0 4 9 0.0	0 5 0 0.0
2	0 5 6 00	0 6 0 0.0	0 6 6 0.0	0 7 0 0.0	0 7 6 0.0	0 8 0 0.0	0 8 6 0.0	0 9 0 0.0	0 9 6 0.0	0 10 0 0.0
3	0 8 3 0.0	0 9 0 0.0	0 9 9 0.0	0 10 6 0.0	0 11 3 0.0	0 12 0 0.0	0 12 9 0.0	0 13 6 0.0	0 14 3 0.0	0 15 0 0.0
Perches										
1	0 0 0 3.3	0 0 0 3.6	0 0 0 3.9	0 0 1 0.2	0 0 1 0.5	0 0 1 0.8	0 0 1 1.1	0 0 1 1.4	0 0 1 1.7	0 0 1 2.0
2	0 0 1 2.6	0 0 1 3.2	0 0 1 3.8	0 0 2 0.4	0 0 2 1.0	0 0 2 1.6	0 0 2 2.2	0 0 2 2.8	0 0 1 3.4	0 0 3 0.0
3	0 0 2 1.9	0 0 2 2.8	0 0 2 3.7	0 0 3 0.6	0 0 3 1.5	0 0 3 2.4	0 0 3 3.3	0 0 4 0.2	0 0 4 1.1	0 0 4 2.0
4	0 0 3 1.2	0 0 3 2.4	0 0 3 3.6	0 0 4 0.8	0 0 4 2.0	0 0 4 3.2	0 0 5 0.4	0 0 5 1.6	0 0 5 2.8	0 0 6 0.0
5	0 0 4 0.5	0 0 4 2.0	0 0 4 3.5	0 0 5 1.0	0 0 5 2.5	0 0 6 0.0	0 0 6 1.5	0 0 6 3.0	0 0 7 0.5	0 9 7 2.0
6	0 0 4 3.8	0 0 5 1.6	0 0 5 3.4	0 0 6 1.2	0 0 6 3.0	0 0 7 0.8	0 0 7 2.6	0 0 8 0.4	0 0 8 2.2	0 0 9 0.0
7	0 0 5 3.1	0 0 6 1.2	0 0 6 3.3	0 0 7 1.4	0 0 7 3.5	0 0 8 1.6	0 0 8 3.7	0 0 9 1.8	0 0 9 3.9	0 0 10 2.0
8	0 0 6 2.4	0 0 7 0.8	0 0 7 3.2	0 0 8 1.6	0 0 9 0.0	0 0 9 2.4	0 0 10 0.8	0 0 10 3.2	0 0 11 1.6	0 1 0 0.0
9	0 0 7 1.7	0 0 8 0.4	0 0 8 3.1	0 0 9 1.8	0 0 10 0.5	0 0 10 3.2	0 0 11 1.9	0 1 0 0.6	0 1 0 3.3	0 1 1 2.0
10	0 0 8 1.0	0 0 9 0.0	0 0 9 3.0	0 0 10 2.0	0 0 11 1.0	0 1 0 0.0	0 1 0 3.0	0 1 1 2.0	0 1 2 1.0	0 1 3 0.0
11	0 0 9 0.3	0 0 9 3.6	0 0 10 2.9	0 0 11 2.2	0 1 0 1.5	0 1 1 0.8	0 1 2 0.1	0 1 2 3.4	0 1 3 2.7	0 1 4 2.0
12	0 0 9 3.6	0 0 10 3.2	0 0 11 2.8	0 1 0 2.4	0 1 1 2.0	0 1 2 1.6	0 1 3 1.2	0 1 4 0.8	0 1 5 0.4	0 1 6 0.0
13	0 0 10 2.9	0 0 11 2.8	0 1 0 2.7	0 1 1 2.6	0 1 2 2.5	0 1 3 2.4	0 1 4 2.3	0 1 5 2.2	0 1 6 2.1	0 1 7 2.0
14	0 0 11 2.2	0 1 0 2.4	0 1 1 2.6	0 1 2 2.8	0 1 3 3.0	0 1 4 3.2	0 1 5 3.4	0 1 6 3.6	0 1 7 3.8	0 1 9 0.0
15	0 1 0 1.5	0 1 1 2.0	0 1 2 2.5	0 1 3 3.0	0 1 4 3.5	0 1 6 0.0	0 1 7 0.5	0 1 8 1.0	0 1 9 1.5	0 1 10 2.0
16	0 1 1 0.8	0 1 2 1.6	0 1 3 2.4	0 1 4 3.2	0 1 6 0.0	0 1 7 0.8	0 1 8 1.6	0 1 9 2.4	0 1 10 3.2	0 2 0 0.0
17	0 1 2 0.1	0 1 3 1.2	0 1 4 2.3	0 1 5 3.4	0 1 7 0.5	0 1 8 1.6	0 1 9 2.7	0 1 10 3.8	0 2 0 0.9	0 2 1 2.0
18	0 1 2 3.4	0 1 4 0.8	0 1 5 2.2	0 1 6 3.6	0 1 8 1.0	0 1 9 2.4	0 1 10 3.8	0 2 0 1.2	0 2 1 2.6	0 2 3 0.0
19	0 1 3 2.7	0 1 5 0.4	0 1 6 2.1	0 1 7 3.8	0 1 9 1.5	0 1 10 3.2	0 2 0 0.9	0 2 1 2.6	0 2 3 0.3	0 2 4 2.0
20	0 1 4 2.0	0 1 6 0.0	0 1 7 2.0	0 1 9 0.0	0 1 10 2.0	0 2 0 0.0	0 2 1 2.0	0 2 3 0.0	0 2 4 2.0	0 2 6 0.0
21	0 1 5 1.3	0 1 6 3.6	0 1 8 1.9	0 1 10 0.2	0 1 11 2.5	0 2 1 0.8	0 2 2 3.1	0 2 4 1.4	0 2 5 3.7	0 2 7 2.0
22	0 1 6 0.6	0 1 7 3.2	0 1 9 1.8	0 1 11 0.4	0 2 0 3.0	0 2 2 1.6	0 2 4 0.2	0 2 5 2.8	0 2 7 1.4	0 2 9 0.0
23	0 1 6 3.9	0 1 8 2.8	0 1 10 1.7	0 2 0 0.6	0 2 1 3.5	0 2 3 2.4	0 2 5 1.3	0 2 7 0.2	0 2 8 3.1	0 2 10 2.0
24	0 1 7 3.2	0 1 9 2.4	0 1 11 1.6	0 2 1 0.8	0 2 3 0.0	0 2 4 3.2	0 2 6 2.4	0 2 8 1.6	0 2 10 0.8	0 3 0 0.0
25	0 1 8 2.5	0 1 10 2.0	0 2 0 1.5	0 2 2 1.0	0 2 4 0.5	0 2 6 0.0	0 2 7 3.5	0 2 9 3.0	0 2 11 2.5	0 3 1 2.0
26	0 1 9 1.8	0 1 11 1.6	0 2 1 1.4	0 2 3 1.2	0 2 5 1.0	0 2 7 0.8	0 2 9 0.6	0 2 11 0.4	0 3 1 0.2	0 3 3 0.0
27	0 1 10 1.1	0 2 0 1.2	0 2 2 1.3	0 2 4 1.4	0 2 6 1.5	0 2 8 1.6	0 2 10 1.7	0 3 0 1.8	0 3 2 1.9	0 3 4 2.0
28	0 1 11 0.4	0 2 1 0.8	0 2 3 1.2	0 2 5 1.6	0 2 7 2.0	0 2 9 2.4	0 2 11 2.8	0 3 1 3.2	0 3 3 3.6	0 3 6 0.0
29	0 1 11 3.7	0 2 2 0.4	0 2 4 1.1	0 2 6 1.8	0 2 8 2.5	0 2 10 3.2	0 3 0 3.9	0 3 3 0.6	0 3 5 1.3	0 3 7 2.0
30	0 2 0 3.0	0 2 3 0.0	0 2 5 1.0	0 2 7 2.0	0 2 9 3.0	0 3 0 0.0	0 3 2 1.0	0 3 4 2.0	0 3 6 3.0	0 3 9 0.0
31	0 2 1 2.3	0 2 3 3.6	0 2 6 0.9	0 2 8 2.2	0 2 10 3.5	0 3 1 0.8	0 3 3 2.1	0 3 5 3.4	0 3 8 0.7	0 3 10 2.0
32	0 2 2 1.6	0 2 4 3.2	0 2 7 0.8	0 2 9 2.4	0 3 0 0.0	0 3 2 1.6	0 3 4 3.2	0 3 7 0.8	0 3 9 2.4	0 4 0 0.0
33	0 2 3 0.9	0 2 5 2.8	0 2 8 0.7	0 2 10 2.6	0 3 1 0.5	0 3 3 2.4	0 3 6 0.3	0 3 8 2.2	0 3 11 0.1	0 4 1 2.0
34	0 2 4 0.2	0 2 6 2.4	0 2 9 0.6	0 2 11 2.8	0 3 2 1.0	0 3 4 3.2	0 3 7 1.4	0 3 9 3.6	0 4 0 1.8	0 4 3 0.0
35	0 2 4 3.5	0 2 7 2.0	0 2 10 0.5	0 3 0 3.0	0 3 3 1.5	0 3 6 0.0	0 3 8 2.5	0 3 11 1.0	0 4 1 3.5	0 4 4 2.0
36	0 2 5 2.8	0 2 8 1.6	0 2 11 0.4	0 3 1 3.2	0 3 4 2.0	0 3 7 0.8	0 3 9 3.6	0 4 0 2.4	0 4 3 1.2	0 4 6 0.0
37	0 2 6 2.1	0 2 9 1.2	0 3 0 0.3	0 3 2 3.4	0 3 5 2.5	0 3 8 1.6	0 3 11 0.7	0 4 1 3.8	0 4 4 2.9	0 4 7 2.0
38	0 2 7 1.4	0 2 10 0.8	0 3 1 0.2	0 3 3 3.6	0 3 6 3.0	0 3 2 2.4	0 4 0 1.8	0 4 3 1.2	0 4 6 0.6	0 4 9 0.0
39	0 2 8 0.7	0 2 11 0.4	0 3 2 0.1	0 3 4 3.8	0 3 7 3.5	0 3 10 3.2	0 4 1 2.9	0 4 4 2.6	0 4 7 2.3	0 4 10 2.0

TABLES FOR TURNING ROODS AND PERCHES INTO MONEY—*concluded.*

Roods	At L.2 per acre.	At L.3 per acre.	At L.4 per acre.	At L.5 per acre.	At L.6 per acre.	At L.7 per acre.	At L.8 per acre.	At L.9 per acre.	At L.10 per acre.	At L.11 per acre.
	L S D F P	L S D F P	L S D F P	L S D F P	L S D F P	L S D F P	L S D F P	L S D F P	L S D F P	L S D F P
1	0 10 0 0.0	0 15 0 0.0	1 0 0 0.0	1 5 0 0.0	1 10 0 0.0	1 15 0 0.0	2 0 0 0.0	2 5 0 0.0	2 10 0 0.0	2 15 0 0.0
2	1 0 0 0.0	1 10 0 0.0	2 0 0 0.0	2 10 0 0.0	3 0 0 0.0	3 10 0 0.0	4 0 0 0.0	4 10 0 0.0	5 0 0 0.0	5 10 0 0.0
3	1 10 0 0.0	2 5 0 0.0	3 0 0 0.0	3 15 0 0.0	4 10 0 0.0	5 5 0 0.0	6 0 0 0.0	6 15 0 0.0	7 10 0 0.0	8 5 0 0.0
Perches										
1	0 0 3 0.0	0 0 4 2.0	0 0 6 0.0	0 0 7 2.0	0 0 9 0.0	0 0 10 2.0	0 1 0 0.0	0 1 1 2.0	0 1 3 0.0	0 1 4 2.0
2	0 0 6 0.0	0 0 9 0.0	0 1 0 0.0	0 1 3 0.0	0 1 6 0.0	0 1 9 0.0	0 2 0 0.0	0 2 3 0.0	0 2 6 0.0	0 2 9 0.0
3	0 0 9 0.0	0 1 1 1.2	0 1 6 0.0	0 1 10 2.0	0 2 3 0.0	0 2 7 2.0	0 3 0 0.0	0 3 4 2.0	0 3 9 0.0	0 4 1 2.0
4	0 1 0 0.0	0 1 6 0.0	0 2 0 0.0	0 2 6 0.0	0 3 0 0.0	0 3 6 0.0	0 4 0 0.0	0 4 6 0.0	0 5 0 0.0	0 5 6 0.0
5	0 1 3 0.0	0 1 10 2.0	0 2 6 0.0	0 3 1 2.0	0 3 9 0.0	0 4 4 2.0	0 5 0 0.0	0 5 7 2.0	0 6 3 0.0	0 6 10 2.0
6	0 1 6 0.0	0 2 3 0.0	0 3 0 0.0	0 3 9 0.0	0 4 6 0.0	0 5 3 0.0	0 6 0 0.0	0 6 9 0.0	0 7 6 0.0	0 8 3 0.0
7	0 1 9 0.0	0 2 7 2.0	0 3 6 0.0	0 4 4 2.0	0 5 3 0.0	0 6 1 2.0	0 7 0 0.0	0 7 10 2.0	0 8 9 0.0	0 9 7 2.0
8	0 2 0 0.0	0 3 0 0.0	0 4 0 0.0	0 5 0 0.0	0 6 0 0.0	0 7 0 0.0	0 8 0 0.0	0 9 0 0.0	0 10 0 0.0	0 11 0 0.0
9	0 2 3 0.0	0 3 4 2.0	0 4 6 0.0	0 5 7 2.0	0 6 9 0.0	0 7 10 2.0	0 9 0 0.0	0 9 1 2.0	0 11 3 0.0	0 12 4 2.0
10	0 2 6 0.0	0 3 9 0.0	0 5 0 0.0	0 6 3 0.0	0 7 6 0.0	0 8 9 0.0	0 10 0 0.0	0 11 3 0.0	0 12 6 0.0	0 13 9 0.0
11	0 2 9 0.0	0 4 1 2.0	0 5 6 0.0	0 6 10 2.0	0 8 3 0.0	0 9 7 2.0	0 11 0 0.0	0 12 4 2.0	0 13 9 0.0	0 15 1 2.0
12	0 3 0 0.0	0 4 6 0.0	0 6 0 0.0	0 7 6 0.0	0 9 0 0.0	0 10 6 0.0	0 12 0 0 0	0 13 6 0.0	0 15 0 0.0	0 16 6 0.0
13	0 3 3 0.0	0 4 10 2.0	0 6 6 0.0	0 8 1 2.0	0 9 9 0 0	0 11 4 2.0	0 13 0 0.0	0 14 7 2.0	0 16 3 0.0	0 17 10 2.0
14	0 3 6 0.0	0 5 3 0.0	0 7 0 0.0	0 8 9 0.0	0 10 6 0.0	0 12 3 0.0	0 14 0 0.0	0 15 9 0.0	0 17 6 0.0	0 19 3 0.0
15	0 3 9 0.0	0 5 7 2.0	0 7 6 0.0	0 9 4 2.0	0 11 3 0.0	0 13 1 2.0	0 15 0 0.0	0 16 10 2.0	0 18 9 0.0	1 0 7 2.0
16	0 4 0 0.0	0 6 0 0.0	0 8 0 0.0	0 10 0 0.0	0 12 0 0.0	0 14 0 0.0	0 16 0 0.0	0 18 0 0.0	1 0 0 0.0	1 2 0 0.0
17	0 4 3 0.0	0 6 4 2.0	0 8 6 0.0	0 10 7 2.0	0 12 9 0.0	0 14 10 2.0	0 17 0 0.0	0 19 1 2.0	1 1 3 0.0	1 3 4 2.0
18	0 4 6 0.0	0 6 9 0.0	0 9 0 0.0	0 11 3 0.0	0 13 6 0.0	0 15 9 0.0	0 18 0 0.0	1 0 3 0.0	1 2 6 0.0	1 4 9 0.0
19	0 4 9 0.0	0 7 1 2.0	0 9 6 0.0	0 11 10 2.0	0 14 3 0.0	0 16 7 2.0	0 19 0 0.0	1 1 4 2.0	1 3 9 0.0	1 6 1 2.0
20	0 5 0 0.0	0 7 6 0.0	0 10 0 0.0	0 12 6 0.0	0 15 0 0.0	0 17 6 0.0	1 0 0 0.0	1 2 6 0.0	1 5 0 0.0	1 7 6 0.0
21	0 5 3 0.0	0 7 10 2.0	0 10 6 0.0	0 13 1 2.0	0 15 9 0.0	0 18 4 2.0	1 1 0 0.0	1 3 7 2.0	1 6 3 0 0	1 8 10 2.0
22	0 5 6 0.0	0 8 3 3.0	0 11 0 0.0	0 13 9 0.0	0 16 6 0.0	0 19 3 0.0	1 2 0 0.0	1 4 9 0.0	1 7 6 0.0	1 10 3 0.0
23	0 5 9 0.0	0 8 7 2.0	0 11 6 0.0	0 14 4 2.0	0 17 3 0.0	1 0 1 2.0	1 3 0 0.0	1 5 10 2.0	1 8 9 0 0	1 11 7 2.0
24	0 6 0 0.0	0 9 0 0.0	0 12 0 0.0	0 15 0 0.0	0 18 0 0.0	1 1 0 0.0	1 4 0 0.0	1 7 0 0.0	1 10 0 0.0	1 13 0 0.0
25	0 6 3 0.0	0 9 4 2.0	0 12 6 0.0	0 15 7 2.0	0 18 9 0.0	1 1 10 2.0	1 5 0 0.0	1 8 1 2.0	1 11 3 0.0	1 14 4 2.0
26	0 6 6 0.0	0 9 9 0.0	0 13 0 0.0	0 16 3 0.0	0 19 6 0.0	1 2 9 0.0	1 6 0 0.0	1 9 3 0 0	1 12 6 0.0	1 15 9 0.0
27	0 6 9 0.0	0 10 1 2.0	0 13 6 0.0	0 16 10 2.0	1 0 3 0.0	1 3 7 2.0	1 7 0 0.0	1 10 4 2.0	1 13 9 0.0	1 17 1 2.0
28	0 7 0 0.0	0 10 6 0.0	0 14 0 0.0	0 17 6 0.0	1 1 0 0.0	1 4 6 0.0	1 8 0 0.0	1 11 6 0.0	1 15 0 0.0	1 18 6 0.0
29	0 7 3 0.0	0 10 10 2.0	0 14 6 0.0	0 18 1 2.0	1 1 9 0.0	1 5 4 2.0	1 9 0 0.0	1 12 7 2.0	1 16 3 0.0	1 19 10 2.0
30	0 7 6 0.0	0 11 3 0.0	0 15 0 0.0	0 18 9 0.0	1 2 6 0.0	1 6 3 0.0	1 10 0 0.0	1 13 9 0.0	1 17 6 0.0	2 1 3 0.0
31	0 7 9 0.0	0 11 7 2.0	0 15 6 0.0	0 19 4 2.0	1 3 3 0.0	1 7 1 2.0	1 11 0 0.0	1 14 10 2.0	1 18 9 0.0	2 2 7 2.0
32	0 8 0 0.0	0 12 0 0.0	0 16 0 0.0	1 0 0 0.0	1 4 0 0.0	1 8 0 0.0	1 12 0 0.0	1 16 0 0.0	2 0 0 0.0	2 4 0 0.0
33	0 8 3 0.0	0 12 4 2.0	0 16 6 0.0	1 0 7 2.0	1 4 9 0.0	1 8 10 2.0	1 13 0 0.0	1 17 1 2.0	2 1 3 0.0	2 5 4 2.0
34	0 8 6 0.0	0 12 9 0.0	0 17 0 0.0	1 1 3 0.0	1 5 6 0.0	1 9 9 0.0	1 14 0 0.0	1 18 3 0.0	2 2 6 0.0	2 6 9 0.0
35	0 8 9 0.0	0 13 1 2.0	0 17 6 0.0	1 1 10 2.0	1 6 3 0.0	1 10 7 2.0	1 15 0 0.0	1 19 4 2.0	2 3 9 0.0	2 8 1 2.0
36	0 9 0 0.0	0 13 6 0.0	0 18 0 0.0	1 2 6 0.0	1 7 0 0.0	1 11 6 0.0	1 16 0 0.0	2 0 6 0.0	2 5 0 0.0	2 9 6 0.0
37	0 9 3 0.0	0 13 10 2.0	0 18 6 0.0	1 3 1 2.0	1 7 9 0.0	1 12 4 2.0	1 17 0 0.0	2 1 7 2.0	2 6 3 0.0	2 10 10 2.0
38	0 9 6 0.0	0 14 3 0.0	0 19 0 0.0	1 3 9 0.0	1 8 6 0.0	1 13 3 0.0	1 18 0 0.0	2 2 9 0.0	2 7 6 0.0	2 12 3 0.0
39	0 9 9 0.0	0 14 7 2.0	0 19 6 0.0	1 4 4 2.0	1 9 3 0.0	1 14 1 2.0	1 19 0 0.0	2 3 10 2.0	2 8 9 0.0	2 13 7 2.0

The foregoing Tables for converting roods, perches, poles, or falls, into money, will be found useful to a surveyor or a land-valuator for ascertaining the value of an estate.

Example 1st. Suppose it is required to turn 3 roods and 36 perches or falls into cash, valued at 18s. per acre.

Look for 18s. at the top column, and for 3 roods in the left hand column, and under 18, and opposite 3 roods, is	£0	13	6
Again, look for 36 perches or falls in the left hand column, and under 18s. in the top column, is	0	4	0½
Value, . . .	0	17	6½

Example 2d. Suppose 2 roods and 39 perches or poles, valued at 11s. per acre, is required to be turned into money.

Look at the top column for 11s., and in the left hand column for 2 roods, and opposite it under 11s., is . . .	£0	5	6
Again, look for 39 perches in the left column, and opposite it, under 11, is	0	2	$8\frac{7}{12}$
Value, . . .	0	8	$2\frac{7}{12}$

Example 3d. Suppose 15 acres 3 roods and 14 perches, valued at 12s. per acre, is to be turned into cash.

First multiply 15 acres by 12s., the quotient is 180s., equal to	£9	0	0
Then look at the top column for 12s., and opposite 3 roods, is	0	9	0
Then opposite 14 perches, and under 12s., is . . .	0	1	0½
Value, . . .	9	10	0½

Example 4th. Suppose 20 acres 1 rood and 35 perches or falls, valued at L.3, 10s. per acre.

First, £3, 10s. is = 70s., which, multiplied by 20 acres, is 1400s. = ...	£70	0	0
Then look into the table for £3, and opposite 1 rood, is .	0	15	0
Again, look into the table for £3, and opposite 35 perches, is	0	13	1½
Then look into the table for 10s., and opposite 1 rood, is .	0	2	6
Again, look into the table for 10s., and opposite 35 perches, is	0	2	2¼
Value, . . .	71	12	9¾

Another method for turning acres, roods, perches, poles or falls, into money, I frequently accomplish by the table for reducing acres, roods, &c., into square links, page 224, and multiply by the amount per acre the ground is valued at, suppose 70s. per acre.

Thus 20 acres, when turned into square links, is	. .	20.00000
1 rood, . . . do. . is	. .	25000
35 perches, poles, or falls, do. . is	. .	21875
		20.46875

The calculation thus, 20.46875
70s. the amount per acre.

Shillings 1432.81250
12

Pence 9.75000
4

Farthings 3.0000

	£	s.	d.
1432 shillings, is	£71	12	0
.81250 decimals of shillings, is .	0	0	9
.75000 decimals of pence, is . .	0	0	$0\frac{3}{4}$
	£71	12	$9\frac{3}{4}$ the same as by the

Tables.

Example 5th. Required the value of 1 acre 0 rood 7 poles; Alexander Buckham's lot in the following measurement, at £10, 10s. per acre:—

		£	s.	d.
1 acre at £10, 10s.	=	£10	10	0
7 poles at £10, ...	=	0	8	9
7 poles at 10s., ...	=	0	0	$5\frac{1}{4}$
1 acre, 0 rood, 7 poles, at £10, 10s., . .	=	£10	19	$2\frac{1}{4}$

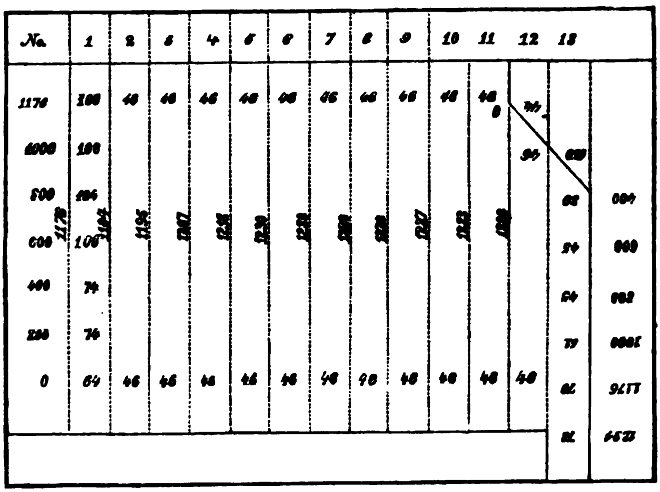
In like manner the values of the remainder of the lots may be found.

I.—FIELD NOTES

FOR COMPUTING THE MEASUREMENT AND VALUE OF CERTAIN LOTS OF CLOVER AND RYE-GRASS, SOLD BY MR JAMES HOGARTH, FARMER, ECCLES-NEWTOWN, TO PERSONS IN AND ABOUT THE VILLAGE OF BIRGHAM,

BY WILLIAM GALBRAITH, SURVEYOR.

DIMENSIONS IN IMPERIAL LINKS.



It may be remarked that when a field is measured in lots, as above, the area of the whole field should be found in the usual way, from independent dimensions, as a check upon that obtained from the sum of the lots,—a method practised by the editor for a period of ten years, when he was annually employed in pretty extensive similar measurements, &c.

II.—MEASUREMENT AND VALUE

OF THE FOREGOING LOTS OF CLOVER AND RYE-GRASS, SOLD BY MR JAMES HOGARTH, TO THE FOLLOWING PERSONS RESIDING IN AND ABOUT THE VILLAGE OF BIRGHAM.

Lots.	Names of Purchasers.	No. of Ridges.	Measure.	Price.	Value.
			A. R. P.	L. S. D.	L. S. D.
1	Alexander Buckham, . .	2	1 0 7	10 10 0	10 19 2¼
2	James Crichton, . . .	1	0 2 7½		5 14 10
3	John Turner, ...	1	0 2 8½		5 16 2
4	Edmund Dodds, 1st, . . .	1	0 2 9¼		5 17 1¾
5	Thomas M'Dougal, . . .	1	0 2 10		5 18 1½
6	Thomas Pringle, ...	1	0 2 10½		5 18 9½
7	Thomas Johnston, . . .	1	0 2 10½		5 18 9½
8	James Merton, ...	1	0 2 10½		5 18 9½
9	John Wood, ...	1	0 2 10½		5 18 9½
10	Edmund Dodds, 2d, . . .	1	0 2 10		5 18 1½
11	Robert Storie, ...	1	0 2 10	11 0 0	6 3 9
12	James Service, ...	1	0 2 1	11 15 0	5 18 11¾
13	James Lighterns, ...	2 butts	0 2 2	11 15 0	6 0 5¼
	Amount of the whole, . . .		7 2 27¼		£82 1 11

Measured and Calculated by

WILLIAM GALBRAITH, SURVEYOR.

Eccles, June 18—

JAMES HOGARTH, Esq.

It may be remarked that the preceding table, page 212, &c., will be useful in computing the above values, and in all similar cases.

The following remarks and principles are in some parts of the country generally attended to, especially where land is valuable; because when a field or more is to be surveyed, it is necessary to have a proper knowledge of what is required, otherwise the surveyor may be liable to take his dimensions wrong, which will, of course, produce an erroneous result, since it would not be that exactly wanted.

1. If the crop on any field has been sold at a certain price per acre, the station or signal staves must be put down exactly at its exterior boundary, so as to take in no more land than what is in actual tillage or crop.

2. If the field be in pasture, the signal poles ought to be put

down at the exterior boundary, or that edge of the fence or ditch, (if there is one,) so that, when the proper dimensions are taken, the true and just area of the whole pasture-land may be obtained.

3. If the proprietor of an estate wants a survey and plan of it, the following particulars should be attended to.

4. First, to divide the estate into such *large triangles* as may be necessary to give a correct outline of the whole, and from which the total amount of acres contained in it may be determined.

5. Then, to proceed to particular fields, observing to measure to the *interior* edge of the ditch; for although when in tillage it cannot be ploughed and cropped exactly to it, yet when in pasture the grass can, by cattle, be eaten to it, or farther—consequently it is frequently a rule to survey *estates* in this way, but not *fields* for the sale of crops.

6. Next in the progress of the survey, proper attention should be paid to the fences, so as to obtain their just area, whether they be dikes, ditches, or of any other denomination.

7. If a field should have a double fence about it—that is, a dike having a hedge and ditch on each side of it—then one half of this fence belongs to the one field, and the other half to adjacent field.

8. Again, if there are roads intersecting the estate, they must be taken into account also; for although they cannot be reckoned arable land, yet they render an estate much more valuable, by these means having free access to any field, and an easy communication to markets, lime, coals, manure, &c.

9. When a road is bounded on both sides by the same proprietor's land, the whole road is measured from ditch to ditch, (if such are the boundaries,) or the side or edge next the road;* for each ditch must be occasionally, as it is technically called, *scoured*, the hedge cleaned, and the dike repaired. Of course, the landlord, or his tenant, must have a power to do this independent of the road trustees, though, for the sake of the road, it is their interest, as well as his, to preserve the road dry and clean, by having the ditches on each side always in proper order.

10. If the land on opposite sides of the road belong to different proprietors, then one half of the road is supposed to belong, or is rather assigned to one, though kept separate, and the other half to the other proprietor—of course not strictly their property, but that

* In this case the dike, and both ditches on each side of it, are to be reckoned the fences of the field to which they belong.

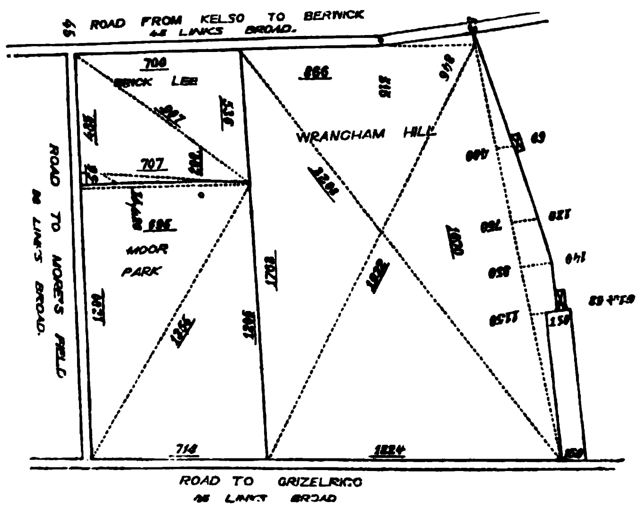
of the public. It is thus easy to show the amount of road accommodation in a property.

11. Lastly, the plantations, meadows, commons, &c., must be all separately measured, and their contents given for the satisfaction of the proprietor; and the grand total resulting from all these particulars will be the whole superficial content of the estate, which must, if both measures are correct, agree nearly with the whole measurement already obtained from the general outline formerly mentioned in No. 4, which affords a satisfactory proof of the truth of all the operations.

This check or proof, however, is seldom attended to by practical surveyors, who think their work pretty correct if within one acre in a hundred of each other.

12. Survey of a small farm or estate in conformity with the preceding principles, in which the arable land, fences, roads, plantations, meadows, commons, ponds, &c., are kept separate. The following is an outline of a few fields surveyed by the editor on this plan for the late James Dickson, Esq. of Antonshill, Berwickshire.

The lines representing the breadth of the ditches should have been dotted lines very close to the exterior continued lines; but, for want of room, cannot be inserted in the reduced plan. From the arable ground to the root of the hedge, the distance generally measured, as a mean, about 10 links, though in the real measure-



ment of the fields the exact distance in every case was carefully measured.

MEASUREMENT.

No.	Names of fields.	Arable land.	Fences.	Amount.
		Acres.	Acres.	Acres.
1	Wrangham Hill, ...	19.710	0.614	20.324
2	Brick Lee, ...	3.698	0.248	3.946
3	Moor Park, ...	8.506	0.388	8.894
	Sums, ...	31.914	1.250	33.164
4	Plantation in Wrangham Hill, ...			1.038
5	Roads on the west side of Moor Park, ...			0.530
	Total, ...			34.735

The road from Kelso to Berwick, and from Eccles to Grizelrigg, being public or parish roads, are not here included, though their contents might, from the data, have been computed.

Surveyed by

W. GALBRAITH.

Eccles, 18—.

MEASURING TIMBER.

As in the measuring either standing or cut timber, a land-surveyor is often employed to ascertain the solid contents in feet, the following Table will be found very expeditious for that purpose. The Table needs little explanation. As every person knows that the length and girth must be taken before the contents can be known, I shall here only mention what way the girth is taken.

Suppose a tree from the bottom to the top tapers equally all the way,—take the girth or circumference in the middle, and divide it by 4, which gives what is called, in customary measure, the quarter girth, or *side of the square*—that is to say, when the bark is cut off the tree, and squared, it has four equal sides; and the girth being taken in the middle of the tree gives a mean thickness: but as many trees are unequal, in that case it will be necessary to take the girth or circumference of the tree in three, four, five, or six places, and add all the different girths into one sum, suppose five girths; dividing the sum by five, gives a mean, and by dividing the mean girth by four, gives the side of the square commonly called the quarter girth. Whatever number of times the girth has been taken, the sum of the girths must be divided by that number; and be particular in taking the girths nearly at equal distances.

TABLE FOR MEASURING TIMBER.

Quarter girth.	Area.	Quarter girth.	Area.	Quarter girth.	Area.
Inches.	F. Dec.	Inches.	F. Dec.	Inches.	F. Dec.
6	0.250	16½	1.890	26½	4.876
6½	0.294	17	2.006	27	5.062
7	0.340	17½	2.126	27½	5.252
7½	0.390	18	2.250	28	5.444
8	0.444	18½	2.376	28½	5.640
8½	0.501	19	2.506	29	5.840
9	0.562	19½	2.640	29½	6.040
9½	0.626	20	2.717	30	6.250
10	0.694	20½	2.917	30½	6.459
10½	0.766	21	3.062	31	6.739
11	0.840	21½	3.209	31½	6.902
11½	0.918	22	3.362	32	6.111
12	1.000	22½	3.516	32½	7.335
12½	1.085	23	3.673	33	7.569
13	1.174	23½	3.835	33½	7.792
13½	1.265	24	4.000	34	8.022
14	1.361	24½	4.168	34½	8.260
14½	1.460	25	4.340	35	8.507
15	1.562	25½	4.516	35½	8.707
15½	1.668	26	4.692	36	8.930
16	1.777				

N.B.—The number of the area opposite the quarter girth, in the right hand column in the Table, must be multiplied by the length of the plank or tree in feet, and the product will be the content in feet and parts of a foot.

Suppose a plank to be 22 feet long, and the mean quarter girth 14 inches—look into the table for 14 inches, and opposite, in the right hand column, and under area, is 1.361; which, multiplied by 22, is equal to 29 feet, 942 decimals; and if you multiply the decimals by 12, is 29 feet, 11 inches, $\frac{304}{1000}$.

Again, suppose a tree to be 30 feet long, 60 inches girth at the thick end, 40 inches girth in the middle, and 20 inches girth at the small end—add up all the girths into one sum, which is 120; which sum, divided by three, the quotient is 40 inches for a mean girth; then divide 40 by four, which is 10 for the quarter girth: lastly, look into the table for the number opposite 10 inches, and you will find .694; which number, multiplied by 30, the length of the tree, the quotient is 20 feet, 820 parts, or 20 feet, 9 inches, $\frac{84}{100}$.

ART. IV.—OF THE DIVIDING OF LAND.

The dividing of land may be considered as a principal part of a land-measurer's business, as he is frequently employed in dividing such amongst sundry tenants and proprietors, according to the proportion of their claims. In many instances, the division relates only to the quantity that each claimant is entitled to; and, in others, the quality as well as quantity must be taken into account. The first operation that a land-surveyor has to perform is, to ascertain accurately, by some of the methods before mentioned, the content or area of the whole land; and a correct draught of it is to be made out upon a large scale.

The following Table will be found very useful for such divisions, as acres, roods, falls, or perches, for reducing them into square links.

TABLE

FOR CONVERTING ROODS AND POLES INTO SQUARE LINKS, AND, CONVERSELY, TO CHANGE SQUARE LINKS, OR THE DECIMALS OF AN ACRE, INTO ROODS AND POLES.

Poles.	0 Rood.	1 Rood.	2 Roods.	3 Roods.	Dec. of Pole.	Links.
0	0	25000	50000	75000	0.0	00.0
1	625	25625	50625	75625	0.1	62.5
2	1250	26250	51250	76250	0.2	125.0
3	1875	26875	51375	76875	0.3	187.5
4	2500	27500	52500	77500	0.4	250.0
5	3125	28125	53125	78125	0.5	312.5
6	3720	28750	53720	78750	0.6	372.0
7	4375	29375	54375	79375	0.7	437.5
8	5000	30000	55000	80000	0.8	500.0
9	5625	30625	55625	80625	0.9	562.5
10	6250	31250	56250	81250		
11	6875	31875	56875	81875	0.01	6.25
12	7500	32500	57500	82500	0.02	12.50
13	8125	33125	58125	83125	0.03	18.75
14	8750	33750	58750	83750	0.04	25.00
15	9375	34375	59375	84375	0.05	31.25
16	10000	35000	60000	85000	0.06	37.50
17	10625	35625	60625	85625	0.07	43.75
18	11250	36250	61250	86250	0.08	50.00
19	11875	36875	61875	86875	0.09	56.25
20	12500	37500	62500	87500		
21	13125	38125	63125	88125	0.001	0.625
22	13750	38750	63750	88750	0.002	1.250
23	14375	39375	64375	89375	0.003	1.875
24	15000	40000	65000	90000	0.004	2.500
25	15625	40625	65625	90625	0.005	3.125
26	16250	41250	66250	91250	0.006	3.720
27	16875	41875	66875	91875	0.007	4.375
28	17500	42500	67500	92500	0.008	5.000
29	18125	43125	68125	93125	0.009	5.625
30	18750	43750	68750	93750		
31	19375	44375	69375	94375	0.0001	0.0625
32	20000	45000	70000	95000	0.0002	0.1250
33	20625	45625	70265	95625	0.0003	0.1875
34	21250	46250	71250	96250	0.0004	0.2500
35	21875	46875	71875	96875	0.0005	0.3125
36	22500	47500	72250	97500	0.0006	0.3720
37	23125	48125	73125	98125	0.0007	0.4375
38	23750	48750	73750	98750	0.0008	0.5000
39	24375	49375	74375	99375	0.0009	0.5625

Since 1 acre is 100000 square links, any number of acres is equal to that number with five ciphers on its right, to which the number for roods and poles from the preceding table being added, the sum will be the whole number of square links in the given area.

1. Thus 2½ acres, or 2 acres and 2 roods, will be in square links—

	Square Links.
2 acres, ...	200000
2 roods, ...	50000
2 acres 2 roods in square links are . .	250000

2. Required the number of square links in 4 acres 3 roods and 25 poles—

	Square Links.
4 acres, ...	400000
3 roods and 25 poles, . . .	90625
4 acres 3 roods 25 poles in square links = .	490625

3. Required the acres, roods, and poles in 756345 square links—

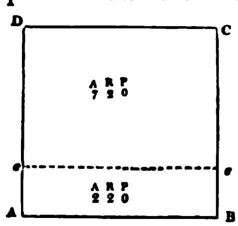
First there are, by pointing off 5 figures to the right, 7 acres.
Then 56345 square links are equal to 2 roods 10 poles, with 95 square links more.

But	62.5 give	0.1 pole
	32.5	
	31.25 give	0.05 „
	1.25	
	1.25 give	0.002 „
	Therefore 95 links give . . .	0.152 pole.

Hence 756345 square links = 7 ac. 2 ro. 10.152 po.

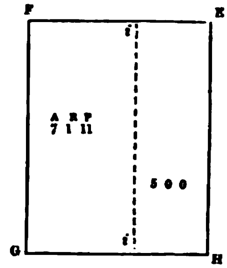
This last operation may generally be omitted as aiming at unnecessary precision.

Suppose the square A B C D consists of 10 acres—2½ acres were sold; how far will it reach up the enclosure, parallel with the side A B? Look into the table, p. 224, for showing how many square links are in 2 acres 2 roods, which by the table is 250000; which divide by 1000, the length of the side A B, and the quotient is 250 links, which must be measured from A to *e* on the side A D. The same distance must be measured on the line B C to *e*, and the dotted line *e e* is the line of division, which is 2½ acres parallel with the side A B, and leaves 7 acres 2 roods in the field *ee* DC.

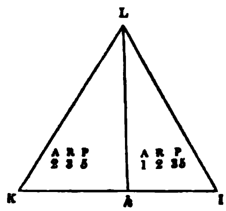


P

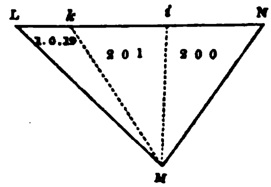
The four-sided figure E F G H consists of 12 acres 1 rood 11 perches; and it is required to measure off 5 acres parallel with the side H E, which is 1270 links long. Look into the Table how many square links are in 5 acres, which by the Table is 500000; which divide by 1270, the length of the side H E, the quotient is 393¾ links; measure off that distance from H to *i* on the side H G, and the same distance on the side E F to *i;* the dotted line *i i* is the division, and is parallel with H E; the land next H E being 5 acres, and the land next G F is 7 acres 1 rood 11 perches.



The triangle I K L contains 4 acres 2 roods, which has to be divided between two men, one to have 1 acre 2 roods 35 perches, and the other to have 2 acres 3 roods 5 perches. The length of the line K I is 1000, and the division to be made from L, to fall upon the base L K, 2 acres 3 roods 5 perches, by the Table is 278125 square links; which divide by 500, half the length of the base K I; the quotient is 556 links, which must be laid off perpendicular from the line K L upon I K to *h;* then draw the line L *h*, which makes the division. Or it may be done thus: If 4 acres 2 roods, which by the Table is 450000 square links, what will 278125 give upon the line K I? Answer, 618 links, which lay off from K on the line towards I, which will reach to *h;* draw in the line from L to *h*, which makes the division next K 2 acres 3 roods 5 perches, and the division next I 1 acre 2 roods 35 perches.

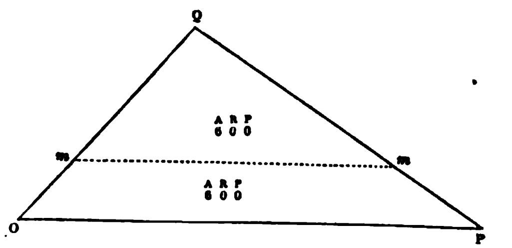


The triangle L M N consists of 5 acres, 20 perches, and has to be divided betwixt three proprietors from the angle at M; the person next N to get 2 acres, and the person next L to get 1 acre, 19 perches—the length of the line M N is 930 links. By the Table 2 acres is 200000 square links; which divided by 465, half the length of M N, the quotient is 432, which must be laid off perpendicularly from the line M N till it intersect the line N L at *i;* then draw in the dotted line from M to *i*, which finishes the 2 acre

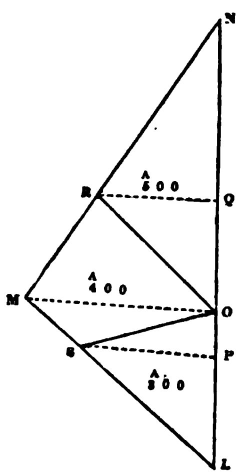


share; then measure the length of M L, which is 1060. One acre and 19 perches or falls by the table is 111875 square links; which divide by 530, half of the length of the line M L; the quotient is 211, which must be laid off perpendicularly with the line M L, and will intersect the line L N at K; draw in the dotted line from *k* to M, which divides it into 3 triangles, viz. M N *i* = 2 acres, M *i k* = 2 acres, 1 perch, and the triangle M *k* L is 1 acre, 19 perches, laid down upon a scale of ten chains in an inch. Or, since the triangles are in the ratio of their bases, the altitude being the same, the points *i* and *k* may be found by distributive proportion, when L N is known.

The triangle O P Q consists of 12 acres, and has to be divided into halves parallel with the line O P. First, from 12 acres, deduct 6, there remain 6 acres; which turned into square links by the Table (see *page* 224,) amounts to 600000; the length of the line O Q is 1330, the square of which is 176900, the half is 884450, the square root of which is 940; which must be measured off from O to *m* on the O Q line. The dotted line *m n* laid off parallel with O P divides the triangle in halves.

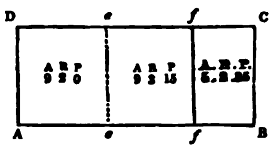


The triangular field L M N, which has to be divided betwixt three men, consists of 12 acres; one man to have 3 acres, a second to have 4 acres, and the third to have 5 acres, and the division of all three to commence from O. First, draw an obscure line from O to the angle M: the length of the base L N is 2400 links; then say, if 12 acres give 2400 links, what will 3 acres give? the result will be 600: which lay off from L to P, and leave a mark; then say, if 12 acres give 2400, what will 4 acres give? the result will be 800; which lay off upon the base line from P to Q, and



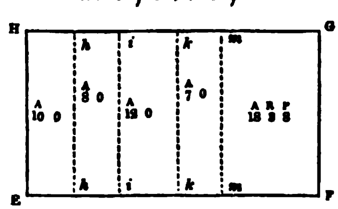
leave a mark. Again say, if 12 acres give 2400, what will 5 acres give? the result will be 1000; which distance lay off from Q, and if it coincides with the angle at N, you are certain no mistake is made. You then lay a parallel ruler upon the obscure line O M, and move it parallel to the mark left at P; and where it intersects the line L M, make a mark at S; draw the line O S, which finishes Lot 1st. You then lay the parallel ruler again upon the obscure line O M, and move it parallel to the mark at Q, and make a mark upon the line M N where the edge of the ruler crosses it at R. Lastly, draw the line O R, which divides the triangle according to the proportion required. A variety of other examples might be given to divide triangles into any number of shares; but as a land-measurer seldom meets with land lying in such regular forms, it will be necessary to give a few examples, which may be of service to him when he is called upon to divide irregular pieces of land. 1st, The general method used by land-measurers is, after having measured the ground proposed to be divided with great accuracy, and having plotted it upon a large scale, to draw a line (by guess) upon the protracted rough draught, and calculate how much is contained between the promiscuous line and the boundary; if too little, what is required must be added, suppose it wanted 1 acre, 2 roods, which, turned into square links by the table, is 150000. This number, divided by the length of the guessed line, which suppose is 900 links, the quotient is 166 links, and is to be laid off parallel with the guessed line, which should be drawn in upon the rough plan, and the quantity of acres and decimals inserted. 2d. Draw another line by guess for the 2d division, and cast up the contents. If it is too large it must be made less; suppose it is 3 roods, 5 perches, turn it into square links by the table, which is 78125, and divide it by the length of the last guessed line, which measures upon the plan 750; the quotient is 104 links; which distance must be laid off upon the plan parallel with the guessed line. This will divide the field into three shares, which must now be staked off upon the ground. This may be very expeditiously done in the following manner: take the rough plan to the field, and with scale and compasses measure from any corner on the plan you know to the first division on the plan, suppose it is 5 chains; measure the same distance on the ground, and drive in a stake for the first division; then measure and see how far it is with the scale and compasses upon the plan to the next division, from any point you know, both on the plan and the ground, suppose 150 links; measure off that distance on the ground from the corner, and drive in a stake for the next division; then go to the other side of the field, and find out some place you know, both upon the plan and on the ground, and measure upon the plan how many links it is to the first division line, which suppose 850; measure off that distance upon the ground, and drive in a stake, and, if you think it necessary, you may drive in three or four additional stakes in a line from the one stake to the other; then measure on your plan a distance of 20 links that the 2d division line is from an angle you know; measure 20 links from that angle on the ground, and drive in a stake; and if you think proper, you may drive in three or four more stakes in a line from one stake to another, which finishes the division; and the number of acres in each division should be inserted on the plan.

A B C D is an oblong field laid down from a scale of 20 chains in an inch, and contains 25 acres. The proprietor sold 9 acres 2 roods, and wished it staked off parallel with the line A D. The distance from A to D is 10 chains: look into the Table p. 224, how many square links there are in 9 acres 2 roods, which is 950000;



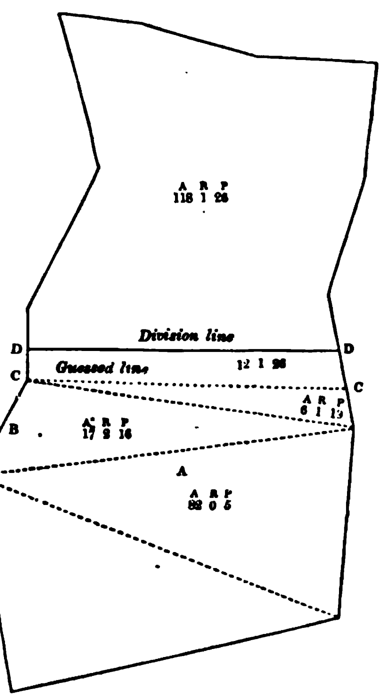
when divided by 1000 links, the length of A D, the quotient is 950; which distance lay off with the scale and compasses upon the plan, which will reach from A D to *e e*. He likewise sold 5 acres 2 roods 25 perches parallel with the line B C, which is 10 chains or 1000 links in length, which by the table is 565625 square links; which number divided by 1000, the length of B C, gives 566; which lay off with the scale parallel with the line C B, which will extend to *f f*; let the same distances be measured off upon the ground as are laid off with the scale and compasses on the plan, and the divisions are finished.

Suppose the rectangular field E F G H, containing 55 acres 3 roods 8 perches, was to be divided into 10 acres, 8 acres, 12 acres, and 7 acres, and the remainder next G F to be another lot of 18 acres 3 roods 8 perches. First look into the table, and see how many square links there are in ten acres, which will be found to contain 1000000; divide it by the shortest side E H



1800, and the quotient is 550; which distance take off from the scale of 20 chains in an inch, and lay it off upon the plan parallel with the line E H, which will extend to *h h*; then look into the table for the square links in 8 acres, which is 800000, and divide it by 1800, the length of *h h*, and the quotient is 444; which distance being laid off from *h h*, will reach to *i i*; then look into the table for the square links in 12 acres, which is 1200000; this divided by 1800, the length of *i i*, the quotient will be 666; which lay off, and it will extend from *i i* to *k k*. Lastly, look into the table for the square links that there are in 7 acres: this is found to be 700000; that number divided by the length of *k k*, 1800, the quotient is 388, which lay off from the scale from *k k*, and it will extend to *m m*, and the remaining lot will be 18 acres 3 roods 8 perches.

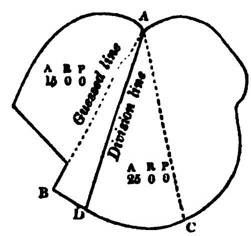
This figure represents the outline of a farm consisting of 236 acres 3 roods 12 perches, which the proprietor wished to divide into two farms of equal dimensions. After an exact survey and plan made out upon a large scale, it was divided into triangles and trapeziums. The area of each was inserted upon the large plan, merely to give an idea where a promiscuous line could be drawn, not far from where the division would run. Accordingly a line was drawn from C C, represented in the plan with wide dots, to distinguish it from the other dotted lines drawn for the purpose of dividing it into triangles and trapeziums for ascertaining the area. The trapezium A is 82 acres 5 perches, the triangle B is 17 acres 2 roods 16 perches, and the triangle D is 6 acres 1 rood 19 perches; when added together, the sum is 106 acres, which is short of half the farm



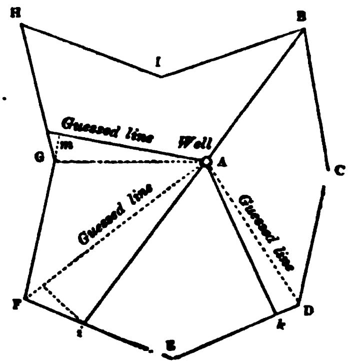
by 12 acres 1 rood 26 perches. The next thing to be done is to measure the length of the guessed line C C upon the plan, which is 3300; then look into the table how many square links are in 12 acres 1 rood 26 perches, which is 1235200; when this is divided by 3300 (the length of the guessed line C C) the quotient is 374; which lay off parallel with the line C C, which will divide the farm into two equal parts, by drawing in the division line D D.

In the dividing of land, the measurer in general keeps an account of the number of links wanting to make up the division, as he has to drive stakes in the ground, and to measure off the number of chains and links, from some angle that he knows upon the field, to the line of division.

Suppose 15 acres were to be staked off from the irregular field A B C D, which consists of 40 acres: from the point A first measure the boundary carefully round from A by B towards C, till you think you have been nearly half way round to C; take an angle, and measure from C to A; then plot all you have measured, and lay it down upon a large scale, and cast up the contents, which you find is considerably more than you want; you then draw a line by guess from A to B upon the plan and, after calculating it, you find it is only 11 acres 2 rood 32 perches, which is short of 15 acres by 3 acres 2 roods 8 perches. Look into the table for the number of square links contained in 3 acres 2 roods 8 perches, viz, 355000; then measure the length of the line A B, which is 1950, and divide 355000 by that number, the quotient is 182½ for the breadth of a parallelogram; which distance must be doubled on account of its being a triangle; then lay off from the scale 365, being twice 182½, from B to the boundary at D; then draw the line D A upon the plan, and measure off the distance 365 on the ground, and drive in stakes from D to A, and it is finished, leaving 25 acres on one side, and 15 on the other of the division line.

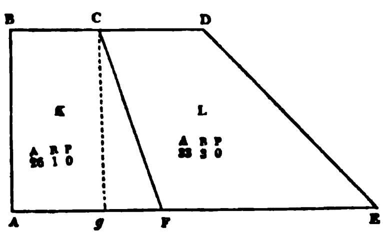


This figure is to be divided into four equal parts from a spring at A, and each division to have the benefit of the water. First measure the whole field correctly, and also the situation of the spring; plot the field with great care from a large scale, and calculate it by some of the methods, (before mentioned in Areas,) which is found to consist of 90 acres, the fourth of which 22 acres 2 roods for each share. Draw a line from A to B and another line by guess, which is dotted, to D; then calculate the trapezium, A B C D; which is short of 22 acres 2 roods, by 2 acres 2 roods 3 perches; when turned into square links by the table it is 251875;



divide it by 910, half the length of the dotted line A D, the quotient is 276; which distance lay off by the scale to *k*; then draw the line from A to *k*, which is one enclosure of 22 acres 2 roods; then draw a line by guess, which is dotted, from A to F, and calculate upon the plan the trapezium A *k* E F, which is found too large by 476320 square links; which sum divide by 1215, half the length of the dotted line A F, the quotient is 555, which distance lay off from the scale upon the line E F, perpendicular from the line A F, which will finish the division A *k* E *i*; then draw a line by guess, which is dotted, from A to G, and calculate the trapezium *i* F G A upon the plan, which is short of 22 acres 2 roods by 22180 square links; which sum divide by 330, half the length of the line A G, the quotient is 267; which lay off with the scale perpendicular to *m*; then draw the line A *m*, which finishes the division. However, it will be convenient to calculate the division A *m* H *i* *b*. If it answers to 22 acres 2 roods, you are certain of its accuracy; if it does not agree, the calculations must be made over again.

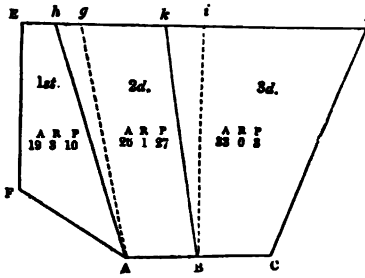
Suppose the piece of land represented by A B C D E, containing 60 acres, is to be divided betwixt K and L, the division to be made from C, K having a right for the value of L.70 a-year, and L having a right to L.90 a-year. First add L.70 and L.90 together, the sum of which is L.160; turn the acres into square links by the table, which is 6000000;



Sq. Links. Sq. Links. A. R. P.
Then say, if £160 give 6000000, what will £70 give? *Ans.* 2625000=26 1 0, K's share.
if £160 give 6000000, what will £90 give? .... 3375000=33 3 0, L's share.

Then from the point C draw the dotted line by guess to *g*, which calculate. Now, suppose you find it is short of K's share by 625000 square links, divide this by 1000, half the length of C *g*, and the quotient is 625, which distance lay off upon the line A E from g, which will reach to F; then draw in the line F C, which divides the field according to the required valuation, giving quantity proportionable to quality.

This figure represents a large field consisting of 79 acres 1 rood, which has to be divided into 3 divisions, giving quantity for quality; and the divisions are to go from A and B, and to fall upon the line E D. The first is valued at 30s. per acre, the second at 40s. per acre, and the third at 50s. per acre; the land was accurately measured, and a plan made out upon a large scale.

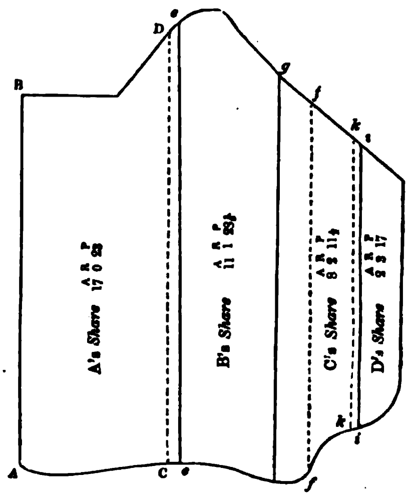


The different shares were made out thus: by adding 30s., 40s., and 50s. into one sum, they amount to 120; and 79 acres 1 rood, when turned into square links by the table, is 7925000.

	Sq. Links.		Sq. Links.	A.	R.	P.	
Then, if 120s. gives 30s. what will	7925000 give?	*Ans.*	1981250 =	19	3	10,	First Share.
„ 120s. gives 40s. what will	7925000 give?	„	2641666 =	26	1	27,	Second Share.
„ 120s. gives 50s. what will	7925000 give?	„	3302083 =	33	0	3,	Third Share.

To make out the division upon the plan, draw a line by guess from A to g. After calculating, it is found too much by 215000 square links; this sum divided by 1300, half the length of the line A g, the quotient is 165; lay off that distance upon the line E D towards E from g, and it will extend to h; draw in the division line A h, which finishes the first lot. A line is drawn from B to i by guess: after calculating A g i B, it exceeded the proportionate share by 457000 square links; which, divided by 1250, half the length of the line B i, the quotient is 360, which distance when laid off from i extends to k; a line drawn from k to B finishes the second and third divisions. If it is thought proper, lot third may be calculated as a check; if it amounts to 33 acres 9 roods, it is right.

This figure is a common where there are four claimants, A, B, C, and D; each of whom have a right to feed cattle upon it. A had a right to feed 6, B 4, C 3, and D but 1. And it was mutually agreed amongst them, that each claimant should have his proportion of the common according to the number of cattle he kept. The grazing of each was valued worth five pounds per year. The



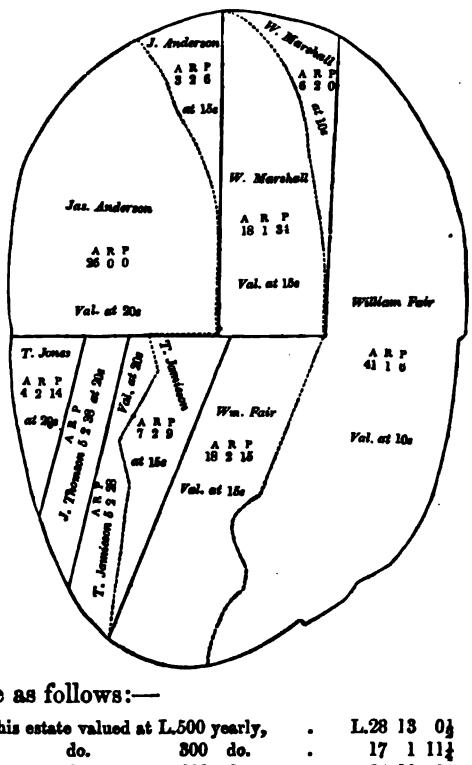
common was carefully measured, and a plan of the same made out upon a large scale, the contents of which amounted to 40 acres = 4000000 square links. Now,

	L.	Sq. Links.	L.	Sq. Links.	A. R. P.	
A has 6 at £5 each is £30, then as	70 :	4000000 : :	30 :	1714285 =	17 0 23,	A's share.
B has 4 at do. „ is £20, „	70 :	4000000 : :	20 :	1142857 =	11 1 28½,	B's share.
C has 3 at do. „ is £15, „	70 :	4000000 : :	15 :	857144 =	8 2 11½,	C's share.
D has 1 at do. „ is £5, „	70 :	4000000 : :	5 :	285714 =	2 3 17,	D's share.
Total amount per year, £70			Amount,	40.00000 =	40 0 0	

By adding up the different shares, the sum is 40 acres; which proves the calculation is right.

To lay off each share upon the plan, and also upon the ground, first draw a line by guess upon the plan, suppose from C to D, parallel with the fence A B; cast up the contents, which is 202400 square links too little, which divide by 2300, the length of C D, the quotient is 88; which lay off with the scale parallel with the dotted line C D, and it will extend to *e e;* then draw in the division line *e e*, B's lot; draw the line *ff*, and cast up the contents betwixt that line and the division line of A's share *e e*, which is too large by 335000, which divide by 2060, the length of *ff*, the quotient is 162, which lay off from *ff* to *g g*, and draw in B's division, which finishes B's lot. We now come to C's lot: draw a line by guess *k k*, and cast up the contents, which is too little by 90000 square links, which divide by the length of *k k* 1630, the quotient is 55 links, which lay off from *k k* to *i i*; draw in the division line *i i*, which finishes C's lot; then measure D's lot. If it answer to 2 acres 3 roods 17 perches, it is right. It has been observed before, in staking off the different lots upon the ground, that the surveyor ought to be attentive in having all the distances, &c. that refer to his plan; and by measuring distances from marks that he knows both upon the plan and in the field, he will find very little trouble in laying off the divisions or lots.

This figure also represents a common, consisting of 137 acres 3 roods 30 perches, which was measured, as also the different lots of valuation, and a plan made out thereof, and plotted from a large scale. The commissioners, not aware of the great trouble which would occur to the surveyor, divided the common into three different values, and chiefly in curved lines, as may be seen in the small sketch, which, although containing only three different qualities, includes no less than ten different lots to calculate, which makes a survey of this description very difficult to make out and delineate. This common is to be divided between six proprietors, whose claims are as follows:—



James Anderson, Esq.,	for his estate valued at		L.500 yearly,		.	L.28	13	0½
William Marshall, Esq.,	for	do.	300	do.	.	17	1	11¼
William Fair, Esq.,	for	do.	600	do.	.	34	11	9¼
Mr Thomas Jamieson,	for	do.	200	do.	.	11	6	9
Mr John Thompson,	for	do.	100	do.	.	5	14	9
Mr Thomas Jones,	for	do.	80	do.	.	4	11	9
Total value of the six estates,		do.	L.1780		Total,	L.102	0	0

The method used to ascertain the valuation per acre is: as 1780, the total amount of the value of all the estates, is to 102, the total amount of the annual value, so is the valuation of each estate to its annual worth.

For example, to find out James Anderson, Esq.'s value, as 1780 : 102 : : 500 : £28, 13s. 0 d., and so on with all the others, the sum of each, when added, gives the total amount.

CONTENTS OF THE COMMON.

	A.	R.	P.		Sh.		£	s.	d.	£	s.	d.
James Anderson, Esq.,	26	0	0	valued at	20	per acre	26	0	0	28	13	0½
Do.	3	2	6	„	15	„	2	13	0½			
William Marshall,	18	1	34	„	15	„	13	6	11¼	17	1	11¼
Do.	6	2	0	„	10	„	3	5	0			
William Fair, Esq.,	41	1	6	„	10	„	20	12	10½	34	11	9¼
Do.	18	2	15	„	15	„	13	18	10¾			
Mr Thomas Jamieson,	7	2	9	„	15	„	5	13	3	11	6	9
Do.	5	2	28	„	20	„	5	13	6			
Mr John Thomson,	5	2	38	„	20	„	5	14	9	5	14	9
Mr Thomas Jones,	4	2	14	„	20	„	4	11	9	4	11	9
Amount of acres,	137	3	30				Amount of valuations,			102	0	0

For an idea of the various calculations requisite to make out the above contents, I shall refer the pupil to the methods of calculating arêas, so particularly described in that section.

We shall now suppose the common-field or run-rigs described before (Plate VI., figs. 3 and 5; see page 119,) which are situated in the heart of the lord of the manor's estate, and he is anxious to give quantity for quality. Accordingly, an agreement was concluded betwixt the lord of the manor and the different proprietors, that he would enclose each person's property at his own expense, in a regular form. The business was referred to arbitrators, and a value put upon Robert Brown's lot, which is 266700 square links, at L.3 : 10s. per acre; David Rennie's, 372200 square links, at L.3 : 5s. per acre; Thomas Smellie's, 306200 square links, at L.3 per acre; Mrs George, 347800 square links, at L.3 per acre; George Peacock, 597200 square links, at L.3 : 5s. per acre; Robert Thomas, 206800 square links, at L.4 per acre; Joseph Dice, 186800 square links, at L.4 per acre; and John Wilson's, 400000 square links, at L.3 : 5s. per acre; and the ground that is to be given in exchange is valued at L.2 per acre, which is to be laid off along the side of a straight road, and each division to be eight chains wide.

Below is the length of the respective lot that each proprietor will receive alongside of the road:—

	Sq. links in the common field.	Sq. links that each proprietor will receive.	Divided by.	Length of each lot.
Robert Brown's lot	266700	466775	800	584
David Rennie's do.	372000	604825	do.	758
Thomas Smellie's do.	306200	459300	do.	574
Mrs George's do.	347800	521200	do.	651
Thomas Peacock's do.	597200	1045100	do.	1254
Robert Thomas's do.	206800	413600	do.	517
Joseph Dice's do.	186800	373600	do.	467
John Wilson's do.	400000	700000	do.	875
	2683500	4584400		
	or	or		
Total of the runrigs,	26 3 14	45 1 30 Total land got in exchange.		

The quantity of ground which each proprietor is to receive at the side of the road, also the amount of ground which they give in lieu of it to the lord of the manor, is stated below. The method used for finding out the proportions that each proprietor receives is simply this: If one acre gives £2, what will any number of acres and decimals of an acre give at what they are valued at per acre? Turn each proprietor's acres, roods, and perches, into square links by the Table in page 224.

THE CALCULATION AS UNDER.

	A. R. P.		Sq. links.		£ s.		A. Dec.			A. Dec.		A. R. P.
Robert Brown,	2 2 26	=	266700	×	3 0	=	9.33450	÷	2	4.66725	=	4 2 25
David Rennie,	3 2 35	=	372000	×	3 5	=	12.09650	÷	2	6.04825	=	6 0 7
Thomas Smellie,	3 0 9	=	306200	×	3 0	=	9.18600	÷	2	4.59900	=	5 2 14
Mrs George,	3 1 36	=	347800	×	3 0	=	10.42400	÷	2	5.21200	=	5 0 34
Thomas Peacock,	5 3 36	=	597200	×	3 10	=	20.19600	÷	2	10.45100	=	10 1 32
Robert Thomas,	2 0 21	=	413600	×	4 0	=	8.27200	÷	2	4.13600	=	4 0 21
Joseph Dice,	1 3 18	=	186800	×	4 0	=	7.37200	÷	2	3.73600	=	3 2 37
John Wilson,	4 0 0	=	400000	×	3 10	=	14.00000	÷	2	7.00000	=	7 0 0

What has now been related of dividing land, includes the principles on which all other divisions are made; and with a little practice, a land-surveyor will overcome all the obstacles the commissioners often give them, by dividing the different qualities in curved and irregular lots, in place of making every division run as straight as the nature of the ground will admit of.

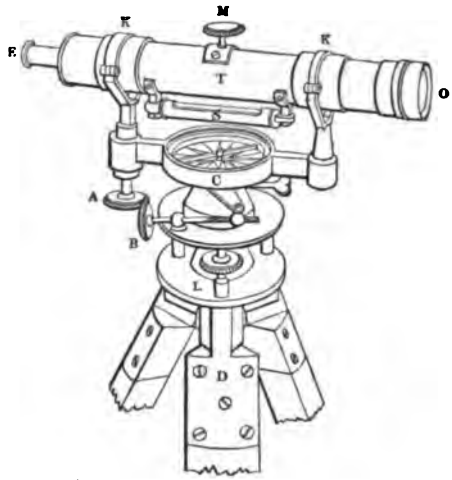
SECTION FOURTH.

ON LEVELLING.

Levelling is the art of finding a line parallel to the horizon at one or more stations, in order to determine how much one place is higher or lower than another, for regulating descents, cutting canals, forming railways, conveying water, &c. Five or more places are said to be on a level when equally distant from the earth's centre, or a line equally distant from that centre in all its points is a line of true level. The apparent line of level is the line of sight given by the repeated operations of levels, and rises always higher above the true line of level the greater the distance. However, by the help of tables constructed for the purpose, we can find the difference between the apparent and true line of level; it is by this assistance also we can level to almost any distance by a single operation. I shall not attempt to go over the properties of the circle upon which those tables are constructed, but shall merely give the following position—viz. That the difference between the true and apparent level is equal to the square of the distance between the two places or stations divided by the earth's diameter, and consequently is always proportional to the square of the distance; and by proportioning the excesses of height as the squares of the distances, we obtain the useful table inserted in page 205.

The common methods of levelling are sufficiently correct for mason's and pavier's work, &c.; but in extensive operations, such as canal levelling, for the purpose of conveying water for a number of miles, too much attention with regard to accuracy cannot be paid to such operations; and in such cases it is necessary to take the difference between the true and apparent level into the account.

The most complete level is the *Spirit Level*, invented by the celebrated artist Ramsden, and latterly much improved by Troughton, Simms, Adie, &c. I shall not take up the surveyor's time with a long and tedious description of this instrument, being well aware that a few minutes' inspection of the instrument itself will give a better idea of its construction and use than the most lengthened detail. Its principal parts is a telescope T, in the tube of which cross wires are fixed and adjusted by means of four small screws. The horizontal wire cuts the object at the time of an observation; under the telescope is fixed a brass tube S, enclosing one of glass containing spirits of wine; this tube is hermetically sealed, and the spirits occupy the whole of its internal cavity all to



a bubble of air, which settles exactly in the middle of the glass when the instrument is level. The telescope and level are supported by two forked uprights called Ys, owing to their resemblance to that letter, near K; under the Ys are milled headed screws A, working against collars, for the purpose of raising or depressing the telescope and level. The level is fastened to the telescope by means of capstan screws, which adjust it exactly parallel with the axis of the telescope; two bars at right angles to the Ys support a compass C, for the purpose of taking magnetic bearings. The instrument turns round on a socket fastened to the legs. There are two parallel plates, fastened together by means of a ball and socket; these plates contain four screws, L, which screw the upper plate thereof, and act against the under, for the purpose of levelling the whole instrument; which is done by raising and lowering the screws till you find the bubble in the glass tube exactly in the middle; which done, turn the instrument at right angles to its former situation. If it is not level, the screws must be raised or depressed till it is so, until it appears to be level in any situation the instrument is placed in. There is a clamp screw for the purpose of keeping the instrument tight in the centre, when required, and a tangent screw B for moving the instrument easily round with a slow motion. The whole is fixed to three legs, either round or triangular; if the latter, the three legs form into one round pole, and are kept together by means of rings. The whole is generally packed in a box, for the purpose of being conveyed safely from place to place when not at work in the field.

I shall now describe the best and easiest method of adjusting the spirit level.

Go to some field or meadow that is nearly level; there set up the instrument, and measure out eight or ten chains, leaving a mark, by driving into the earth a small wedge or stake; return to the instrument, and measure out the same distance the opposite way from the centre of the instrument, and drive in a stake to set the level staff on; there leave one of your assistants with the staff, which should have a vane upon it to slide up and down; send another assistant to the other stake with his staff and vane, and let him place it upon the wedge or stake; then set the instrument level by the four screws L on the parallel plates, and look through the telescope, and make signs for the assistant you first take an observation to, to move his vane up or down the staff till you see the horizontal wire and the black line drawn across the vane to coincide exactly; then make a sign for him to mark upon his staff with a piece of chalk where the under part of the vane is upon the staff; then turn the telescope round, and observe if the telescope is level; if not, set it level as before to the other assistant, and sign to him to put up the staff upon the stake; then cause him to move the vane up or down till you perceive the horizontal wire and the mark on the vane to coincide, and cause him to mark the under part of the vane with chalk upon the staff; for although the instrument should not be adjusted, the vanes upon the staffs are exactly level. You then remove the instrument to the assistant, and place it up within six or eight yards of him, and set it level as before, and order him to put up the staff, and cause him to move the vane up or down till you see the horizontal hair in the telescope and the mark on the vane to coincide. Whatever space there is between the first and second marks on the staff, the one assistant must go to the other, and cause him to put his vane either up or down upon his staff, the same difference there is upon his, with the addition of the allowance for the earth's curvature; then sign to your assistant to put up his staff upon the stake, and observe that the instrument continues level; then take an observation to him, and if the horizontal hair in the telescope and the mark on the vane agrees, the level is right; but if it should be otherwise, you must move the joint and capstan-headed screw that holds the level to the telescope, either up or down, till you see through the telescope the horizontal hair or wire to coincide exactly; and the instrument will be adjusted. The station or level staffs above-mentioned should be 10 or 12 feet in length and about an inch and a quarter square. It will be convenient to have two shorter ones, about five feet long, to be used occasionally. The vanes which slide up and down upon the staffs are sometimes of thin iron, brass, or copper, about four inches in diameter. The segment of the circle is taken from the under side, and the vane painted white with oil-colour (see Plate XIII. fig. 8;) a black line is drawn, about the eighth of an inch broad, through the centre of the vane, which has a spring on the back part of it, to clamp it gently to the staff, but so contrived, that, with gentle pressure, it can be made to slide along the staff. The staffs should be exactly divided into feet and inches, and the inches divided into eighths. The feet should be marked with large figures, and numbered upwards; the same ought to be done with the five feet staff, which is only used when the descent exceeds the length of your long staff: in this case the weight or vane is put to the top of the staff, and the five feet staff upon the stake, which should be held as perpendicular as possible, sliding the long staff up close by the short one, and adding whatever height the bottom of the long staff cuts upon the short one, suppose 2 feet 10 inches; which shows that the fall is 12 feet 10 inches, if the long staff is 10 feet long. This your foremost assistant marks; but there is to be deducted the height of the instrument, which suppose is 4 feet; which deducted from 12 feet 10 inches, there remain 8 feet 10 inches of declivity.

This figure represents a pocket level, which is very useful for small operations, such as cutting drains,* or taking levels for ascertaining the cutting; also for determining the cubical yards required to be taken out for the foundation of houses, &c. &c. It is com-

monly made about eight inches in length, of mahogany, with a groove taken out from the upper side to admit a glass tube to go in, which is filled with spirits of wine all to a bubble of air, which shows when it is level, which is done by means of a screw. This instrument is so simple in itself as to require no explanation.

The following Table will be found useful in taking an observation when the instrument cannot be placed at equal distances. In that case, an allowance must be made for the curvature of the earth.

* See Stephens' Book on Practical Draining, published by Messrs Blackwood.

Suppose you have a station that is 80 chains or one mile in length; look into the table for 80 chains, and opposite it, in the right-hand column, is 8 inches; which shows that the line of sight is 8 inches higher than the true level. Again, suppose you have another station that is 30 chains in length; opposite 30 chains is 1 inch 12 decimals; which shows that the difference betwixt the apparent and true level is 1 inch and 12 decimals.

TABLE I.

OF THE CURVATURE OF THE EARTH, CALCULATED TO THE APPARENT LEVEL IN INCHES AND DECIMALS OF AN INCH, FROM THREE CHAINS TO EIGHT MILES.

Chains.	In. Dec.	Chains.	In. Dec.	Chains.	In. Dec.	M. Ch.	Ft. In. Dec.
3	0.01	16	0.32	29	1.05	0 50	0 3.12
4	0.02	17	0.36	30	1.12	0 55	0 3.78
5	0.03	18	0.37	31	1.19	0 60	0 4.50
6	0.04	19	0.45	.32	1.27	0 65	0 5.31
7	0.06	20	0.50	33	1.35	0 70	0 6.12
8	0.08	21	0.55	34	1.44	0 75	0 7.03
9	0.10	22	0.60	35	1.53	0 80	0 8.00
10	0.12	23	0.67	36	1.62	2 0	2 7.8
11	0.15	24	0.72	37	1.71	3 0	5 11.6
12	0.18	25	0.78	38	1.80	4 0	10 7.3
13	0.21	26.	0.84	39	1.91	5 0	16 6.9
14	0.24	27	0.91	40	2.00	6 0	23 10.6
15	0.28	28	0.98	45	2.28	7 0	32 5.6
						8 0	42 6.16

TABLE II.

OF THE CURVATURE OF THE EARTH, CORRECTED FOR REFRACTION, TAKEN AT 0.08 OF THE INTERCEPTED ARC.

Chains.	Correction in Feet.	Chains.	Correction in Feet.	Chains.	Correction in Feet.	Chains.	Correction in Feet.
1	0.0001	11	0.0106	21	0.0386	31	0.0840
2	0.0003	12	0.0126	22	0.0423	32	0.0895
3	0.0008	13	0.0148	23	0.0463	33	0.0952
4	0.0014	14	0.0171	24	0.0504	34	0.1011
5	0.0022	15	0.0197	25	0.0547	35	0.1071
6	0.0031	16	0.0224	26	0.0591	36	0.1133
7	0.0043	17	0.0253	27	0.0637	37	0.1197
8	0.0056	18	0.0283	28	0.0686	38	0.1263
9	0.0071	19	0.0316	29	0.0735	39	0.1330
10	0.0087	20	0.0350	30	0.0787	40	0.1399

Levelling, of all parts of a land-measurer's work, is the easiest attained; yet it requires the greatest nicety, not only in his own observations, but that of his assistants. The most certain method to attain the greatest degree of accuracy, is to carry out a quantity of small stakes or wedges, to be stuck in the ground, about seven or eight inches long, and about two inches broad at the top, and and tapered off at the bottom, that they may the more easily enter the ground. Upon these are placed the station staffs. The assistants should be very attentive to hold the staffs as nearly perpendicular as they can.

A surveyor is frequently employed to convey water from a spring to any particular place, either in open cuts or in pipes. To explain this, we shall suppose the water from a spring is wished to be conveyed to a house at about a mile distance, and a trial is to be made if it is possible to do so. First provide paper, pen, and ink, to each of your assistants, and order the hindmost assistant to set up his staff at the spring; then go in the direction towards the house, and plant up the instrument, and set it level by the screws in the brass plates till the bubble in the glass settle exactly in the middle; order the staffman left at the spring to move the vane up or down upon the staff, till you see it and the horizontal hair in the telescope, and the mark on the vane, to coincide; sign to him to mark the feet and inches cut by the bottom part of the vane upon the staff, and to insert them in his field-book; then turn the telescope round to the other assistant, whose pole or staff he holds up as perpendicular as possible, make signs to him to slide the vane up or down upon the staff, till you see the hair in the telescope and the mark on the vane to coincide; and cause him to mark the feet and inches cut upon the staff at the under side of the vane, which he also enters in his field-book; you then order the hindmost assistant to go and place his staff exactly where that of the foremost assistant stood; you desire the foremost assistant to go forward and erect his staff; then plant the instrument as near half-way as you can guess between them, and set it level as before; then, by signs, cause the hindmost assistant to raise or depress the vane till you see it and the hair coincide; he then marks in his field-book, as before, the feet and inches cut on the staff: then turn the telescope about to the foremost assistant, and sign also to him to raise or depress the vane till you perceive through the telescope the hair and the mark drawn across the vane to coincide; then sign to him to mark the feet and inches in his field-book cut by the lower edge of the vane upon the staff. Go on exactly in the same manner all the way to the house. This done, the running level is finished.

In comparing each assistant's notes, they turn out as follows:—

Hindmost Assistant.	Ft.	In.	8ths.	Foremost Assistant.	Ft.	In.	8ths.	Difference of each Station. Ft.	In.	8ths.
1. Station	3	4	1	1. Station	4	10	5	1	6	4
2. do.	2	6	0	2. do.	4	4	2	1	10	2
3. do.	4	7	3	3. do.	5	7	6	1	0	3
4. do.	4	1	1	4. do.	6	10	4	2	9	3
5. do.	3	10	0	5. do.	6	1	1	2	3	1
6. do.	4	6	0	6. do.	4	7	7	0	1	7
7. do.	3	11	7	7. do.	4	11	1	0	11	2
8. do.	4	10	6	8. do.	5	8	6	0	10	0
	31	9	2		43	2	0	11	4	6

This shows the difference from the spring-head to the house, by the stations, to be only 11 feet 4 inches 6 eighths; that is to say, the fall is 11 feet 4 inches 6 eighths, from the spring to the house, which is sufficient declivity for the water to run to the house, being only about a mile distant from the spring-head; and it is well known that water will run along a pipe at little less fall than five inches in a mile. In that case the water may run with too great a current, which requires that some of the pipes should be laid nearly level, which in some measure would impede the progress; or in some places the pipes may be laid in a curved direction, which will have a similar effect. There is sometimes a necessity for laying the pipes a little uphill before they reach to the appointed place, to avoid the water running with too great a velocity.

In levelling, it will be proper to observe that, if the ground will admit of it, if the instrument is planted at equal distances between the back and fore assistant, although the level should be out of its adjustment, the observations will be right, and there will be no occasion for making any allowance for the curvature of the earth; but if you have to take levels where the ground is very uneven, and you have an opportunity of observing a rising ground a little more depressed than where you stand, and a valley betwixt you and that ground, which would take a long time in levelling over, the instrument ought to be very correctly adjusted, and an allowance made for the curvature of the earth. Suppose the ground which is across the valley to be half a mile, then (by the table,

p. 242) 2 inches must be allowed for the curvature, and for which 2 inches the vane must be raised upon the staff above its former situation.

PLATE XIII. (Fig. 1.)

Levelling for a railway, which requires to be made as level as the ground will admit, the first thing that ought to be done is to take a running level in the same way as described in page 242, in levelling from a spring-head to a house, to ascertain the practicability of conveying water to it.

After having ascertained the number of feet in the fall, measure the distance in feet from the place you began to level at to the place you left off at, which suppose 7600 feet, and the fall 10 feet; then divide 7600 by 10, and the quotient is 760; which shows that the level can be brought by cutting and filling up to one in 760, which will make a remarkably easy railway. Set up the instrument level at any convenient distance, suppose 800 feet from where the railway is to begin; sign to the staffman to move the vane up or down till you see the mark on the vane and the horizontal hair in the telescope to coincide; he then marks with chalk the bottom part of the vane on the staff; the chainmen measure the distance, 800 feet; which distance, being doubled, is 1600; then say, if 7600 gives 10 feet, what will 1600 give? which, when calculated, is 2 feet 1 inch 2 eighths, which the vane must be raised upon the staff above the mark. The chainmen then measure 800 feet farther, and the staffman drives in a stake, and erects the staff upon it; if it is too high, he must drive it a little lower with a hand-mallet, which he ought to have for that purpose. The observer looks again through the telescope till he sees the horizontal hair and the mark on the vane to coincide. The instrument is then moved and placed at any convenient distance in the line of direction forward, and placed exactly level as before; the staffman puts up his staff upon the stake, and is caused to move the vane higher or lower till the observer sees the mark upon the vane and the hair in the telescope to coincide; he is then to mark with chalk as before. The chainmen then measure from the stake to the instrument, which is 790 feet, which distance doubled is 1580; then 7600 : 10 : : 1580 : 2 feet, 1 inch; this shows that the vane must be raised upon the staff that height: the chainmen then measure 790 forward, and the staffman drives in a stake and places his staff upon it; if the stake is too high, it must be driven lower, and raised if too low. After repeated trials, the observer at last perceives the cross hairs in the telescope and the mark made by the vane to coincide; which finishes the 2d level. The instrument is then set up, and being levelled, the staffman moves the vane up or down, according to the signs given him, till the hair in the telescope and the mark in the vane coincide. The chainmen then measure the distance to the instrument, which is 720; which doubled, is 1440; which multiplied by 10 and divided by 7600, shows that the vane must be raised 1 foot 10 inches 6 eighths higher than the last mark. The chainmen then measure forward 720, and the staffman drives in a stake and sets his staff upon it, which finishes the 3d level. Go on in the same manner, whatever may be the number of levels to take. It is no matter at what distance the instrument is placed from the staffman, if you measure the same distance forward that you measured to the instrument. Suppose the instrument was placed 200 feet from the staff, then measure 200 feet forward in the line of direction of the railway; when the two distances are added, the distance from one stake to another is but 400 feet; that number multiplied by 10 feet, which is the whole of the fall of the railway, is 4000, which is less than the whole distance, 7600, and shows that the 4000 must be reduced to inches by multiplying it by 12, which amounts to 48,000; this, when divided by the length of the railway, gives 6 inches, 2400 parts, × 8 to bring it to the eighth of an inch is 19,200 ÷ 7600 is 2 eighths and some odd fractions of the eighth of an inch; which shows that the vane is to be raised upon the staff 6 inches 2 eighths above the mark on the staff. Here it will be proper to observe, whatever height the vane is raised upon the staff, the ground is so much lower from one stake to the other. The levelling for turnpike roads is done exactly in the same manner, when the road is required to be brought to an equal ascent.

Plate XIII. (Fig. 2.)

Is a continuation of the same railway as Fig. 1, but upon more uneven ground, where there is some cutting and banking required to bring it to an equal descent. First take a running level as correctly as possible, and find out what fall there is from C to D, which is 10 feet, and the distance 7510 feet, which is equal to 1 in 751; that is to say, for every 751 feet the railway falls 1 foot. In this method care must always be taken to measure the height of the instrument from the ground, and mark it in a field-book. This will point out the lowest ground in making out the section, and the staff will point out the highest ground, by taking the difference from one station to another, which should also be marked in the field-book. After having taken all the different levels and all the distances from one station to another, you have a section of the ground to make out, showing all the heights, hollows, &c., which ought to be done very correctly. To do this, first draw a line C *b* to represent a level; the height of the vane is 1 foot 6 inches above the ground at No. 1; the distance from thence to the instrument is 830 feet, and the height of the telescope above the ground is 5 feet; which shows that the fall from No. 1 to the instrument is 3 feet, 6 inches; which protract and lay off from a scale. The distance from the instrument to No. 2 is 580 feet, and the rise is 3 feet; protract this also from the same scale, and it will give the representation of a hollow between 1 and 2. The distance from No. 2 to where the instrument is placed is 260, the fall 4; the distance from the instrument to No. 3 is 370, and the rise is 3; the distance from No. 3 to the instrument is 380, and the fall 5; the distance from the instrument to No. 4 is 220, and the rise 3; protract this also, which will give the hollow between Nos. 3 and 4. The distance from No. 4 to the instrument is 320, and the fall 5; the distance from the instrument to No. 5 is 500, and the rise 3 feet 9 inches, which gives the representation of the hollow from 4 to 5; the distance from No. 5 to the instrument is 500, and the fall is 4 feet 6 inches; and the distance from the instrument to No. 6 is 530, and the rise is 3 feet 3 inches; protract this, which represents the hollow between 5 and 6. The distance from No. 6 to the instrument is 500, and the fall is 4; the distance from the instrument to No. 7 is 538 feet, and the rise is 3, which represents the hollow between 6 and 7. The distance from No. 7 to the instrument is 450, and the fall 5; the distance from thence to No. 8 is 400; and the rise is 4 feet 6 inches; protract this, which will represent the hollow between 7 and 8. Again, the distance from the instrument to No. 9 is 610 feet, and the rise 2 feet to D; protract this, and you have a representation of the whole section from C to D. Draw a line from the lowest ground at D to the highest ground at C, which shows what cutting and banking will be required to make the railway from C to D of an equal descent.

N.B.—One station staff and one staffman answers the same purpose as making use of two, and it is equally the same in taking the levels for a canal. In levelling, the staff is now generally divided into feet and tenths, so distinctly as to be easily read through the telescope, and smaller parts may be estimated and recorded by the surveyor independent of his assistants.

PLATE XIII., Fig. 3,

Shows the method of levelling for a canal. Adjust the level as has already been described, and place the instrument as near the summit as you can guess; send the staffman the contrary way from where you begin, and let him drive in one of his wedges or stakes and set his staff upon it; sign to him to move the vane higher or lower, till you perceive the horizontal hair in the telescope and the mark across the vane to coincide; the staffman then marks the staff at the bottom part of the vane, and the chainmen measure the distance from the stake to the instrument, which is 720 feet; when the staffman comes up, look how much higher the vane upon the staff is above the height of the instrument, which suppose 4 feet; this shows that there is a cut of 4 feet on the summit. The chainmen then measure about the same distance forward, and the staffman puts up his staff; if he is too low, the observer signs to him to move upon higher ground till he sees him nearly upon the same level with the first stake. The observer then makes motions for him to drive in one of his stakes into the ground, and to set his staff upon it. If it is too high, the observer signs to him to drive the stake farther into the ground, and to put his staff upon it, till such time as the horizontal hair and the mark on the vane coincides. The staffman ought to be very attentive, at every time he places the staff upon the stake, that the vane is exactly at the mark, as the vane is apt sometimes to shift a little in carrying from one station to another. The instrument is now removed past the staffman, (the first level being finished), and placed at any convenient distance; and being levelled, the staffman rubs out all the marks, and puts the staff upon the stake: he then moves the vane up or down till such time as the hair in the telescope and the mark on the vane coincide, and marks the staff as before with chalk. The chainmen then measure the distance, which is 720 feet, and also the same distance forward from the instrument; the staffman puts up his staff to try if it is near the level; he is then ordered to drive in a stake, which he sets the staff upon; and after repeated trials, moves the stake up and down till such time as the observer sees the hair and the mark on the vane to coincide; which finishes the second level. All others are done in the same way, by placing the instrument at any convenient distance from the man left at the staff. Sometimes you may find it convenient to place it at 5, 6, 7, or 800 feet; but observe particularly, at whatever distance the instrument is placed from one stake, the next stake should be placed nearly at the same distance from the instrument. You may go on in this way for many miles upon a dead level, till you come to some fall in the ground, which must be descended by locks of any number of feet that the ground will admit of, suppose 8 feet each lock. Before proceeding farther, as stakes were driven in at every place the levels were taken, if a plan and section is required, the ground must be again carefully measured with a theodolite and chain, the first measure that was taken being only done in a rough way, merely for erecting the instrument at nearly equal distances. The surveyor will have to return to where the second stake was placed, and plant up the theodolite, and take a bearing to the first stake that was driven into the ground, past the cut of 4 feet upon the summit; also a bearing to the third stake, and cause the chain-men to measure those distances carefully; and if there are any towns, villages, or farm-houses seen, take bearings to them as in Figs. 1 and 2, page 124; so that these distances may be ascertained by intersection from the line of the canal on either side of it. Also, in crossing a road or brook, or any little rivulet, insert on a sketch not only the distance where they are crossed, but also bearings both to the right hand and the left; mark every enclosure you enter into, and leave a mark by digging a hole with a spade at or near every fence. If the canal is to be made, every enclosure will have to be measured, and the quantity of ground ascertained on each side of the canal, and also the quantity of ground occupied by the canal.

I now come to show the best method of taking locks, suppose of 8 feet each. Order the staffman to place his staff upon the stake at the end of the level, and plant the instrument level at any convenient distance, suppose 430 feet, from the staff, and order the staff-man to slide the vane up or down till you see the hair in the telescope and the mark across the vane to coincide, and cause him to mark with chalk where the under side of the vane is upon the staff; when he comes down to the instrument look how far the mark is upon the staff, and you will find it 1 foot 3 inches; then slide the vane up the staff to 9 feet 3 inches, and order the staff-man to go down 6 or 700 feet, and there to put up his staff to see if he be nearly right. If he is too low, call him nearer you, and place his staff, and look again; you then order him to put in one of his stakes, and keep driving the stake till such time as the vane and hair in the telescope coincide; then remove the instrument 3 or 400 feet below the stake and set it level; order the staffman to pull down the vane till such time as you see the hair in the telescope to coincide; he should then mark where the vane is upon the staff, which is 1 foot 6 inches; then raise the vane to 9 feet 6 inches, which is 8 feet above the mark, and cause him to go 400 or 500 feet down on the line of direction, and put up his staff; if you see the vane, order him to drive in a stake. After several trials, you will see the hair in the telescope to coincide, which finishes the second lock. Every lock required to be taken is done in the same way.

I shall only observe two things more in canal surveying; namely, the method of measuring cuts and banking across glens, gulleys, ravines, or valleys, as they are termed in different parts of the country.

Plate XIII., Fig. 3,

Represents the section of a canal A A, where the instrument was placed; at each station B B is the staff where it was placed; C C, Fig. 8, is the representation of the staff and slider, commonly called the weight or vane, which the staffman holds up as perpendicular as he can when an observation is taken; E is a cutting of 8 feet at the deepest part, and is 850 feet in length; F is another of 11 feet deep, and 920 feet in length; G is another cut of 4 feet deep, and 1440 feet in length; H is a glen or ravine that is almost perpendicular on one side of the brook or burn which was not levelled, being impracticable, but the depth taken from a level stake on the opposite side, and the distance across was fixed by intersection, and calculated by logarithms, to prove if the intersection was right; which, by both methods, was found to be 920 feet wide. The cutting on each side might have been avoided by going across the brook farther down the bank; but as earth is wanted to fill up the ravine or glen, it is best to make a cut, if the ground will admit of it, before crossing any hollow that cannot be avoided, on purpose to get earth to fill it up.

The cutting, banking, building, and calculation of the probable expense required in cutting a canal, is, however, more the profession of an engineer than a land-surveyor; but as a land-measurer is generally employed under the former, his knowledge of a little of this department, joined to his own, is certainly a great acquisition. I shall now point out the method I have hitherto practised myself.

Suppose the section of the canal, Plate XIII., fig. 3, the cutting of which is to be calculated, the canal being proposed to be 12 feet wide at bottom, 26 feet wide at top, and 4 feet deep.

First, add 12, the width at bottom, to 26 feet, the width at top, the sum is 38; the half is 19 for a mean, which multiplied by 4, the depth is 76; which sum multiplied by 7430 feet, the whole length of the canal, the product is 564680 feet; which sum, divided by 27, the number of feet in a cubic yard, is 20914 yards nearly, at 8d. per yard, ...	L.697	2	8

The cut of 4 feet deep at the summit is 1440 feet long, 26 feet wide at bottom, and 40 feet wide at top.

Add 26 and 40, which is 66, the half of which is 33 for a mean width; which multiplied by 2, being a mean depth, is 66; which again multiplied by 1440, the whole length of the cut, is 95040; which, divided by 27, the cubic feet in a yard, is 3520 yards, at 8d. per yard, is . .	117	6	8

The cut F at the steep side of the glen is 26 feet wide at bottom, 62 feet wide at top, and 11 feet deep, and 920 feet long.

Add 26 and 62, the sum is 88, the half of which is 44, being a mean width; which multiplied by 5 feet 6 inches, half the depth, being a mean, is 242 feet; which again multiplied by 920, the length of the cut, is 222640; which, divided by 27, the cubic feet in a yard, is 8246 yards, at 8d.	274	17	4

The cut E at the opposite side of the glen is 26 feet wide at bottom, 52 feet wide at top, 8 feet deep, and 850 feet long.

Add 26 to 52, the sum is 78, the half is 39 for a mean width; which, multiplied by 4, half the depth, is 156; which, multiplied by 850, the length, is 132600; which, divided by 27, is 4911 yards, at 8d., ...	163	14	0
To two locks, at L.800 each, ...	1600	0	0
To an aqueduct across the glen, ...	150	0	0
To towing path, &c. &c.	180	0	0
To ten per cent allowed for unforeseen accidents, . .	318	6	0
Total expense of cutting a canal 1 mile 3 furlongs and 45¼ yards in length, . .	L.3501	6	8

Plate XIII., Fig. 4,

Represents part of a lake, which the proprietor wished drained either by an open cast or a mine. The levels being taken, the summit was found to be 34 feet high above the bed of the lake, and the cut is 1200 feet long, the ground a stiff clay. In calculating the expense, it was found as follows:—

6 feet wide at bottom + 30 feet wide at top, is 36 feet; the half is 18 × by 1200 = 21600, which × 17, half the depth, is 367200 cubic feet, which ÷ 27 cubic feet in a yard, is 13600 cubic yards, at 1s., . . .	L.680 0 0
Two men took in hand to cut a mine 4 feet deep and 3 feet wide at 9s. per running foot, and to put up the sluice, &c. at their own expense, for	540 0 0

This last method was adopted, not only for being less expensive, but because it saved the land from being broke into a deep and steep ravine.

Plate XIII., Fig. 5,

Represents the section of a road, which has pulls of 1 in 15, 1 in 34, and 1 in 9, in going from the bottom to the top—the height is 30 feet, and the length 480 feet: 480, divided by 30, is 16; which shows that the road can be brought to an equal pull of 1 in 16, by cutting 2 feet deep at *a*, and banking 3 feet at *b*. In going down the hill on the other side, the road descends at 1 in 16, 1 in 60, and 1 in 10—the length is 480; which sum divide by 30, the height of the hill, and the quotient is 16; which shows that the hill can be brought on that side to an equal descent of 1 in 16, by cutting 6½ feet at *c*, to the dotted line drawn upon the section.

Plate XIII., Fig. 6,

Is the section of a hollow, which has a descent of 1 in 25, another of 1 in 16, and another of 1 in 13; and it is proposed to make the road of an equal declivity. The length is 610 feet, and the fall is 36 feet. Divide 610 by 36, the quotient is 17 nearly; which shows that the descent can be brought to a fall of 1 in 17, by cutting 4½ feet from the surface at *a*, to the dotted line marked on the section. In ascending the hill from the hollow there is a rise of 1 in 25, another of 1 in 12, another of 1 in 10, and another of 1 in 28: the height is 31 feet, and the whole length is 620 feet; which divided by 31, the quotient is 20; which shows that the ascent can be brought to an equal rise of 1 in 20, by banking 2 feet at *b* to the dotted line, and cutting 7 feet deep from the surface at *c*.

Plate XIII., Fig. 7,

Represents an excellent little instrument, which was invented by Messrs Adie, mathematical-instrument makers in Edinburgh, which enables the inspector and contractor of roads to ascertain the ascents and descents of a road with greater facility and precision than has hitherto been practised. A and B are two rulers of mahogany a foot in length, 1 inch broad, and half an inch thick, joined together with a joint in the same manner as a common foot-rule. The leg A has a glass tube filled with spirits all to a bubble, which is fixed on the upper side; the brass arch *c* folds into the leg B when put into the pocket; D D is a rod 12 feet in length, which is laid upon the surface of the road, and the instrument is laid upon it; there is a rack and pinion to bring the leg A to a level; the arch is divided into equal distances of 12 in an inch. That this road instrument may be rendered as easy and expeditious as possible, the inspector and contractor ought to be furnished with a table the same as the one annexed, that the declivity or acclivity of any road they wish to know the pull of (as it is termed) may be exactly ascertained.

Divisions on the Arch.	One in	Divisions on the Arch.	One in	Divisions on the Arch.	One in
1	144	7½	19	14	10
1½	96	8	18	15	9½
2	72	8½	17	16	9
2½	58	9	16	17	8½
3	48	9½	15	18	8
3½	41	10	14½	19	7½
4	36	10½	13¾	20	7¼
4½	32	11	13	21	7
5	29	11½	12½	22	6¾
5½	26	12	12	23	6¼
6	24	12½	11½	24	6
6½	22	13	11	25	5¾
7	20	13½	10½		

The following table, by the help of the arc on the theodolite, is found to be very useful to a practical surveyor in ascertaining the pull that a road can be brought to, by a single observation, if the top of the rise from the bottom can be seen, or *vice versa*, if the bottom can be seen from the top.

Degrees.	One in	Degrees.	One in	Degrees.	One in	Degrees.	One in
0¼	229	2¾	21	5¼	11	10	5¾
0½	115	3	19	5½	10½	11	5¼
0¾	76	3¼	18	5¾	10	12	5
1	57	3½	16	6	9½	13	4½
1¼	46	3¾	15	6½	9	14	4¼
1½	38	4	14	6¾	8½	15	4
1¾	33	4¼	13	7	8⅓	16	3¾
2	29	4½	13	7½	7½	17	3½
2¼	25	4¾	12	8	7⅓	18	3¼
2½	23	5	11½	9	6½	19	3

Explanation of the Table.—Set the theodolite level by the help of the screws between the brass plates, after having set the index at *o* on the arc; send one of your assistants to the top of the hill, or rising ground, with a piece of paper, or any other mark, which he holds to that part of his breast that the height of the telescope reached to when he was at the bottom; then elevate the telescope till you see the paper and the cross hairs in the telescope to coincide; look to the arc, and see how many degrees and minutes the index cuts, which is 8 degrees; look for 8 degrees in the table, and opposite it, on the right-hand column, is 1 in 7; that is to say, for every 7 feet you go upon the road, you ascend 1 foot in perpendicular height: again, if the index cut 2½° upon the arc, look into the table for 2½°, and opposite it is 1 in 23. This shows that every 23 feet you go upon the surface, you rise 1 foot in perpendicular height. If you wish to know the perpendicular height of the hill, measure the length from the bottom to the top of the rise, which suppose is 2162 feet; divide the number by 23, and the quotient is 94 feet for the height of the hill.

It will be necessary to observe here, that in the different sections on Plate XIII., representing the railway, &c., two scales are used, one for the lengths, and the other for the perpendicular heights. If they had been laid down upon the same scale, the perpendiculars on some of the sections would not have been appreciable on so small a scale; but to be very correct, it will be proper to lay them down upon a very large scale, that the perpendicular height may be ascertained by applying a pair of compasses to the scale the section is plotted by.

As an example of the method of keeping a field-book of levelling nearly as now generally practised, the following has been supplied. The editor has given it in full from his note-book, with the bearings by the compass needle, and the distances in links of the imperial chain. This will afford the means of plotting and sectioning it, if thought advisable, in an approximate manner. In the case of railways, and other works requiring great accuracy, the plan must be protracted from measures taken with the theodolite and chain, as has been previously shown; and those in great practice have a set of books ruled in a commodious form expressly for this purpose. For rapidly executing flying levels of moderate accuracy, a notch in the diagram of a given magnitude will subtend a certain number of divisions on the staff at a given distance, by recording which divisions in an appropriate column, the horizontal distances, by reference to a table for the purpose, become readily known, even in cases difficult to be obtained by direct measurement.

FIELD-BOOK.

FIELD-BOOK OF THE LEVELLING OF THE QUEEN'S ROAD ROUND ARTHUR-SEAT,

COMMENCING AT THE FOOT OF ARTHUR STREET.

No.	Back.	Fore.	Rise.	Fall.	Result.	Name.	Bearing. Back.	Bearing. Fore.	Distance in links of imp. chain.
	Feet.	Feet.	Feet.	Feet.	Feet.		Deg.	Deg.	
1	0.10						222		444
2		7.75		7.65	7.65			51	212
3	3.10						235		222
4		4.95		1.85	9.50			63	227
5	5.68						249		316
6		5.25	0.43		9.07	Fall.		79	319
7	4.50						263		406
8		5.94		1.44	10.51			89	332
9	4.94						271		450
10		1.90	3.04		7.47			96	400
11	11.65						279		800
12		1.05	10.60		3.13	Rise.		98	650
13	5.56						277		734
14		9.05		3.49	0.36	Fall.		79	809
15	11.10						292		484
16		0.35	10.75		10.39			77	167
17	11.70						249		190
18		0.40	11.30		21.69			95	110
19	11.97						280		238
20		0.09	11.88		33.57			119	105
21	12.05						297		225
22		0.50	11.55		45.12			124	100
23	12.05						341		240
24		0.35	11.70		56.82			161	120
25	11.30						328		180
26		0.54	10.76		67.58			143	119
27	9.40						329		164
28		0.72	8.68		76.26			157	100
29	11.64						346		225
30		0.25	11.39		87.65			165½	113
31	11.86						339		198
32		0.11	11.75		99.40			152	100
33	11.50						347		190
34		0.08	11.42		110.82			172	87
35	10.55						341		143
36		0.56	9.99		120.81			158	58
37	11.04						349		100
38		0.17	10.87		131.68			190	60
39	11.10					Rise.	22½		100
40		0.64	10.46		142.14			204½	50
41	11.56						20		128
42		0.46	11.10		153.24			194	77
43	11.99						12		161
44		0.36	11.63		164.87			195	100
45	11.50						18½		187
46		0.36	11.14		176.01			203	85
47	10.35						22½		150
48		1.30	9.05		185.06			210½	73
49	10.31						23		146
50		0.02	10.29		195.35			207½	100
51	11.70						28		174
52		0.27	11.43		206.78			210	100
53	11.97						43		200
54		0.07	11.90		218.68			247	100
55	11.70						80		174
56		0.35	11.35		230.03			267½	90
57	11.82						95½		240
58		1.90	9.92		239.95			278	58
59	9.45						101		253
60		0.01	9.44		249.39			260½	381
	295.14 45.75	45.75							
	249.39	Carried over.							

No.	Back.	Fore.	Rise.	Fall.	Result.	Name.	Bearings. Back	Bearings. Fore.	Distance in links of imp. chain.
	Feet.	Feet.	Feet.	Feet.	Feet.		Deg.	Deg.	
	295.14...	45.75...	Brought	forward.	249.39				
61	5.00						42		318
62		4.52	0.48		249.87			176½	618
63	0.27						354		457
64		7.26		6.99	242.88			189	496
65	4.69						43½		193
66		5.91		1.22	241.66			258½	507
67	4.70						75½		636
68		0.00	4.70		246.36			267	748
69	7.52						83		493
70		4.40	3.12		249.48			275½	430
71	8.65						109		475
72		0.67	7.98		257.46			294½	179
73	10.22						128		355
74		3.92	6.30		263.76			317	225
75	1.23						144		158
76		11.45		10.22	253.54			334	246
77	0.37						170		130
78		11.65		11.28	242.26			5	215
79	0.30						185		122
80		11.00		10.70	231.56			16	102
81	0.45						143		102
82		11.70		11.25	220.31			328	110
83	0.52						127		64
84		11.40		10.88	209.43			302½	90
85	0.18						114		64
86		9.95		9.77	199.66			232½	70
87	0.15						108		63
88		10.95		10.80	188.86			293	88
89	0.15						116		63
90		11.76		11.61	177.25			302	100
91	0.05						123		68
92		11.90		11.85	165.40			307½	110
93	0.07						128½		83
84		11.63		11.56	153.84			314	125
95	0.38					Rise.	132		81
96		11.70		11.32	142.52			309	202
97	0.60						127		130
98		10.20		9.60	132.92			304½	150
99	0.80						127		100
100		11.00		10.20	122.72			303	150
101	0.25						124		110
102		11.00		10.75	111.97			298	150
103	0.70						116½		100
104		10.80		10.10	101.87			293	150
105	0.00						117		130
106		8.75		8.75	93.12			300	100
107	0.75						132		110
108		8.35		7.60	85.52			316½	100
109	1.65						147		100
110		8.30		6.65	78.87			338	100
111	0.20						171		150
112		10.50		10.30	68.57			360	200
113	0.10						185		190
114		9.40		9.30	59.27			4	200
115	0.30						186½		170
116		10.90		10.60	48.67			7	200
117	0.50						190		150
118		7.95		7.45	41.22			11	100
119	2.15						195½		100
120		8.50		6.35	34.87			17	100
121	0.95						203		100
122		9.00		8.05	26.82			23	100
123	0.30						189½		108
124		9.90		9.60	17.22			32½	90
125	0.20						211		107
126		10.15		9.95	7.27			37½	150
127	0.20						215		143
128		7.47		7.27	0.00			40	218
	349.69	349.69							26106

RAILWAY SURVEYS.

In surveying a tract of country for a railway, some of the methods previously given may be readily applied under any given circumstances that can occur in actual practice; it will only be necessary to attend to the conditions of the railway, and the standing orders of parliament for the time, to accommodate the plans, sections, and transits, or cross-levels, to every object which the projectors have in view. These may vary at different periods, and it therefore becomes necessary for every engineer to make himself thoroughly acquainted with them at the time he is so employed. In general, it will be seen from an inspection of *Plates* XIII. XIV. and XV., that great attention must be paid to an accurate survey of the country extending to some distance on each side of the railway, pointing out every county, parish, town, village, farm, or other remarkable place throughout its extent, near which the railway passes—indicating the position of the rails by a *strong dark line;* while the limit of deviation, to which the railway company may be empowered to go is indicated by dotted lines on each side, at a certain distance from the proposed line of rails. The different portions, fields, &c., must be all numbered, as in *Plate* XIV., for the sake of reference, either before parliament or elsewhere. The crossing of public roads, the passages of buildings, &c., must be drawn on a larger scale, fixed by parliament. At the present time, 1848, the plans must not be less than *four* inches to the mile, and one hundred feet to the inch for sections.

In the ordnance survey, the maps are at present drawn on a scale of six inches to the mile. Now, if on these the sections are 100 feet to an inch, the ratio of the plan to the section would be 1 to 8.8, or nearly 1 to 9—that is, the section scale would be nearly nine times that of the plans; and on this scale, in important and closely-built positions, the plans and sections must be given. The rates of inclination must likewise be all carefully marked on the section, with references to well-known bench marks, as the mean level of the sea, &c.

See the plates already referred to for more minute information, since an inspection of them will give, at a glance, more distinct ideas than a lengthened description in words.

The working sections must be given on a much larger scale than those intended for parliament, so that an accurate estimate may be made of their contents, and consequent expense of execution. This will be readily comprehended by an inspection of *Plate* XV., which is a part of the Radstock branch of the Wilts and Somerset Railway, furnished me by Mr James Geddes, under Mr Brunel.

It was drawn originally on paper faintly ruled with yellow ink; making the horizontal scale two chains, to one inch, and the vertical scale *twenty* feet to one inch. For cross sections, cuttings, and embankments, the scale was *ten* feet to an inch. This ruled paper can be procured from Messrs W. and A. K. Johnston, engravers in Edinburgh, as well as in London, and will be found remarkably convenient for the purpose of drawing plans and working sections, because the ruled lines on the paper itself formed the scales.

When the plan and sections are obtained, the extent of cuttings and embankments can be determined. For this purpose, various collections of tables have been computed. Those of Sir John MacNeill are the most copious, though their price is considerable. Smaller collections have been published by Kelly and Baker. The most convenient for general purposes are those of Sibley and Rutherford, which I have great pleasure in recommending to the notice of practical men. Most of them are founded on what is called the *prismoidal formula*, which is very generally applicable; there are however, occasionally, instances in which it cannot be confidently employed, especially when the ground is very irregular, and the surface of the ground, as in cross section No. 10, slopes considerably across the line. In such cases, no mathematical formula will give results sufficiently exact, and it then becomes necessary to take the area of the cross sections, as often as may be considered necessary, and multiplying each by the length chosen, according to the judgment of the engineer. The prismoidal formula, however, will be frequently useful, and it is accordingly given here.

Let l be the length, and b the breadth of the base; H the height of the higher, and h that of the lower end, d the difference of the heights, and s the ratio of the side slopes — the whole dimensions being given in *feet*, the result in *cubic yards* will be,

$$y = \frac{1}{27} l \left\{ b \frac{1}{2} (\mathrm{H} + h) + s \mathrm{H} h + \frac{1}{3} s d^2 \right\} \quad . \; . \; . \quad (1.)$$

This will give the cubic content in square yards, when the surface of the ground across is level, but will require a modification when that is not the case.

For the application of this formula to practical examples, see the explanation of the tables already referred to, and the explanation of Table XXXII. of this work, which, when properly applied, will serve most general purposes.

ON DETERMINING THE EFFECTS OF GRADIENTS ON DIFFERENT LINES OF RAILWAY.

1. In the present times, when railways are constructing in all parts of the United Kingdom, as well as abroad, on account of their great importance and general utility, a recommendation in favour of their adoption cannot now be necessary. However evident this general proposition may be, yet it requires much caution and considerable professional knowledge to select the lines in such a manner as to enable the country, as well as the general public and shareholders, to derive the full advantage they are certainly calculated to produce. The surveyor undertaking such a work ought first to ascertain, by personal observation, the nature of the country through which it is intended to pass, with regard to its localities, its structure, and geological character. This might lead him to the choice of several lines apparently equally favourable, as far as a cursory inspection of the ground by the eye, chiefly through the medium of its lakes, rivers, and mountain ranges, could determine.

2. In the selection of railways, too, the amount of traffic, to a certain extent, ought to regulate the nature of the construction. If there is a certainty of great traffic, the expenditure in tunnelling, cutting, embanking, and viaducts, may be very considerable, in order to improve the gradients; but if a moderate trade only is to be expected, such an expenditure must be injudicious, because it increases the charges, or diminishes the profits of the original shareholders, and thus, unless at the expense of the public, they must receive an inadequate remuneration for their capital.* Circuitous lines, to serve inferior provincial towns, are not to be recommended, except to a limited extent, because the greatest amount of the whole traffic may be expected from the large towns at the extremities, for whose use chiefly the railway would be constructed, and through whose influence the bill for such a purpose must chiefly be carried.

The passengers between these towns would thus be compelled to pay fares for these additional miles so thrown into the railway,

* It has sometimes been insinuated that the managers or directors of railways have paid high per-centages not derived from actual profits, and then reduced them low. This system might suit the purposes of artful speculators in railway shares. In fact, however, the extravagant formations of many railways are such as to prevent a good return for invested capital. I do not understand on *what principles of equity preferable shares* can be created. This seems to demand legal or parliamentary inquiry.

while, at the same time, the duration of transit would be increased, by this means entailing a positive loss on the majority of the passengers, both in money and time. It would generally, therefore, be better to connect these inferior towns with the main line by short branches.

The advantages, therefore, to be derived from the use of railways, are, rapidity of transit, and economy of charge; to accomplish which, the following principles should be kept constantly in view:—

1°, One of the conditions which must not be departed from in laying out great lines of railway is, that those lines may be traversed throughout their whole extent by locomotive engines; and in order to avoid, as much as possible, interruptions and delays, that the same engine should draw the same train.

2°, Another condition is, to diminish, as far practicable, the time of transit between two given points, by reducing the length of the railway. In this case the straight line, either horizontal or having one uniform slope, will be the most advantageous. It is this line which ought to be selected, or the nearest practicable one to it, both horizontally and vertically.

3°, If two lines may be chosen equally advantageous in these respects, then that which passes through the most populous and richest country in minerals ought to be chosen.

4°, It would be a great error to suppose that the line may be lengthened circuitously, because by that means, by getting easy gradients, the velocity will be much increased; since what is gained in velocity, it is obvious, may be easily lost in greater distance.

5°, It was formerly supposed (and this hypothesis has been acted upon by many engineers) that the entire line of railway should, as nearly as possible, have one uniform slope, with very good gradients, however circuitous almost the line might be made to obtain them.

6°, Now, within certain limits, this is doubtless true; but it requires great care and considerable science to be able to determine these with tolerable accuracy in practice.

7°, It has been a maxim with some engineers, that if a uniform slope is impracticable, or if it requires too great a deviation from the straight or direct line, it is necessary, at least, to endeavour to rise progressively from one extremity of the line to another, and never to ascend where it must descend again.

8°, But it is clear that such views are, within certain limits, incorrect; for if the traction be increased by gravity, when a train or engine is impelled *up* an inclined plane, in proportion to the rate of rise, it will be diminished in nearly the same proportion when it descends, especially when the gradients are very good, never exceeding 1 foot in 300, and generally much less, in which circumstance the acceleration from gravity requires no check.

9°, On this principle, the loss of velocity in ascending one side of a rising ground or inclined plane will be nearly, but not exactly, compensated by the gain in descending the other, when the slopes are equal, and some aliquot part of it regulated by the difference, if they are unequal, and this compensation will be the more nearly equal the better the slopes are, and the more perfect our engines become. In this last case, the *ratio* of the friction on the inclined plane to that on the horizontal plane may increase, *though the total effect will be diminished.*

10°, Hence, in tracing a line of railway, there is no inconvenience in rising higher to redescend afterwards, so long as that does not render it necessary to extend the limit of the slopes. Thus, for example, several lines uniting two given extreme points, upon which it is admitted that the same locomotive engine draws throughout the same train, will be perceptibly equal in respect to the expense of transit, whatever be the height to which they rise or to which they descend, if their lengths be equal, and if, upon any of these lines, the steepest slopes do not surpass 1 in 200, so as to produce an inconvenient or dangerous acceleration.* Hence it appears that special care should be taken to diminish the length of the line of transit, to lower the limit of the slopes; and that it is unnecessary, for the sake of remarkably favourable gradients, to involve a railway company in extravagant expenses, in order to complete tunnels, make embankments, and construct viaducts—the interest of the money required for which frequently exhausting a considerable portion of the revenue of the speculation, and diminishing the dividends of the shareholders, who ought, in all cases, to receive a fair remuneration for the money they may have advanced.

11°, To select the cheapest and most efficient line of railway, depends upon the following proposition:—*To combine the distance between two given points with the gradients in such a manner as to produce the greatest effect at the least expense.*

* Since this paragraph was originally written, the power of locomotive engines has been greatly increased, so that a train can be moved up an inclined plane of 1 in 40, or less, and that 1 in 80 or 1 in 100 is now much introduced, and reckoned tolerably good. In fact, many stationary engines are now replaced by more powerful locomotives, though certainly good gradients should always be adopted when convenient, and can be obtained at moderate expense.

Though this proposition, in general, cannot be solved directly, yet, by attending to the preceding principles, an approximate solution may be obtained, by the aid of the tables accompanying this work, sufficiently accurate for all ordinary purposes.

12°, In estimating the mean value of the gradients throughout a line, the value of each, with its proper sign, must be multiplied by its length, and the algebraical sum of the products divided by the length of the whole line, including the levels in the same measure, will be the mean value of the gradients, in which the signs of the ascents must be reckoned positive, and those of the descents negative.

13°, If the force of traction obtained in this way on two lines connecting the same two extreme points be inversely as their lengths; or if the product of the length of one line, multiplied by its force of traction, be equal to the product of the length of another line multiplied by its force of traction, the effects of those two lines would be equal, or equal tonnage would, by equivalent locomotive engines, be transported along each line in equal times. This follows from the fact that, if the traction on a *unit* of the line—such, for example, as *one* mile—be multiplied by the whole length in miles, the product will be the total traction throughout the line, and it will *express* the power expended in propelling an engine throughout the whole line. Hence the relative effective powers of two lines of railway may be easily estimated, and their respective advantages and disadvantages readily determined.

14°, As the length of a line of railway is one of the elements employed to compute the expense of transit, it is clear it should be as short as convenient and sound principles will admit, because it will also reduce the time of transit. It would be committing a great error to suppose we may lengthen the line because the velocity of transport over it is great. The same principle which rendered the establishment of a railway necessary or desirable, in order to obtain a mode of transport quicker than any other, requires that the *shortest lines* should be sought after, and even to prefer them, when sometimes they appear disadvantageous in other respects.

15°, In order to ascertain the effects of slopes, experiments have been instituted to determine the amount of tractive force necessary to propel a ton of burden on a level plane or horizontal line of a well constructed railway. This, of course, varies a little with the quality of the railway, as well as with the construction of the carriages, and depends on the total amount of friction. In general it varies from 8 lb. to 9 lb. per ton, and is therefore very generally assumed at 8½ lbs. per ton, an approximation, in the present state of railway carriages, not far from the truth. Now, in one ton there are 2240 lbs., consequently, if 2240 lbs. be divided by 8.5 lbs., the quotient is 264, an abstract number, from which it is inferred that the traction on the level plane is equal to 1-264th part of the weight drawn. But, by the principles of mechanics,—*The weight moved upon an inclined plane, is to the power by which it is moved as the length of the inclined plane is to its height.*

Suppose, for example, that a waggon enters upon an inclined plane rising 20 feet in an English mile of 5280 feet, or 1 foot in 264 feet, it follows, from the preceding analogy, that an additional 8½ lbs. will be combined with that on the level, or that *twice* the force will be necessary to propel the carriage with its load *up* this ascent at the same velocity as on the level—that is, if 8½ lbs. per ton be required to propel a carriage or train of waggons at the rate of 30 miles an hour on the *level*, it would require *double* that force of traction, or 17 lbs. per ton, to keep up that velocity on an inclined plane or slope rising 1 foot in 264, or 20 feet in a mile.

It also follows, from the same process of reasoning, that a velocity of 30 miles an hour might be kept up on ascending that inclined plane, if the train of waggons carried a part of the load only. It is frequently observed that an undulating line, having considerably steep slopes, limits the load to what the locomotive engine can propel *up* these gradients,—a fact undoubtedly true. But no slopes so steep as to nearly stop the trains proceeding at the rate of 30 miles an hour on the level should be admitted on any railway, unless from unavoidable necessity; and in that case a stationary, or, better still, a powerful assistant locomotive engine must be employed at those points where they may be required. In all lines where the gradients are not more than 1 in 300, no such occurrence can take place; and to expend large sums of money on tunnels, cuttings, embankments, and viaducts, or circuitous lines for better, must be considered a useless expenditure of the public money.

Again, if the rise be 1 in 2000, it will require an additional force of 1.12 lbs. per ton, which, added to 8.5 lbs., that on the level, gives 9.62 lbs., the necessary tractive force *up* this inclination, similarly as before. In this way we arrive at a distinct knowledge of the exact amount of tractive power necessary to propel any load *up an inclined plane*, whatever be its rise per mile, or its inclination.

If, on the contrary, the train be moving *down* the descending plane, then the tractive force necessary on the level plane will be diminished by the effects of gravity, to keep up the same velocity on the inclined plane as on the level. Hence, if the power be constant, there will be a *retardation* in ascending the inclined plane, and a corresponding *acceleration* in descending, which will, in well-constructed railways, whose gradients do not exceed 1 in 300, nearly counterbalance each other. The modifications on this account may be obtained from the accompanying tables.

Indeed, absolute accuracy is hardly to be expected in such cases, since a sufficient number of experiments on all sorts of inclinations, in different circumstances, to be combined with mathematical investigations, have not yet been completed. In the first table, by Mr Barlow, I believe, though it may give a good approximation to the truth, it appears singular that there should be such disruption of *the law of continuity* on the descending plane at about 1 in 140. It appears somewhat strange that a change from 1 in 140 to 1 in 160 should change abruptly the equivalent horizontal plane from 1.00 to 0.83; while from 1 in 160 to 1 in 180 it does not change at all, and even continues the same to 1 in 500; while, by the experiments of Dr Lardner, he finds a complete compensation of velocity from 1 in 177 to the dead level, and there is no dangerous acceleration on inclined planes of considerable steepness, so that, after acquiring a certain velocity, the motion becomes uniform. No doubt this must be true when the friction, combined with the resistance of the atmosphere, become equivalent to the acceleration from gravity. More numerous experiments, I suspect, are yet required to ascertain the precise limits, in given circumstances, within which this compensation takes places. Though I am disposed to put greater confidence in Mr Barlow's views on most points than in Dr Lardner's; yet, in the present case, from the remarks I have made above, and what has occurred to my own knowledge, it would appear that there is some foundation for Dr Lardner's results.

On the preceding principles will be compared the relative merits of two assumed lines of railway, in which the values of the respective gradients are given in a column adjacent to the corresponding measured distances of the slopes, &c., for a passenger train of 50 tons only, by way of example.

RESULTS OF LINE A, for a Passenger-Train of 50 Tons.

No. of Slopes.	Character.	Measured Distance in Miles.	Gradients.	Equivalent Hor. Dist. Forward.	Equivalent Hor. Dist. Backward.	Mean Hor. Distance.
1	Level	0.800	0	0.800	0.800	0.800
2	Descent	1.530	1 in 2000	1.484	1.576	1.530
3	Ascent	2.950	1 in 2000	3.227	2.673	2.950
4	Level	1.736	0	1.736	1.736	1.736
5	Ascent	6.250	1 in 600	7.450	5.250	6.362
6	Ascent	1.143	1 in 1200	1.251	1.036	1.163
7	Level	1.143	0	1.143	1.143	1.143
8	Descent	1.143	1 in 1200	1.036	1.251	1.163
9	Ascent	2.270	1 in 2000	2.483	2.057	2.270
10	Level	0.760	0	0.760	0.760	0.760
11	Descent	2.480	1 in 1200	2.247	2.713	2.480
12	Level	0.470	0	0.470	0 470	0.470
13	Ascent	8.455	1 in 800	9.639	7.271	8.455
14	Level	2.600	0	2.600	2.600	2.600
		33.730		36.326	31.336	33.882

Hence 33.882^m—33.730^m = 0.152 mile, or about ⅐ of a mile, the loss upon the gradients.

RESULTS OF LINE B.

No. of Slopes.	Character.	Measured Distance in Miles.	Gradients.	Equivalent Hor. Dist. Forward.	Equivalent Hor. Dist. Backward.	Mean Hor. Distance.
1	Level	0.875	0	0.875	0.875	0.875
2	Ascent	1.000	1 in 528	1.212	0.830	1.030
3	Level	1.500	0	1.500	1.500	1.500
4	Ascent	2.000	1 in 480	2.460	1.660	2.060
5	Ascent	4.875	1 in 422	6.138	4.046	5.115
6	Ascent	3.625	1 in 440	4.531	3.009	3.770
7	Ascent	6.625	1 in 330	8.811	5.499	7.155
8	Descent	4.000	1 in 330	3.320	5.320	4.320
9	Descent	2.500	1 in 660	2.100	2.938	2.675
10	Descent	1.759	1 in 406	1.452	2.222	1.837
		28.750		32.399	27.899	30.337

Hence 30.337^m—28.750^m = 1.587, or about 1½ mile, the loss upon the gradients.

On comparing the results of these two lines, designated by A and B, it appears that A loses only about *one-seventh* of a mile, while B loses about *a mile and a-half*, *in steam-power or in time*, by means of the gradients *alone*, when the effects of the slopes are estimated by Mr Barlow's tables; but this does not give a proper estimate of the relative expenses of the lines. This is obtained from a comparison of the mean horizontal distances in the right-hand columns. Thus A has 33.882 miles of mean horizontal distance, while B has only 30.337 miles. The mean difference of these is 3.545 miles, the loss of A above B, in passing once along the line, and of course double of this, or about 7 miles, in one trip forward and back, of *steam-power or of time*. These conclusions are independent of 5 miles of actual measured distance, for the construction of which additional miles funds must be provided, which causes an immense loss, or useless expenditure of money to the shareholders of A's line, while that of B is more effective.

Besides entailing the expenses of construction on the shareholders for these additional useless 5 miles, the expenses of transit over them must be charged on goods and passengers, thus compelling those who use the railway to suffer a severe annual loss, without any equivalent advantage. These injudicious schemes will no longer be tolerated, as the legislature now (1848) employ, very properly, men of competent science to examine them before an act for their execution can be obtained. So much for the benefit conferred on the public by injudicious speculators in railways. *In fact, they materially injure the public as well as themselves.**

Hence, from this investigation, great care should be taken to avoid extending the lines too much for the sake of good slopes, whereby more may be easily lost in distance than gained by good gradients. Hence, too, the policies, or fancy grounds and parks of noblemen and private gentlemen are indiscriminately assailed without any reason. The public are, indeed, greatly interested in the proper selection of the cheapest and most economical line of railway in every respect, and ought to make every exertion to obtain it. For this purpose, it appears that a national system of railways ought to be adopted, and that parliament ought to exercise great care in examining the nature and qualities of all railways, before passing bills for their completion.

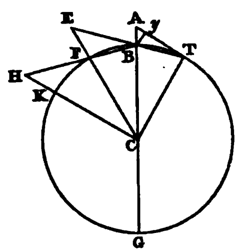
In conclusion, it ought to be an object with the engineer to render, as nearly as possible, the cuttings and embankments *equal*, so that little ground will be required for superfluous earths to be deposited in *spoil banks*, as they are technically called. For making the necessary calculations, MacNeill's Tables will be found very

* These are the results from the comparison of two lines actually surveyed and proposed for adoption. The disadvantageous line was adopted and the *advantageous rejected!* Can shareholders expect a fair return for capital so expended?

useful. If these are considered too expensive, some of the smaller tables, as our Table XXXII., &c., may be easily obtained, accompanied by directions for their use.*

3. *To lay off points in a circle, such as in the curves of railways.* In the figure we have, by the principles of geometry, $AB = \frac{AT^2}{BG}$ nearly.†

Here let the radius CB of the curve be one mile, hence BG = 2 × 8000 links = 16,000 links. Now, let a point for each degree round the centre be set off. Then the natural tangent for one degree is 0.017455, whence 0.017455 × 8000 = 139.64 links = AT = Ty nearly, in this case. Hence $\frac{AT^2}{BG} = \frac{139.64^2}{16000} = 1.225$ links = AB or y B, because, when the circle is great compared with TA or T y, then AB and y B must be nearly equal.



Again, produce the chord TB to subtend another degree, making BE = TA, or rather T y, then EF = 2 y B = 2 × 1.225 = 2.45 nearly. Again, through B and F produce the chord BF to H, so as to subtend another degree, and make HK = 2.45 links as before, and K will be another point in the circle; continue this process till the whole curve is completed by points, and then soften the points into a continuous curve by any practical method that may readily occur.

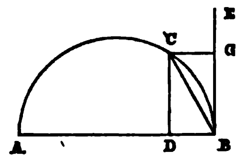
The curves may be set off in the following manner, where the arc, as used, is small; and it is sufficiently near the truth for practice.

By geometry, AB : BC : : BC : BD; hence $BD = \frac{BC^2}{AB}$. When the arc BC is small, it differs little from its chord; and since AB

* See Explanation of Railway Tables.

† From the same principles it is also obvious that BG or 2 BC $= \frac{AT^2}{AB}$ or BC$= \frac{\frac{1}{2}AT^2}{AB}$; that is, the radius of curvature varies as half the *square* of the length of a rail divided by the deviation; hence great care is required in laying rails to preserve the adopted curvature, since a *small deviation* of the rail causes a *great change* in the radius of curvature.

is constant, then BD varies as BC^2. Now, commencing at B, the deviation of the curve BC, from the straight line BE, as in levelling, increases as the square of BC. Taking, therefore, BC for the whole length of the curve, and assuming any number of points in BE, as C', C'', C''', &c., these deviations will be as the squares of BC', BC'', BC''', &c.



Setting out from C, and CG being known, then at *one-half* GB the deviation will be *one-fourth* of CG; at one-third of GB, it will be one-ninth, &c. Hence any number of points in the curve may be found when the operations commence either at B or G, as may be considered most convenient.

The curves of railways may be also determined by means of the following table of co-ordinates of the circle computed to a radius of unity, and to each hundredth part of that radius, through the whole diameter, to six places of figures.

TABLE I.

CO-ORDINATES OF THE CIRCLE TO RADIUS UNITY FOR EVERY HUNDREDTH PART OF THE DIAMETER.

Absciss.	Ordinate.	Absciss.	Absciss.	Ordinate.	Absciss.
0.00	0.000000	2.00	0.20	0.600000	1.80
0.01	0.141067	1.99	0.21	0.613107	1.79
0.02	0.198997	1.98	0.22	0.625780	1.78
0.03	0.243105	1.97	0.23	0.638044	1.77
0.04	0.280000	1.96	0.24	0.649923	1.76
0.05	0.312250	1.95	0.25	0.661438	1.75
0.06	0.341174	1.94	0.26	0.672607	1.74
0.07	0.367560	1.93	0.27	0.683447	1.73
0.08	0.391918	1.92	0.28	0.693974	1.72
0.09	0.414608	1.91	0.29	0.704202	1.71
0.10	0.435890	1.90	0.30	0.714143	1.70
0.11	0.455961	1.89	0.31	0.723809	1.69
0.12	0.474974	1.88	0.32	0.733212	1.68
0.13	0.493052	1.87	0.33	0.742361	1.67
0.14	0.510294	1.86	0.34	0.751266	1.66
0.15	0.526783	1.85	0.35	0.759934	1.65
0.16	0.542586	1.84	0.36	0.768375	1.64
0.17	0.557763	1.83	0.37	0.776595	1.63
0.18	0.572364	1.82	0.38	0.784602	1.62
0.19	0.586430	1.81	0.39	0.792401	1.61

[*Continued.*

TABLE I.—*continued.*

Absciss.	Ordinate.	Absciss.	Absciss.	Ordinate.	Absciss.
0.40	0.800000	1.60	0.70	0.953939	1.30
0.41	0.807403	1.59	0.71	0.957027	1.29
0.42	0.814616	1.58	0.72	0.960000	1.28
0.43	0.821645	1.57	0.73	0.962860	1.27
0.44	0.828493	1.56	0.74	0.965609	1.26
0.45	0.835165	1.55	0.75	0.968246	1.25
0.46	0.841665	1.54	0.76	0.970773	1.24
0.47	0.847998	1.53	0.77	0.973191	1.23
0.48	0.854166	1.52	0.78	0.975500	1.22
0.49	0.860174	1.51	0.79	0.977701	1.21
0.50	0.866025	1.50	0.80	0.979796	1.20
0.51	0.871722	1.49	0.81	0.981784	1.19
0.52	0.877268	1.48	0.82	0.983667	1.18
0.53	0.882666	1.47	0.83	0.985444	1.17
0.54	0.887919	1.46	0.84	0.987117	1.16
0.55	0.893029	1.45	0.85	0.988686	1.15
0.56	0.897998	1.44	0.86	0.990152	1.14
0.57	0.902829	1.43	0.87	0.991514	1.13
0.58	0.907524	1.42	0.88	0.992774	1.12
0.59	0.912086	1.41	0.89	0.993932	1.11
0.60	0.916515	1.40	0.90	0.994987	1.10
0.61	0.920815	1.39	0.91	0.995942	1.09
0.62	0.924986	1.38	0.92	0.996795	1.08
0.63	0.929032	1.37	0.93	0.997547	1.07
0.64	0.932952	1.36	0.94	0.998198	1.06
0.65	0.936750	1.35	0.95	0.998749	1.05
0.66	0.940425	1.34	0.96	0.999200	1.04
0.67	0.943981	1.33	0.97	0.999500	1.03
0.68	0.947418	1.32	0.98	0.999800	1.02
0.69	0.950737	1.31	0.99	0.999950	1.01
0.70	0.953939	1.30	1.00	1.000000	1.00

By this table the co-ordinates of an ellipse may be obtained, by multiplying those for the circle by the ratio of the axes of the ellipse.

APPLICATION.

Rule.—Divide the *given* absciss by the radius of the given circle, and find the tabular number corresponding to the quotient. Multiply this number by the radius of the given circle, the product will be the corresponding ordinate.

Let a be the given absciss, r the given radius, or o the required ordinate, then from $\frac{a}{r} = n$ the tabular number will be found

$o = nr$ the given ordinate.

Example 1.—Let the radius of the circle r be 450 feet, and the absciss, or distance on the diameter, from one of its extremities, 200 feet, required the ordinate, or perpendicular from this point on the diameter to the circumference?

First $\frac{a}{r} = \frac{200}{450} = \frac{4}{9} = 0.444444$.

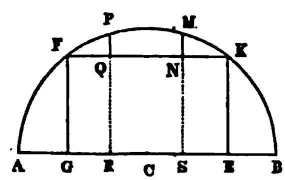
From the table, 0.444444 gives n = 0.831458, the corresponding number from the table.

But $o = nr$, then the ordinate is equal to $0.831458 \times 450 =$ 374.16 feet nearly.

Example 2.—Let the radius of the circle be 60 chains, and the absciss 40 chains, required the ordinate?

$\frac{40}{60} = \frac{2}{3} = 0.66\dot{6}$ gives as absciss $=$ 0.942796, and $0.942796 \times 60 = 56.568$ chains the absciss.

Example 3.—By this table any number of points may be found in an arc of a circle when the radius is known, by setting off distances from its *chord*, which will be the case most commonly required in railway curves.



Suppose the length of the chord FK to be 600 feet, the points G and E will be 600 feet distant, and let the diameter of the circle be 1000 feet, and radius 500 feet. Whence if the ordinate GF or EK be subtracted from those of the points P, M, &c., then the portions remaining, PQ, MN, &c. of these ordinates become known, which set off from the points Q, N, &c., will give the points P, M, &c. in the arc, and through which it may be readily traced, when sufficiently numerous. Thus from 1000 feet AB, subtract 600 feet FK = GE, the remainder will be 400 feet, one-half of which is 200 feet, equal to AG or EB. Then $\frac{200}{500} = \frac{2}{5} = 0.4 =$ AG to radius unity. Whence to 0.4 the ordinate by the table is 0.800000. Again let AR = 400 feet from A, in like manner, $\frac{400}{500} = 0.8$ to radius unity; therefore the ordinate 0.8 is 0.979796. From 0.979796 subtract 0.800000, the remainder is 0.179796, the value of PQ or MN to radius unity, which multiplied by the given radius, 500 feet, gives 89.898 feet, the length of PQ or MN in feet, because these are equidistant from the extremities of the diameter, or from the centre C of the circle.

Now, the table being computed to every hundredth part of the radius, the perpendiculars PQ, MN, &c., may be computed at every 5 feet to a radius of 500, as in this example—a number of points quite sufficient in ordinary practice, which being set off from the chord F K, will enable the engineer to trace the arc F P M K with all the requisite accuracy.

In setting off road curves, and more especially railway curves, the radius should be as great as convenient, to prevent, as far as possible, the danger from centrifugal force tending to carry the engines off the rails. In the *Quarterly Journal of Agriculture* of 1830, the writer endeavoured, by a part of the following table, to point out the bad effects of rapid curves on common roads for the usual wheel carriages, which is here again introduced, but must be modified, as Pambour has attempted to do for railways, for reasons that will be immediately given. Let F be the centrifugal force, v the velocity of the moving body in feet in one second of time, r the radius of the circle or curve in the road, and g the force of gravity equal to 32.2 feet, then

$$F = \frac{v^2}{r} \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

by which the centrifugal force F is determined in feet. If the centrifugal force be required in terms of the weight, w, of the moving body, then

$$f = \frac{v^2}{gr} = \frac{v^2}{32.2r} \quad . \quad . \quad . \quad . \quad (2.)$$

which, multiplied by w in known terms will give the centrifugal force in the same terms, or denomination of weight. These formulæ are in general terms, in which any value may be given to r. In my first table, intended to be applied to common roads, for the usual wheel carriages, r was assumed equal to 100 feet. In railways it is more appropriate to assume $r = 1000$ feet. Its results may be, however, modified so as to suit other values of r. By taking r at $\frac{1}{n}$ the effect will be increased about n times, and by taking it n times, the effect will be $\frac{1}{n}$ only, that is, the effect is *inversely* as the radius of curvature, as the formulæ show. Whence the numbers in the table may be made applicable to various values of r within reasonable limits, and those nearly correct in our present views.

On common roads the correction must be made by the curvature and inclination of the road itself; but in railways, the correction may be, and generally is, subdivided between the inclination of the rails to the horizontal plane, and the conical shape of the *tires* of the wheels, in such proportion as the engineer may think advisable.

For this purpose, formula (2) requires a modification in the results derived from it depending upon the conical shape of the tires of the wheels, as generally now constructed, which is half an inch in three inches and a-half, with play of from half an inch to an inch of the flanges of the wheels on each rail.

Hence, by this play between the rails, the interior axis will fall while the exterior rises, which in part counteracts the effects of centrifugal force, as well as the sliding of one interior wheel by the greater velocity of the exterior, since they are both *fixed* on the same axle—and one end of the axis falls, while the other rises,

3.5 : 1 : : 0.5 : 0.14286 for a half inch of play.

Hence, for a play of one half inch between the rails, an elevation of 0.14 inch of the exterior wheel is obtained from the conical shape of the wheels, and the remainder must be derived by the elevation of the exterior rail, from the accompanying table. If the lateral motion of the wheels be limited to a quarter of an inch, the elevation from the tires will be 0.07 inch only—the rest must be obtained from the elevation of the exterior rail. This may be determined with tolerable accuracy.

Formula (2) gives the inclination of the road or rails to the horizontal plane, in order to counteract the effects of gravity, the numbers being, in fact, the natural cotangents of the inclination, when not multiplied by w, the weight of the moving body.

TABLE II.

Velocity in English miles per hour.	Velocity in feet in one second of time.	$F = \frac{v^2}{r}$ in feet, when $r = 1000$ feet.	$f = \frac{v^2}{32.2\,r} =$ nat. tang. of inclination when $r = 1000$ feet.	I = inclination of the road to the horizontal plane.		
				°	′	″
10	14.667	2.151	0.006681	0	22	58
15	22.000	4.840	0.015031	0	51	40
20	29.333	8.604	0.026722	1	31	50
25	36.667	13.444	0.041753	2	23	27
30	44.000	19.360	0.060124	3	26	26
40	58.667	34.418	0.106887	6	6	4
50	73.333	53.778	0.167012	9	28	53
60	88.000	77.440	0.240497	13	31	21
100	146.667	215.111	0.668047	33	44	41

USE OF THE TABLE.

Example 1.—Let the velocity, v, be 30 miles an hour, the radius of curvature 1000 feet, the play of the tires between the rails half an inch, the conical shape of the tires half an inch in three and a half, or one in seven, and the distance between the rails 4 feet 8½ inches; required the rise of the exterior rail above the interior, to counteract the effect of centrifugal force?

Opposite the velocity, 30 miles an hour, in column first, will be found 0.060124; in column fourth, then

0.060124 × 56½ inches, gives . . .	3.398 inches.
For half an inch play of the tires, subtract .	0.143 "
Rise of exterior rail above the interior, . .	3.255 "

Example 2.—Suppose the radius of curvature to be 2000 feet, in place of 1000, then the value of f must be half of its former amount, thus—

0.030062 × 56½ inches, ...	1.699 inches.
Deduct as before, ...	0.143 "
Rise of exterior tire above the interior, . .	1.556 "

Because, when the radius of curvature increases, the value of f decreases, though that derived from the bevelling of the tires remains about constant.

Example 3.—The same thing may be done by means of the fifth column, with an instrument for measuring angles accurately. It is clear also, from this column, that the radius should be great when the velocity is great, such as 50 or 60 miles an hour. Then the radius of the curve cannot, with propriety, be less than a mile or two. On the contrary, when the velocity is small, the radius may be reduced.

It may also be remarked, that the difference of the heights of the rails, as is obvious from columns fourth and fifth, depends upon their distance from one another.

Example 4.—Let the broad gauge of 7 feet be taken for 4 feet 8½ inches, in Example 1, then—

0.060124 × 84 inches, gives . . .	5.050 inches.
Subtract the effect of the conical tires, . .	0.147 "
Rise of exterior rail above the interior, . .	4.903 "

Other minute considerations might give a small correction to these, and they may be considered slightly too great; but this is an error on the safe side, as it is difficult to prevent the rails and wheels from wearing and altering their position slightly.

For adapting railway curves to one another, see Galbraith and Rutherford's edition of *Bonnycastle's Algebra*, 1848.

The effects of the force of the wind upon a railway train must be very considerable, even in calm weather. The enormous resistance of the atmosphere to the motion of shot and shell has been long known, from the experiments of Hutton and Gregory; while the same thing with regard to other bodies has been rendered familiar by the experiments of Smeaton, Rouse, and Borda. If i be the angle of incidence, s^2 the surface struck in feet, v the velocity of the wind per second in feet, then the force of the wind in *avoirdupois pounds* will be

$$f = 0.0024\, v^2 \sin^2 i.$$

The following table has been taken from data giving somewhat smaller results, the velocity being in miles per hour or feet per second:—

TABLE III.

Velocity of Wind.		Direct force in lbs. on 1 square foot.	Common designation of the winds.
Miles.	Feet.	lb. oz.	
1	1.47	0.005	Hardly perceptible.
2	2.93	0.020	
3	4.40	0.044	Perceptible.
4	5.87	0.079	
5	7.33	0.128	Gentle breeze.
10	14.67	0.492	
15	22.00	1.107	Brisk gale.
20	29.34	1.968	
25	36.67	3.075	Wind.
30	44.01	4.429	
35	51.34	6.027	High wind.
40	58.68	7.873	
45	66.01	9.963	Tempest.
50	73.35	12.300	Storm.
60	88.02	17.715	Violent storm.
80	117.36	31.490	Hurricane.
100	146.70	49.200	Violent hurricane.

From this it follows that the expense of steam-power, at high velocities, must in all cases be enormous, especially when opposed to the direction of the wind.

SECTION FIFTH.

COUNTY SURVEYING.

THE surveying of a large district or county is an operation so extensive and complicated, as to require the utmost exertion of the surveyor's abilities in every branch of his department; for in the exercise of it, he will find various difficulties to encounter: errors, if due care is not taken, will arise, and these will continue multiplying throughout the whole survey. The satisfaction, then, that the surveyor will have, both in the progress and termination of his work, must, it is evident, entirely depend upon the correctness, care, and caution he sets out with; and, added to this, the accuracy of his instruments. The choice and measurement of a base line, though in itself it appears a simple operation, is, however, when done with the nicety required, laborious and difficult, as a very small error (more particularly if the base is short) will affect the whole work.*

* The accuracy with which base lines have been measured in this country, is better known by those who are in possession of an invaluable work, entitled, "*The Account of a Grand Trigonometrical Survey of England and Wales,*" carried on by government under the direction of the Board of Ordnance. In this work is contained the result of a series of great triangles, extending over almost all the British isles. These triangles are again filled up by smaller ones. By this means the situations of all the principal objects are ascertained, including their latitudes, longitudes, bearings, and distances from the meridian, and perpendiculars to that meridian, with the heights of the several grand stations, and other remarkable hills, &c., besides scientific information concerning the measurement of the degree in this country. In a short time, then, a skeleton map, as it were, of the whole kingdom will be accomplished; however, in that part already completed, the surveyor will find bases already measured, and objects intersected, from which he can find no difficulty in surveying or filling up a district, however large. The public, besides, being possessed of such a valuable work, derive another advantage; maps are constructed on the basis of the trigonometrical survey, and regularly published at the Tower of London, by the same gentlemen under whose direction the various branches of such a laborious undertaking are performed. These maps are published in large districts, on a scale of one inch to a mile; and being constructed on trigonometrical principles, they have attained the greatest degree of perfection in respect to accuracy, besides their superior beauty of engraving. To a skilful surveyor this work may be termed an invaluable treasure. The general method of procedure will be given in a subsequent part of this work, under the title of "Trigonometrical Surveying and Levelling." (See Plate XXIX.)

A person who undertakes the survey of a county is apt to imagine, that to measure his base, and take his angles with expensive instruments, calculating his triangles by logarithms, by which means, laying down the objects by their distances instead of protracting them by their angles, &c., will take more time and expense than the advantage, if any, that may accrue from his performance will compensate for. It is perhaps more on this account than any other, that the proper method has seldom, till government took it in hand, been pursued—and of course the reason why most of our maps are so erroneous—and though some, in the course of various editions, get rid of the most glaring errors, the skeleton, or boundary work, often remains the same. However, in the few remarks (elucidated by example) I have to make, I will attend as much as possible to the advantage of the surveyor as to the accuracy of his work.

First, I would advise the surveyor to make himself well acquainted with the district or county he is about to survey. This may be effected by riding or walking over it in various directions with a person well informed of every particular part. By this means the names of the several towns, villages, seats, hills, and other remarkable objects, may be obtained, which will prove of the most essential service in the progress of the survey. Being possessed of this previous information, the surveyor will be enabled to choose a proper piece of ground whereon to measure a base; he will also be acquainted with the names of objects he sees at a distance from any of his stations. Having measured a base of a convenient length, bearings must be taken from its extremities, &c., all round, by this means intersecting every object of note with at least three or four intersections. Those objects being accurately determined with regard to situation, bearings must be taken from those that will be convenient for stations, to the former objects, and to as many more of note as may appear in view. The surveyor may find it convenient to measure the turnpike roads, &c., as he goes on; and, by this means, make use of his stations in the roads, to intersect objects also. This will be more clearly comprehended by an example.

Measure a base line, with your chain, of a convenient length, as accurately as you possibly can; but in order to have the utmost certainty of such accuracy, it will be proper to measure the base, at least two or three times over, in a contrary order. Having thus completed it, suppose its length 400 chains, or 5 miles, as in Plate XVI. Being provided with a good theodolite, which ought to be at least 6 or 7 inches diameter, divided to 20″ or 10″, to take the principal bearings with, (as for the road surveying, an instrument divided into every minute will answer all the purpose of one larger,) plant the theodolite at the eastern extremity of the base, and, having levelled it exactly, which must be carefully done at every station you intend taking principal bearings from, take a bearing to the windmill, and a circle of bearings all round; repeat the bearing to the windmill; should it answer to what it was formerly, it proves the instrument has not shifted.

The theodolite is then to be removed to the west end of the base line. To put the instrument in the same position it was at the east end, the surveyor must be very attentive to observe what was the bearing from the east to the west extremity, viz. 270°, from which must be subtracted 180°; there remains 90°; set the index on the limb over 90°; turn the head of the instrument round, till you see the mark left at the east end; when the mark and the cross hairs in the telescope is seen to coincide, screw the instrument fast; take first a bearing to the windmill; if it answers to 270°, it is right as to the line; then commence taking your observations. In the following page is the field-book, pointing out the bearings taken from both ends of the base. (See Plate XVI.)

OBSERVATIONS TAKEN FROM THE EAST END.

	° ′	
Windmill, . .	270.00	The base.
	278.00	Old Church.
	290.00	Billonhouse.
	293.30	Catcraig & Davieston.
	299.48	Haggs Castle.
	303.36	Pease Cairn.
	310.40	Usie Church.
	317.30	Broad Cairn.
	338.24	Tomkins.
	343.48	High Pen.
	353.06	Tippet Law.
	18.48	Huntershill.
	84.54	Wardlaw.
	90.00	Higham Ch.
High Pike, .	134.36	
Parkhouse, .	197.30	
Torshill, . .	238.40	
Windhill, . .	254.00	
Dunscairn, . .	255.30	
Red Church & John's Church.	265.30	

OBSERVATIONS TAKEN FROM THE WEST END.

	° ′	
Higham Ch.,	90.00	And mark at east end.
Wardlaw, .	81.36	
Huntershill,	66.40	
Tippetlaw, .	48.36	
Tomkins and High Pen in a line,	42.00	
Haggs Castle,	23.30	
Broad Cairn,	19.50	
Pease Cairn,	347.00	
Davieston, .	330.30	
Old Church,	315.00	
Windmill, .	270.00	
	94.00	Red Church
	97.50	High Pike.
	111.36	Parkhouse.
	117.48	Dunscairn.
	125.12	Torshill.
	189.48	Windhill.
	237.00	St John's Ch
	90.00	High. Ch.

[*Continued.*

TABLE *continued.*

DISTANCES ON THE BASE.	Chains.	
From the east end to a mark at *a*,	50	
Crosses a brook at,	63	
Crosses a road at,	110	Byre Loan.
Crosses a road at,	130	
Crosses a road near lower bridge,	170	
Crosses Till river,	200	
Crosses road at Longlee houses,	222	
Crosses road at Redhall houses,	271	
Crosses road at Todholes houses,	350	
West end of the base line,	400	Or five miles.

FIELD NOTES OF THE ROAD MEASURED ON PLATE XVI. FROM THE MARK AT *a* TO DAVIESTON.

	Bearings.		Distances.	
	° ′		Chains.	
	24.30	from *a* to *b*	29	Road goes off.
	301.30	do. *b* to *c*	45	
Road goes off,	337.30	do. *c* to *d*	38	Ussie Town.
	313.30	do. *d* to *e*	45	Ussie Village.
	272.00	do. *e* to *f*	50	
Road goes off,	255.30	do. *f* to *g*	28	
Hillhead,	279.00	do. *g* to *h*	54	
	311.30	do. *h* to *i*	62	
	279.00	do. *i* to *k*	38	
	265.24	do. *k* to *l*	44	
Upper bridge,	291.12	do. *l* to *m*	37	
Road goes off,	314.24	do. *m* to *n*	37	
	271.30	do. *n* to *o*	35	
Davieston,	248.48	do. *o* to *p*	32	Davieston.

FIELD NOTES OF THE MEASURE OF THE RIVER TILL, FROM No. 1 TO KING'S SEAT.

	Bearings.		Distances.	
	° ′		Chains.	
	17.00	from No. 1 to 2	61	
Opposite Drembank,	276.00	do. 2 to 3	63	
	242.30	do. 3 to 4	62	
Opposite Bridgend,	358.00	do. 4 to 5	61	Cockfield House at 36.
	248.48	do. 5 to 6	42	
	210.30	do. 6 to 7	57	
	334.30	do. 7 to 8	66	Broadholm at 38.
	64.48	do. 8 to 9	45	
	340.30	do. 9 to 10	49	Tillbank at 24.
	51.00	do. 10 to 11	68	

Use the same method in protracting the above observations as is particularly described in page 94, farm of Tipperty, and Bonnyton, page 97—with this difference, that those farms were laid down with a semicircular protractor divided into 180 degrees, and the above must be protracted with a circular one divided into 360 degrees, or twice 180 degrees, according to the division of your theodolite. Should the surveyor use a semicircular, in this case he must subtract 180 from the amount, and the remainder will be the bearing. For my own part, I would prefer taking the bearings with an instrument divided into twice 180 degrees, as it saves the trouble of altering the index and the limb on the theodolite, when it is removed from one station to another. Besides, two or three verniers can be used on the limb, and a mean taken of their readings. This is useful when the triangles are to be calculated by logarithms, as greater accuracy is thereby attained.*

To protract the observations, use the same scale by which you have laid down the bearings on the base line, suppose one inch to a mile. Should you not use a protractor of your own making, prick off all the bearings from the east and west end of the base, as described in p. 94, (Farm of Tipperty,) and draw them all in with a black lead pencil; the point of intersection marks the distance to the stations where the respective bearings are taken; insert the name of the church, house, or hill, or whatever object you take your bearings to; let the same be done with all the other intersections, which will not only ascertain the horizontal distance from each end of the base, but the distance from one place to another. The next thing to be done is to go to one of the intersected objects, suppose the broad cairn, and there erect the theodolite; look into the field-book for the bearing to the broad cairn, which was 317° 30′ from the east end of the base; subtract 180° from it, and there remains 137° 30′; move the index to that degree and minute on the limb, and turn the head of the instrument round till you see through the telescope the conspicuous mark built at the east end of the base line where you began; then screw the head of the instrument fast to the legs, and loosen the screw a little that holds the telescope and arc fast to the limb; then turn the telescope, and take a range of bearings from the broad cairn, the same as was done from the east and west ends of the base. If you take bearings from it to all the places you know you had bearings to before, when they are protracted, the three lines will meet in a point, which is a proof that your first intersection to those places

* It has already been observed by the editor, page 91, that the division of the arc of the theodolite into 360° is preferable, in extensive geodetical surveys, to one of twice 180°, because errors and uncertainties are thereby avoided.

are certain. For example, the bearing to Hunter's Hill is 108°, that to Tippetlaw 89° 30′, and to Pease Cairn 266° 30′. After protracting those bearings from the broad cairn, should they meet in a point with the former observations, you are certain thus far you are correct. You may then go to Hunter's Hill, or any of the other intersected places where three lines meet in a point, taking another range of bearings all round, particularly to the north and east, and make as many intersections as you can; then go to the Pease Cairn, and take another range of bearings to the north and west. After having subtracted 180° from 266° 30′, the remainder is 86° 30′; set the index to 86° 30′ on the limb, and turn the head of the instrument round with both hands till you see the cross hairs in the telescope to coincide with Hunter's Hill. Go from hill to hill, or any other convenient intersected place, where three or four bearings meet in a point, till you have got intersections to all the principal places in the county. By using a protractor of your own making, you will very much facilitate the plotting, as every bearing is protracted from the centre. The work may be shortened in a great measure by using the T square and its companion, as particularly described in p. 82, by laying the chamfered edge upon the centre, and the degrees and minutes of the bearings you are laying off, and sliding the T parallel, by the help of its companion, to that part of the plan you are laying down the bearing from. In using those protractors made by land-surveyors, the figures denoting the degrees are only supposed to be inserted with a black-lead pencil, which is easily rubbed out when the bearings and distances are all laid off. See Plate III.

From what has been said, it is hoped a surveyor will find very little difficulty in laying down the intersections of an inland county.

I have set down on the plan the base line, which is dotted from one end to the other, and marked where roads were crossed, and also the river Till, and such houses, with their names, as are near the base. After having protracted all the bearings, make marks thus ⊙ where the black-lead lines intersect one another; which may be drawn in if you think proper. This I have avoided, in order that the diagram may appear less complicated. A road is also measured and protracted from a mark left on the base line at *a* to Davieston, and also the river Till, from another mark where the Till was crossed to the village of Kingseat. I have merely inserted the road and part of the river, to give the surveyor of a county a notion of the labour he may expect to meet with. Some surveyors prefer the plain table in filling up the vacant spaces between the great intersections, as they can draw in upon the table all the little angles and distances upon the spot, which indeed saves much time in plotting, as it is very tedious to lay down the bearings and distances with a protractor and scale.

In surveying the roads, the best method that a surveyor can adopt is to measure to one or more fixed points, as the road, when plotted with those points at its extremities, can be extended to the respective points from the same scale on the skeleton work of the plan; it it agree, the stations in the road ought to be pricked through, and the sides of the roads formed, leaving the offsets, &c. to be laid down on the fair plan. This is the most regular method a surveyor can adopt in any survey of a large extent.

SECTION SIXTH.

MILITARY SURVEYING.

As the art of arranging armies in order of battle, regulating their movements in such a manner as may be deemed most proper for attacking, defending, or retreating with the greatest possible advantage, must depend in a great measure upon a perfect knowledge of that part of the country where those movements are to take place, surveying and sketching ought to be made one of the most essential parts of a military education, as it is often necessary to sketch the ground in the neighbourhood of an encampment. I have inserted a military sketch, *Plate* XIX. out of the reach of an enemy. The ground may be regularly measured, and a map made of the same; when otherwise situated, it is sometimes paced.

Although this work does not require so much care and attention as other surveys where the content and area is required, a few lines may be measured, and a number of bearings taken, and intersections made of particular points, with any instrument for taking angles, and the rest of it sketched in by riding or pacing the ground. The plain table is an excellent instrument for a work of this kind, particularly as everything can be sketched in upon it on the spot in better proportion than by the eye. It cannot be expected, that a person belonging to the army can take theodolites or plain tables along with him to the field of battle; but a box sextant, as represented on page 71, being of small compass, can be easily carried, and will take an angle with great exactness.* There is one thing very necessary that an officer in the army should be well acquainted with: he may know how to lay off squares, parallelograms, &c., very well on paper, but may be at a loss to do so in the field. The sextant, if the index is put to 90°, reflects a perpendicular; the optical square does the same. We shall now

* The prismatic compass, represented on page 85, will also be found very useful in making military sketches.

suppose you begin at A, and place a pole there; then go to any other part of the ground, suppose B, and measure the length from A to B; stand at B, and look through the sextant, the index being at 90°, and cause one of your men to go in as straight a line as possible; if you cannot see him at first, cause him to move to the right or the left till you see the pole at A and the man's pole to coincide, which will be exactly perpendicular from B to C; measure the distance to C, and then go to A; then measure the same distance to D as is from B to C, which forms a rectangular long square for the encampment. With the sextant take an angle from A to Hill Pike, and another to a cairn on the hill-top nearest you; then go to B, and take an angle to Hill Pike, and another angle to the cairn that you observed from A; next go to C, and take an angle by reflecting the pole D to the park of artillery; then take an angle by reflecting the park of artillery to the fort. The angles are represented on the sketch by dotted lines, merely to show where they were taken from. Again, take an angle by reflecting the park of artillery to the Jew's farm. The best method of laying those angles off upon the sketch is with a protractor or line of cords (both of which are contained in a case of mathematical instruments,) and where the intersections meet is the distance. The other parts of the plan may be all done by pacing, and other parts sketched in with the eye, or, if necessary, assisted by the prismatic compass.

The following table, for reducing the common pace of $2\frac{1}{2}$ feet into feet, will be found convenient in laying off the distances upon the sketch.

Paces.	Ft. In.	Paces.	Feet.	Paces.	Feet.	Paces.	Feet.	Paces.	Feet.
1	2.6	30	75	230	575	430	1075	630	1575
2	5.0	40	100	240	600	440	1100	640	1600
3	7.6	50	125	250	625	450	1125	650	1625
4	10.0	60	150	260	650	460	1150	660	1650
5	12.6	70	175	270	675	470	1175	670	1675
6	15.0	80	200	280	700	480	1200	680	1700
7	17.6	90	225	290	725	490	1225	690	1725
8	20.0	100	250	300	750	500	1250	700	1750
9	22.6	110	275	310	775	510	1275	800	2000
10	25.0	120	300	320	800	520	1300	900	2250
11	27.6	130	325	330	825	530	1325	1000	2500
12	30.0	140	350	340	850	540	1350	2000	5000
13	32.6	150	375	350	875	550	1375	3000	7500
14	35.0	160	400	360	900	560	1400	4000	10000
15	37.6	170	425	370	925	570	1425	5000	12500
16	40.0	180	450	380	950	580	1450	6000	15000
17	42.6	190	475	390	975	590	1475	7000	15500
18	45.0	200	500	400	1000	600	1500	8000	20000
19	47.6	210	525	410	1025	610	1525	9000	20500
20	50.0	220	550	420	1050	620	1550	10000	25000

The sketch is frequently drawn in with pen and ink in a rough manner, but so as everything can be easily read, and all the different characters perfectly understood. Forts and fortified towns are represented as on the sketch, the outline of which is generally made bold, and the buildings shaded with red, or dark with Indian ink, according to fancy. Batteries are of various kinds—viz., batteries with embrasures, batteries *en barbette*, masked batteries, where the cannon are placed behind hedges, &c., mortar batteries, and open batteries. The embrasures are represented showing the breast work, which is shaded; and the place left white is where the cannon are placed. In batteries *en barbette* the guns or howitzers fire over a breastwork; mortar batteries have a breastwork, and the number of guns they contain is expressed by small circles behind the breastwork; an open battery has no breastwork, and is represented on the sketch with one stroke and two shorter ones on each side, to represent a cannon: an *abattis* is represented by a quantity of trees laid down and securely fixed before cannon, with their branches outwards, to keep off an enemy; pallisades are represented by vertical strokes; fraises are represented by small strokes laid horizontally before a battery; a *chevaux de frize* is represented by a line drawn across a river with oblique crosses. Troops of infantry are represented in small parallelograms crossed by a diagonal, the one half shaded dark and the other left light; the light part is sometimes coloured according to the colour or uniform of the regiment. Cavalry troops are likewise represented by parallelograms, but, for the sake of distinction, are commonly made broader than infantry troops. Encamped troops are represented the same way; but in place of erecting the standards upon the front line, they are placed at a little distance before it. Park of artillery is described by a square, crossed by two diagonals, one half shaded and the other half left white; its front is shown by a strong line. The evolutions of troops are represented by dotted lines and arrows, representing the way they are moving and marching.

From what has been said with regard to making out a military sketch, an inspection of the plate will greatly assist the surveyor in becoming acquainted with the characters. The plate, containing "Signs and Illustrations of Modern Fortification," will also be instructive on this subject.

MILITARY PLANS, &c.

SIDES AND ANGLES OF A REGULAR FORTIFICATION.

PENTAGON.

	Fathoms.
1. S = Exterior side, .	180.00
2. R = Radius of exterior side,	153.12
3. *s* = Interior side, . .	122.78
4. *r* = Radius of interior side,	104.44
5. K = Capital of bastion, .	48.68
6. N = $\frac{1}{6}$ S = normal or perp^r.,	30.00
7. C = Curtain, . . .	56.29
8. F = Flank, . . .	21.00
9. B = Face of bastion = $\frac{1}{3}$ S =	60.00
10. D = Line of defence, .	124.54
11. G = Demigorge of bastion,	33.25
12. *d* = Main ditch, . .	13.55

ANGLES.

	°	′	″
13. Angle of the Centre,	72	0	0
14. „ Polygon,	108	0	0
15. „ Curtain,	103	35	58
16. „ Shoulder,	122	2	5
17. Angle of Bastion, .	71	7	46
18. Diminished Angle, .	18	26	7
19. Exterior Flanking Angle,	143	7	46

RAVELIN.

	Fathoms.
20. Face in = B = $\frac{1}{3}$ S, .	60.00
21. Capital, . . .	56.80
22. Demigorge, . . .	39.00
23. Ditch, ...	10.32

ANGLES.

	°	′	″
24. Angle of Ravelin, .	77	46	52
25. Angle of Shoulder, .	66	7	16
26. Angle at Gorge, .	149	58	36

HEXAGON.

	Fathoms.
1. S = Exterior side, . .	180.00
2. R = Radius of exterior side,	180.00
3. *s* = Interior side, . .	134.54
4. *r* = Radius of interior side,	134.54
5. K = Capital of bastion, .	45.47
6. N = $\frac{1}{6}$ S = normal or perp^r.,	30.00
7. C = Curtain, . .	56.29
8. F = Flank, . . .	21.00
9. B = Face of bastion = $\frac{1}{3}$ S =	60.00
10. D = Line of defence, .	124.54
11. G = Demigorge of bastion,	39.13
12. *d* = Main ditch, . .	13.55

ANGLES.

	°	′	″
13. Angle of the Centre,	60	0	0
14. „ Polygon,	120	0	0
15. „ Curtain,	103	35	58
16. „ Shoulder,	122	2	5
17. Angle of Bastion, .	83	17	46
18. Diminished Angle, .	18	26	7
19. Exterior Flanking Angle,	143	7	46

RAVELIN.

	Fathoms.
20. Face = B = $\frac{1}{3}$ S, . .	60.00
21. Capital, . . .	56.80
22. Demigorge, . . .	39.00
23. Ditch, ...	10.32

ANGLES.

	°	′	″
24. Angle of Ravelin, .	77	46	52
25. Angle of Shoulder, .	66	7	16
26. Angle at Gorge, .	149	58	36

Note.—In many examples of modern fortification the face of the ravelin is about $\frac{4}{9}$ S = 80, instead of 60 fathoms, which renders the ravelin very large, thus extending its faces so as to cut when produced the face of the bastion considerably distant from the shoulder, such as about 3 or 4 fathoms. It would be as well to strengthen the fortification by additional works.

To these may be added tenailles in the main ditch, with three faces, as in the modern system, or with a convex semicircular middle division, casemated, if thought desirable, and mounted with heavy guns to flank the main ditch, and a caponnière reaching to within ten or twelve feet of the gorge of the ravelin. A traverse may also be placed across the ditch of the ravelin near the shoulder, with a passage or crotchet cut out of the counterscarp, at the entrant places of arms, to defend it either by great or small arms.

Redoubts, circular or otherwise, may be placed in the bastions and ravelins, and similar redoubts in the places of arms, the arcs extending from the counterscarp of the main ditch to that of the ravelin. Detached redoubts, or advanced lunettes, may also be placed at a short distance from the extremities of the salient angles of the glacis opposite the ravelins, or counterguards of the bastions accessible by covered ways from the works.

In square redoubts or forts the perpendicular is generally taken at *one-eighth* of the side—that is, much less than in large forts of numerous sides—in order to afford internal accommodation.

The covered way surrounds the whole works. It is generally 5 fathoms wide, with traverses from the counterscarp to the glacis, in the shape of parapets with palisades, opposite and parallel to the faces of the bastions and ravelins at the salient places of arms, and also perpendicular to the counterscarp at the *entrant* places of arms at least, with one or more intermediate, if thought necessary. There are passages cut in the glacis round the exterior ends of the traverses called crotchets, or sometimes *en cremaillère*. The *salient* places of arms are formed by the circular part of the counterscarp, and by the prolongation of the adjacent branches of the covered way. The *entrant* places of arms have demigorges along each adjacent counterscarp of about 25 fathoms, with faces of 30 fathoms when they have a redoubt within them. These redoubts may advantageously be made circular, with a radius of 20 fathoms from the counterscarp to counterscarp. The cavaliers in the bastions might also occasionally be complete circles, in order to be fully enclosed, and have a commanding fire in all directions. Coupures are made from the cavalier up to the scarp of the flank, but not through it, so that in a siege the breach in the face of the bastion may be easily cut off. The parapets of all the salient angles might be made *internally* circular, to permit of a direct fire in their front.

The crest of the glacis rises about 8 feet above the covered way, to afford protection to the troops assembled in it. There is close to the glacis a banquette, rising to within 4½ feet of its crest, to enable the troops to fire along its inclined plane, extending towards the country for about 60 or 80 fathoms, thus falling at the rate of nearly 1 in 50, and effectually prevents the scarps of the works from being seen by an enemy. These remarks are generally applicable to fortifications, though there are considerable variations in the works of different engineers, according to their different opinions in various circumstances, of which they must endeavour to form a just and competent opinion.

Plate XX. gives a general view of military signs, and the common expressions used in fortifications and artillery.

The first given is composed of bastions and ravelins, or, as they are sometimes called, demilunes. The lunettes, however, are wanting, either as attached to the bastions or ravelins, or in advance of the works. The traverses are also wanting in the covered way as well as the sally ports, with perhaps one exception, in a bad position, while the rampart and *terre-pleine* of the rampart are reversed. There is perhaps enough, however, to give the general reader a tolerably correct view of the principles of fortification.

In the preceding tables the shoulder of the ravelin is determined by producing the flank to the counterscarp of the main ditch, rounding off the revêtement in a semicircle, forming a small orillon, and thus protecting the guns in the flank completely.

Plate XXI. illustrates the siege of San Sebastian, and exemplifies the mode of placing batteries in the attack of places, as well as the saps and lines of approach, while its execution will afford a good specimen of military shading and finishing such plans.

Finally, Plate XXII. will furnish the young engineer with an example of the method of giving plans of battle-fields. It exhibits the arrangements of one of the greatest of our captains in overthrowing his talented rival, and may be advantageously considered by our military engineers in the discharge of that portion of their duty.

MARINE SURVEYING.

1. Marine surveying is the art of delineating coasts, bays, and harbours. It requires a knowledge of the methods generally employed in determining the latitudes and longitudes of essential and important stations, the variation of the compass, and the depths of the channels at low-water in spring tides, together with the times of high-water at new and full moon, generally called the *establishment*, and the rise of the tides at syzygies and quadratures. The relative positions of the most important points within the limits of the survey are first accurately fixed trigonometrically,

from an extensive base carefully measured, and having a known inclination to the meridian. At the extremities of this base numerous angles are observed, with a good theodolite or circle, intersecting all the more essential positions and most remarkable points, which, when necessary, must be rendered conspicuous by piles constructed, or staves erected expressly for this purpose. The intermediate parts are next delineated by the aid of the surveying compass and chain, and the more minute and less important features are usually sketched by the eye, and afterwards transferred to their proper places on the chart.

2. The chart most generally employed for this purpose is that of Mercator, especially if the extent of the survey be somewhat considerable. When limited to a small portion of a coast, or to a single bay or harbour, the plane chart is frequently used. The first is constructed from a table of meridian parts, contained in all books on navigation. Those of Garrard and Mendoza Rios are the most extensive and accurate, being carried to two places of decimals. These meridian parts may also be computed by the formula given in another part of this work to a given ellipticity of $\frac{1}{300}$ as the chart will then be more in accordance with the real figure of the earth. The second may be constructed with sufficient accuracy by diminishing the distances between the meridians in the ratio of the cosines of the respective latitudes, or more accurately by the formula involving the ellipticity, in a different part of this work on Trigonometrical Surveying.

3. The soundings are obtained by means of a boat. A steamboat, when it can be employed, would be the most convenient and efficient, which must proceed in different directions, so as to traverse completely the whole of the space within the limits of the survey. It would contribute to accuracy, especially where there are dangers from sunken rocks, shoals, and sand-banks, to intersect the boat's position, intimation for that purpose being given by signal from the extremities of the measured base, or from any other two points well determined trigonometrically, and depending upon that base. In the course of the survey, many of the directions in different parts of this work must be kept in view, and acted upon according to circumstances.

4. The preceding observations relate to the more accurate kinds of marine surveying, in which any confidence can be placed in navigating vessels with safety. There are, however, other more easy, though less accurate methods of sketching a rough plan of

in coasts and harbours, which may sometimes be useful when the other operations cannot be practised. In this case, a base may be marked out by two buoys, and the distance measured by the run of a vessel between them ascertained by the log, and the direction by the compass. Two surveying vessels may also form the extremities of the base, and their distance may be ascertained, approximately, by the time which sound takes to move from the one to the other, by the discharge of guns in each. If the time between seeing the flash and hearing the report be carefully noted in seconds, or the beats counted by a good chronometer, the distance may be obtained, nearly, by allowing 1130 feet for one second of time; and the angles may be measured by the sextant, azimuth compass, or other nautical instruments.

Sketches may also be taken when coasting along shore, especially in reference to new countries, when no better method can be obtained. The survey of a newly-discovered island may be accomplished by circumnavigating it with a boat, and recording the bearings and distances all round.

From these, the plan and area may be ascertained, by some of the methods explained in this work.

See Plates XXIII., XXIV., and XXV.

MARINE CHARTS.

1. Seamen often prefer a graphical construction to determine the courses and distances sailed during a voyage; and though the conclusions are less precise than those by calculation, they are, however, more easy, and sufficiently accurate for general practice, especially as the exact position of a ship can be determined by celestial observations alone.

It is therefore necessary to construct charts on which these problems may be performed with facility, and comparative accuracy; while the adjacent shores, rocks, and other dangers, may be also readily seen and avoided; and, at the same time, the desired position of the ship may be secured. The most convenient chart for these purposes is that of Mercator. In this chart the equator and its parallels in latitude are represented by parallel straight lines, at *unequal* distances, arranged according to a given law. The meridians are also represented by straight lines, perpendicular to them, and *equidistant* from each other, for equal degrees of longitude, or degrees of the equator. In this way the chart is projected in a series of rectangles, each having the same or an equal base, but whose height increases with the latitude in such a manner that the ratio between a degree of latitude and a degree of longitude, at any latitude, is the same as on the globe itself. Hence the *bearing* of one place from another, on this chart, is also the same as on the globe itself, and this constitutes its most valuable property to the practical navigator.

2. This chart is constructed in the following manner:—*First*, let the extent of the chart in latitude and longitude be determined; and *secondly*, consider what scale would be required to keep the whole within proper limits. For example, let it be required to determine the outlines of a chart for the island of Great Britain. A little consideration will show that such a chart will require about ten degrees of latitude, and eight degrees of longitude, and hence the size of the scale for the given limits of the chart can be readily chosen. See PLATE XXVI.

Now, draw a horizontal line for a parallel of latitude passing through the lower latitude of 50° N., on which lay off 8° of longitude from the scale of equal parts previously selected. Or the scale may be drawn a little below, and parallel to, the above mentioned horizontal line, and properly divided, so as to enable the constructor to take off as minute divisions as may be required. Through each point of division, or as many of them as may be thought necessary, draw perpendiculars to represent the meridians of the chart. The circles on the globe, parallel to the equator, are represented by perpendiculars to these, but having intervals proportional to the differences of the *meridional parts* corresponding to each latitude. These numbers being taken from a table of meridional parts,* or computed from the formula given in another part of this work, either for a sphere or spheroid, their successive differences will be the intervals between the parallels, in taking for a scale the space of one degree of longitude divided into sixty equal parts. This is the common practice, which may, however, be perhaps modified occasionally. Thus, from calculation to a spheroid of $\frac{1}{305}$ of compression, we have to—

* A table of meridional parts is given to every degree of latitude, on both the sphere and spheroid, with differences for interpolation, in a succeeding part of this work.

Lat.	Mer. Parts.	Suc. Diffs.	Suc. Sums.	In Deg.
° 50	′ 3456.91	′ 94.09	′ 94.09 =	° ′ 1 34.09
51	3551.00	96.13	190.22 =	3 10.22
52	3647.13	98.33	288.55 =	4 48.55
53	3745.46	100.64	389.19 =	6 29.19
54	3846.10	103.10	492.29 =	8 12.29
55	3949.20	105.70	597.99 =	9 57.99
56	4054.90	108.50	706.49 =	11 46.49
57	4163.40	111.46	817.95 =	13 37.95
58	4274.86	114.63	932.58 =	15 32.58
59	4389.49	118.03	1050.61 =	17 30.61
60	4507.52			
		1050.61		

Now, after having divided the horizontal line, or lower parallel into equal parts or degrees of longitude, and each degree into 60 equal parts to represent minutes, this scale will be that on which we must take 94′.09 or 1° 34′.09. This being set off from 50° N., the first parallel assumed in this chart, towards the north, will give that of 51° N. In like manner, 3° 10′.22 being set off again from 50° as before, will give the parallel of 52°, and this process continued will give the position of all the parallels within the designed chart, to be ultimately drawn in where considered necessary. This method of proceeding is considered preferable to setting first off 1° 34′.09 to get 51°, and then from 51° to set off 1° 36′.13 to get 52°, &c., because any error committed in finding the first point is naturally transferred to the next in succession; whereas, by our method, the position of each point is affected by its own individual error only.

The degrees of longitude being all equal to one of the divisions of the scale, the position of the parallels of longitude, or rather meridians, will be found by setting off, on the top and bottom of the chart, as many of these as will extend to the extremities of the chart; on one or more convenient places of the chart, are put compasses corresponding to both the true and magnetic meridians, by means of which, the bearing of one place from another may be readily found, by means of a parallel ruler or T square, and its companion, as has been already shown in the articles on plotting. The distance between any two places may also be easily found by extending the feet of a pair of compasses from one point or place to another on the chart, and this extent applied to a scale of latitude on the side with one foot, as much below the smaller latitude as the other is above the greater, the intercepted number of degrees and minutes, converted to minutes, will give the distance in geographical or nautical miles, of 6086 feet.

The variation of the compass at the different points within the chart must be determined astronomically, and inserted in the proper position; the times of high water at new and full moon, and the rise of the tides at the springs and neaps, where they are well known, or have been determined. These, together with the principal points determined astronomically and geodetically, having been all inserted, the constructor will proceed to finish his chart, by filling in the coasts and contours of the whole from actual survey, as explained in different parts of this work. The accompanying chart will serve as an example; but the Ordnance maps, the Admiralty charts, both executed in this country and in France, may be consulted with advantage.

CONICAL PROJECTION OF MAPS.

The conical projection is one of the most useful and accurate for maps of a kingdom of moderate extent, such as that of the British Isles. It receives its name from the supposition that the terrestrial globe is enveloped by a cone tangent to the circle of the mean parallel of latitude between the northern and southern extremities, sensibly coinciding with this parallel and those near it on each side, as exemplified by S A B, the cone touching the globe E P Q p, in fig. 1. This cone is developed on the plane of the map, in the form of a circular section, fig. 2, in which the *meridians* converge in straight lines to the summit of the cone as the centre of the sector. The parallels of latitude are arcs of circles, of which the vertex of the cone is their common centre, and are equally distant for equal degrees of latitude.

In fig. 1, let E Q be the equator, P the north pole, p the south pole, and E P Q p a meridian of the globe. Then let E A be the distance from the equator E Q to the parallel A a B $= l$, the latitude of the mean parallel, $\frac{1}{2}$ A B $=$ A c $= \cos l$ when the radius A C $=$ 1. Let D be the number of degrees of longitude contained in the map, and let S designate, in degrees, the angle A S B at the vertex of the developed section in fig. 2. Then will A B equal the development of D degrees of the circumference of the circle of which A c, in fig. 1, is the radius. Now, as is generally done, put $\pi = 3.141593$ the circumference of a circle to diameter

$= 1$, or the semicircumference to radius $= 1$. If the length of an

Fig. 1.

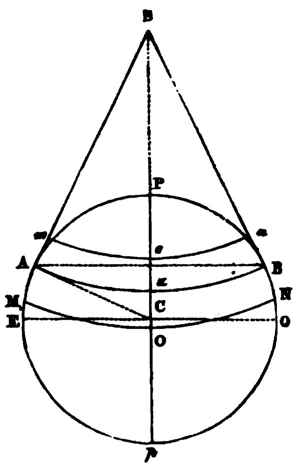
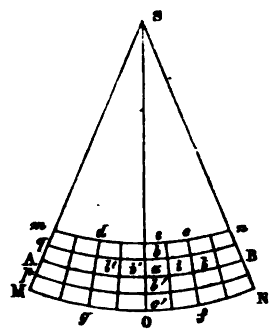


Fig. 2.



arc of a circle, equal to the radius in degrees be denoted by R°, &c., then

$$R^{\circ} = \frac{180^{\circ}}{\pi} = 57^{\circ}.2957795 \quad \log. = 1.7581226$$

$$R' = \frac{10800'}{\pi} = 3437'.74677 \quad \log. = 3.5362739$$

$$R'' = \frac{648000''}{\pi} = 206264''.80625 \quad \log. = 5.3144251$$

numbers of great use in various calculations, as is frequently shown in the course of this work.

From fig. 1 may be readily derived—

$180^{\circ} : \pi \times A\,c : : D^{\circ} : A\,B$ the length of the required arc,

$$\text{Or } 180^{\circ} : \pi \cos l : : D^{\circ} : \frac{\pi\, D \cos l}{180^{\circ}} = \frac{\pi}{180^{\circ}} \times D \cos l = A\,B \quad . \qquad (1.)$$

the developed length on the mean arc of the sector.

But in the triangle S A C, fig. 1, $S\,A = \tan A\,C\,S = \cot l$,

$$\text{therefore } 180^{\circ} : \pi \cot l : : S : \frac{\pi}{180} \times S \cot l = A\,B \quad . \quad . \qquad (2.)$$

Equating these two values, we have, after striking out the common factor, $\frac{\pi}{180}$.

$$D \cos l = S \cot l$$

Therefore $S = D \frac{\cos l}{\cot l} = D \cos l \times \frac{\sin l}{\cos l} = D \sin l$ . . (3.)

Thus we form an angle M S N, fig. 2, of as many degrees as there are of longitude required in the map, and we shall then have the angle S of the developed segment. Next S A must be taken equal to cot l, which will give the radius of the arc representing the mean parallel. Afterwards there must be set off in a straight line from A to M and m, the developed lengths of the meridian E A P, between the limits of the extreme latitudes. If the map should contain d degrees of latitude, Mm will be

$\frac{\pi d}{180°}$, and the length A M $=$ A$m = \frac{\pi d}{360°} = \frac{d}{2 R°} = \frac{1}{2} d$ arc $1° = 30' d$, in minutes. . . . . (4.)

1° being its assumed length in latitude on the scale selected for the map.

Now dividing A M and Am into as many parts in p and q as may be required, we describe through these points of division from the centre S arcs forming the parallels of latitude, while the meridians are straight lines drawn from the centre S, and passing through the equal divisions on the arc A B.

For countries near the equator, however, the centre S becomes too distant to be practicable. In this case Flamsteed's projection may be employed, in which the central meridian is first drawn in a straight line. This meridian is next divided into degrees or less divisions, through which, perpendicularly, the parallels of latitude are drawn. On these parallels, divisions conformable to the convergence of the meridians, according to their respective latitude, are inserted. These operations may be very simply effected by the aid of Table XXV. of the general tables.

If the numbers in the column titled "Minute of Longitude" be divided by 100, or if the decimal point be moved two places to the left, the result will be a degree of longitude in geographical miles nearly.

For more precision, the line S A, in figure 1, is sometimes made to cut the meridian Mm in the middle of Am and A M, or at one fourth of the meridian distance from either of its extremes. This will give—

$$A S = \frac{\cos l}{\sin \frac{1}{2} (l + l')} \quad . \quad . \quad . \quad . \quad (6.)$$

in which l and l' are the latitudes of the two points where S A cuts the meridian.

To apply these we shall select the British Isles, which extend from latitude 50° to 60° N., and therefore the middle latitude, or that of the mean parallel, A B, is 55° N. Also, including Ireland, they extend in longitude from 2° E. to 10° W., and therefore include 10° of latitude and 12° of longitude, that is, the arc M*m* equals 10°, and A B 12°.

Hence, as formerly shown, A S = A C cot l to radius A C = 1. But the length of an arc equal to the radius is 57°.2957795, therefore A S = 57°.2957795 cot l = R° cot l.

Now, as already shown, log R°	. . . .	1.7581226
	l = 55° log cot . .	9.8452268
	A S = 40°.119 log . .	1.6033494

Hence, also, A S + ½ d = 40°.119 ± 5° = 45°.119 and 35°.119, the lengths of S M and S*m* respectively, therefore the radii of the mean and two extreme parallels are determined.

By formula (3) the angle A S B in figure second must also be found, that is, S = D sin l.

Now D = 12° of longitude l = 55° N. the mean parallel.

Hence l = 55° log sine	. . . . .	9.9133645
D = 12° log	. . . . .	1.0791812
S = 9°.8298 log	. . . . .	0.9925457

These values of A S and S have been here computed in degrees, but they may be easily converted into minutes and seconds by multiplying successively by 60, or by using in the computation log R′ or log R″ respectively.

To complete the construction, draw two straight lines, S A, S B, figure 2, making an angle of 9°.8298, or 9° 50′, with each other; then with S as a centre, and 40°.119 or 40° 7′ as radius, describe the arc A B, and divide it into twelve equal parts, each of these will be 1 degree of longitude. In like manner the other arcs M N, *m n*, &c., may be described. Divide M A, A*m*, each into five equal parts, each of these will be 1 degree of latitude, and it is obvious the subdivision in both cases may be carried to minutes, or even seconds, if required.

The lengths of the cords joining the extremities of these parallels of latitudes may also be computed, if thought desirable in peculiar cases, by the formulæ

$$k = 2 \, sine \, \tfrac{1}{2} \, S, \qquad . \quad . \quad . \quad . \quad (7)$$

$$\text{or,} \quad k = a - \frac{a^3}{24 \, r^2} \qquad . \quad . \quad . \quad . \quad (8)$$

in which k is the cord or straight line, A B, fig. 1, a the length of the arc A a B, and r the radius S A. Hence such maps may be readily constructed with ease and accuracy. It was in this manner that the outline map exhibiting part of the triangulation of Scotland was constructed, and any portion of the whole may be readily selected or cut out, as the part $d\,e\,f\,g$, fig. 2, when, as in this case, it becomes necessary to retain a section only.

There are several other projections, besides the above, employed in forming the map of a country, particularly a modification of Flamsteed's, adopted for the national maps of France, whose principles are fully discussed in the *Topographie* of Puissant and the *Geodesie* of Francœur, which may be advantageously consulted. The tables of Plessis very readily compute the magnitude and position of each rectangular portion, but our limits will not permit of entering upon its discussion here.

TRIGONOMETRICAL SURVEY OF A BAY.

In order to fill in properly the coasts in marine surveys, the following example has been given, in which the base was measured with the chain, and the various angles measured with a theodolite or sextant, or a combination of both, assisted by the prismatic compass occasionally in sketching the contours. The whole is completed and delineated on Plate XXV., which will enable students of surveying to follow out all the details successfully.

Observations.	° ′ ″		° ′ ″	Sums. ° ′ ″	180° — Sums. ° ′ ″	
ABC =	125 30 10	BAC =	18 45 15	144 15 25	35 44 35	= ACB
ABD =	101 45 30	BAD =	36 50 30	138 36 0	41 24 0	= ADB
ABE =	109 10 25	BAE =	48 5 40	157 16 5	22 43 55	= AEB
ABF =	81 20 35	BAF =	68 3 20	149 23 55	30 36 5	= AFB
ABG =	69 30 40	BAG =	83 57 36	153 28 16	26 31 44	= AGB
ABH =	50 42 15	BAH =	102 30 44	153 12 59	26 47 1	= AHB
ABI =	39 55 25	BAI =	87 53 18	127 48 43	52 11 17	= AIB
ABK =	12 31 50	BAK =	148 1 36	160 33 26	19 26 34	= AKB

The base A B was carefully measured with a hundred feet chain, and found to be 6524.5 feet, bearing 15° 36′ 20″ N.W. true. The variation of the compass was 28° 35′ W. in 1836.

Latitude of B, 55° 31′ 56″ N. Longitude 5° 7′ 30″ W.

Results.

	° ′ ″		
Now sin ACB	35 44 35	cosec	0.233475
Is to sin CAB	18 45 15	...	9.507192
So is AB	6524.5 feet		3.814547
To BC	3591.0 log.		3.555214

In like manner,	AC = 9092.7 feet.
AD = 9659.0	BD = 5915.7
AE = 15947.7	BE = 12566.2
AF = 12670.6	BF = 11887.9
AG = 13683.6	BG = 14526.5
AH = 11205.2	BH = 14135.5
AI = 5300.1	BI = 8253.0
AK = 4252.6	BK = 10379.4

(See Plate XXV.)

If, therefore, the base be extended indefinitely, the perpendiculars falling upon it may be calculated and set off from an appropriate plane scale, to determine the points C, D, E, F, G, H, I, and K, as in Mr Gale's method of plotting, shown in a preceding part of this work.

Plate XXVII. will give the hydrographic engineer an example of a naval attack on a fortified town, with arrangements both for defence and attack. Plate XXVIII. indicates the arrangement followed in naval battles by one of the greatest of our national heroes, and may be submitted as an instructive example of that portion of a marine surveyor's duty.

Plate XXIV. is the plan of Kingston harbour, at Dublin Bay, and will show the method of planning a modern harbour, though the field-book is not given. The method of proceeding is exactly similar to that in Plate VII., which may be consulted.

Plate XXIII. is a chart of part of the sea-coast, laid down from a scale of one inch to a mile, and the bearings taken with a theodolite divided into twice 180°, and the distances measured with a Gunter's chain of 66 feet. This chart will give a surveyor who is employed in the survey of a county some idea of the labour in a work of that kind, as the whole coast must be measured and planned, as represented by dotted lines from one station to another; but numerous other bearings besides those on the stations, including all the roads, rivers, brooks, towns, farm-houses, and all remarkable objects, must be inserted, in the same way as represented on the chart, in their true situations; and all inaccessible objects must be intersected, as may be seen by examining the chart. For instance, take bearings from station 5 to Broad Island at *a*, and another from station 8 to Broad Island at *a*, and, in order to be certain, take another from station 10 to the same place; do the same to all other inaccessible objects as represented on the chart. If you have a tolerably long base, the intersection of two lines meeting, if not too acute, may be trusted to church steeples, gentlemen's houses, farms, remarkable trees, windmills, &c. &c., till you have an opportunity of checking them in the course of the survey. To get an idea of every little occurrence that is requisite in taking the survey of a large district or map of a county, being a combination of the various branches of surveying, can be attained only from a consultation of the whole work. The hills are generally done by an eye-sketch upon a blank corner of the field-book, or by a sketch in passing them, drawn in a rough manner in the blank leaves of the book; but if you are upon a hill, place the theodolite so as the index and the limb correspond with the needle in the compass-box, and take bearings to two hills you know, and sketch in their likeness. The plain table is an excellent instrument for this purpose, as you can sketch in the likeness of a hill in truer proportion with it than by a guessed eye-sketch. If you prick as many intersections off as the plain table sheet will hold, and, when you plant the table, lay the thin edge of the index upon the hill you are upon, or any other hill or place you know, and turn the table round by the socket upon its axis; the needle will play over the *fleur-de-lis* in the compass-box if you have laid the meridian line of the great triangles parallel with the longest side of the plain table.

But the most accurate method of sketching hills is, after having plotted a part of your plan, trace it through upon oiled paper; and by going over that particular part of the country represented by the plan, you may sketch in the hills from point to point with great accuracy.

This is the field-book of the survey of that part of the sea-coast. See Plate XXIII. The survey was begun at *a*, and bearings and distances measured to *b*, *c*, *d*, *e*, &c. till such time as the letters were all exhausted; recourse was then had to numbers, from No. 1 to No. 17. If the soundings are to be taken, you may proceed in various ways.* One method is by heaving out the log, and noting how far the boat sails in half a minute—the boatmen having got previous notice to row as equably as possible, and to sail in a straight line from one point to another. Suppose the log run out 20 fathoms in half a minute, this is at the rate of 2400 feet in 10 minutes, or 36 chains 36 links; so that if a sounding is taken every 10 minutes you prick off upon the chart 36 chains and 86 links, and insert the depth upon the chart. Again, if you take another depth in 22 minutes from starting, prick off 80 chains, or one mile, and so on till the boat crosses the bay.

Another method, which is much practised, is to sail in a direct line from one point to another, and take a sounding every 4 or 5 minutes; measure the length upon the chart, and divide that length by the number of soundings that are taken, suppose 5, and the distance 4 miles. Another method is, by ordering the boatmen to keep the boat steady, and take a bearing with a sea compass, first to one place, then another bearing to another place, both places being at a considerable distance from one another, and both objects particularly marked on the chart; protract those bearings from the meridian, and, where the intersection meets, insert the sounding at each intersection you make all along the coast and middle of the bays.

The above methods are all liable to error, although they are much practised.

A better method is by placing the theodolite at any part of the shore where you had a station, and order the boatmen to sail from one point to another in a direct line; and, when a sounding is taken, to put up a flag. As soon as you take a bearing to it, set down the hour and minute; the assistant in the boat sets down the sounding, and the minute when it was taken; the same is done at every time the flag is put up. When one line is done, another must be begun in a different direction, and so on with several other lines till one bay is finished; then remove the theodolite, and place it upon any part of the shore where you have a good view, and give the boatmen orders to sound another bay in the same way; to protract the soundings, draw a black lead line upon the chart to represent the line the boatmen sailed on from one point to another;

* The depths given are those at low water in spring tides. The rise and fall of the tide should also be given, or the heights at high and low water. See a subsequent part of this work.

FIELD-BOOK OF A SURVEY OF A SEA-COAST.

(Begins at the bottom of the page.)

	Bearings.		Chains.	Opposite
	° ′ 64.12	from *d* to *a*	54	Closes
	166.12	from *c* to *d*	50	
	65 36	from *b* to *c*	51	
	166.30	from *a* to *b*	53	
Began again	at *a* an	Intersected	point	on Broad Island
	146.00	Long Island	82	Long and 8 wide
	48.18	from 16 to 17		
Broad Island, . .	110.00	from 16		
Broad Island, . .	93.00	from 16		
Long Island, . .	75.30	from 16		
Long Island, . .	24.30	from 16		
	2.30	from 15 to 16	50	
	21.00	from 14 to 15	31	
Long Island, . .	127.00	from 14		
Long Island, . .	68.48	from 14		
	145.30	from 13 to 14	63	
	53.30	from 12 to 13	55	
	120.30	from 11 to 12	67	
	134.00	from 10 to 11	60	
Broad Island, . .	166.00	from 10		
Broad Island, . .	134.00	from 10		
	53.00	from 9 to 10	47	
	121.00	from 8 to 9	45	Dunston
	2.00	from 8		
Broad Island, . .	67.40	from 7 to 8	66	
	20.30	from 6 to 7	82	
	132.00	from 5 to 5	68	Innertiel
Broad Island, . .	64.12	from 5		
	167.30	from 4 to 5	46	
	16.30	from 3 to 4	94	
	173 06	from 2 to 3	95	
	43.40	from 1 to 2	68	Chains
At 22 it is 28, At 31 it is 10, Bay, }	179.00	from *z*	53	Anker Castle
Bay, . . .	108.00	from *z*	32	
	108.00	from *z* to *z*	62	
	30.00	from *y* to *z*	38	
	128.20	from *x* to *y*	88	
Crooked Island, .	167.00	from *x*		
Crooked Island, .	61.00	from *x*		
	97.30	from *w* to *x*	27	
Crooked Island, .	18 00	from *w*		
	76.30	from *w*		
	40.30	from *v* to *w*	36	
	133.15	from *u* to *v*	78	
	109.30	from *t* to *u*	70	
	88.30	from *s* to *t*	105	
	54.40	from *r* to *s*	55	
	15.00	from *q* to *r*	58	
	65.00	from *p* to *q*	52	
	138.30	from *o* to *p*	94	Crow Point
Crow Rocks, . .	85.00	from *p*		
	17.00	from *n* to *o*	61	
	76.30	from *m* to *n*	30	
	13.30	from *m*		
Crow Rocks, . .	151.00	from *m*	70	
	2.24	from *l* to *m*	95	Old Church
	88.36	from *k* to *l*	88	
	91.30	from *h* to *water*	38	
	1.00	from *h* to *k*	27	
	147.48	from *h* to *i*	100	Joppa 52
	118.30	from *g* to *h*	107	
	157.00	from *f* to *g*	37	Sink House
	42.24	from *e* to *f*	27	Town of Dunse
Horse Shoe, . .	66.00	from *e*	51	
Horse Shoe, . .	5.30	from *e*	23	
	88.30	from *d* to *e*	63	
Little Island, . .	55.00	from *d*		
	2.30	from *c* to *d*	108	
Little Island, . .	112.00	from *c*		
Little Island, . .	91.00	from *c*		
	68.36	from *b* to *c*	50	
Little Island, . .	129.00	from *b*		
	177.00	from *a* to *b*	175	Distance in Chains

then lay off the bearings taken to the flag, which will cross that line; then observe the hour and minute the sounding was taken, which will correspond with the time you took the bearing; set down the depth of water; do the same with all the other lines on which the boat sailed till you have finished.

When you cannot sail from a direct point, order the boatman, at each place he puts up his flag, to put himself in a line with two objects upon the land, which you have marked upon the chart. Place the theodolite in the best situation you can find, that the bearings you take may intersect the boat, so as the angles may not be too acute; to protract the soundings, draw a black lead line by laying the edge of the ruler upon the two objects, and lay off the bearings from the meridian, and insert the soundings.

Another method, which is preferable to any of those described, is to have three memorandum books, and three watches, all set to the same minute at the beginning. Your assistant has one theodolite and you have another, which are placed at least a mile separate, or more if it can be got, where each of you have a good view of the bay or coast you intend to take the soundings of: the assistant in the boat sets down the minute and the depth of the sounding the moment the flag is hoisted; you take one bearing, and your assistant takes another, and you both set down in your book the minute of time it was taken. After having taken a number of soundings in all directions, all three compare books. The observations are then to be protracted, and laid down and plotted on the chart, and the soundings all inserted at their respective intersections. The same method of protracting is used as the chart is plotted by, and need not be again repeated. Be very particular in making an allowance for the fall and rise of the tide in all the different methods above specified: for this purpose a man ought to be watching at an index, such as are at most harbours, to set down on paper how much the tide rises or falls every quarter of an hour during the time the soundings were taking.

From the remarks I have made upon county surveying, a surveyor will find very little difficulty in surveying and making out a map, after having had a little practice in both departments. As the survey of a large extent requires great minuteness, it is necessary to have such instruments as will measure angles to a great nicety. An altitude and azimuth circle of large radius, so contrived as to read at least every 10″ distinctly, to observe the angles with the greatest accuracy, to extend triangles formed to distant objects calculated by logarithms, so as to have distances very correctly, and from them a range of bearings taken to various other distant objects, is indispensable.* Few surveyors are at so much pains, resting satisfied when three bearings meet in a point. It frequently happens, that from two stations, another that you wish to make a station of cannot be seen from the one you are at, and it is necessary to intersect it; in that case, measure a distance from where you are to where it can be seen; there plant up the theodolite, and take a bearing to it, and another at some convenient place, and so intersect it. It is necessary that a land-surveyor should take the latitude of some of the principal places by observation. This may be done by observing the altitude of the sun at 12 o'clock exactly; and, by a table of the sun's declination, you find the elevation of the equator in that place, and the complement of this angle is the latitude required. Unless you have a well-regulated watch, set by a time-piéce, the hour of 12 cannot be exactly ascertained; therefore it is necessary that a surveyor ought to know how to find a true meridian line by observation taken with his theodolite. Choose a clear day, an hour or two before 12, take the sun's altitude, and in the afternoon you have to observe when the altitude is the same.

In the forenoon, suppose at 10 o'clock, the instrument is set level, the index over *o* on the limb, and the needle set over the *fleur-de-lis* in the compass box; move the index horizontally and the arc vertically, till you see through the telescope the cross hairs and the limb of the sun, upper or under, exactly in contact; observe what degrees and minutes are cut on the horizontal arc by the index, which note in a memorandum book, and also the angle of elevation cut by the index on the vertical arc. In the afternoon, observe that the instrument is not moved from the level, and that the index and the arc correspond with the angle of elevation it was in the forenoon. Then move the index on the limb horizontally, and watch it till you see the cross hairs in the telescope touching the same limb of the sun as before, and note the degrees and minutes cut by the index on the limb, suppose 38° 30′; from the afternoon's observation subtract that made in the forenoon, which was 7° 30′; there remains 31°, the half of which is 15° 30′; to this half sum add the morning's observation, 7° 30′, and the sum is 23°. Let the theodolite remain in the same position, and turn the telescope about till the index cuts 23° on the limb; look through the telescope, and if the cross hairs coincide with any particular mark at a distance, it is

* See articlo Trigonometrical Surveying in a subsequent part of this work.

in the line of the meridian. If no mark is seen that you are certain of, you must then set up a pole, or something else, by way of a mark; put also a mark where the instrument stands. By this means you may also know the variation of the magnetic needle, by observing what degree it cuts in the compass box when the index is placed exactly on the meridian.*

The latitude of the place may be taken thus:—After you have found out a true meridian, plant the theodolite level; bring the vertical arc and telescope into the line of the meridian, and let the index remain at the same angle on the limb; then elevate the telescope towards the sun, and when the sun is in a line with the meridian, and the cross hairs in the telescope appear as if they were touching the sun's upper or lower limb, note down for the sun's meridian altitude the degrees and minutes cut by the index on the arc, which suppose is 42° S., allowing 16′ for the sun's semidiameter, and for the sun's declination by an ephemeris for the same day, suppose 4° 2′; if it be a north declination, subtract this from 42°, the meridian altitude, and the remainder will be 37° 58′ the co-latitude, and 52° 2′ N. the latitude; but if the altitude be 33° 56′ S., and the sun has a south declination, 4° 2′, it must be added to the meridian altitude, and the sum will be the co-latitude; by substracting which from 90° you have the latitude of the place, 52° 2′. (See the method of finding the latitude afterwards given.)

* This method, without applying the equation of equal azimuths for the change of the sun's declination during the interval, is not far from the truth near the solstices, and may in that case answer tolerably well; but that equation must always, as shown in the explanation of Table XVIII., be applied where accuracy is required.

SECTION SEVENTH.

OF REDUCING AND DELINEATING PLANS.

ART. I.—OF REDUCING.

THE *reduction* of a figure, design, or draught, is the making a copy of it either larger or smaller than the original, still preserving the form and proportion. This may be accomplished in various ways; but it may be necessary to mention that each has, more or less, its defects. Plate XVIII., fig. 1, represents a plan which is to be reduced by means of squares. This is performed as follows: divide the original into little squares, and divide a piece of paper of the dimension required into the same number of squares, which are to be larger or smaller than the original as the map is to be enlarged or diminished. This done, in every square of the second figure draw what you find in its correspondent one in the first, as in fig. 2, same plate, which is reduced to one-half of the original, or one-fourth in extent.

The proportional compasses, or, as they are frequently termed, (from their use,) compasses of reduction, are of great use in reducing. Plate XVIII., fig. 3, represents a pair of those compasses, which are made use of as follows: move the slider A till such time as, by trials, you find the number of extents of the one end of the legs C C contained in one extent of the other end B B. The number of extents indicate the proportion in which you are to enlarge or diminish. From this the application must appear evident. The instrument is generally sold divided to your hand, which renders it still more convenient.

PLATE XVIII., (Fig. 4,)

Represents a proportional scale or fan, which answers exactly the same purpose as the proportional compasses. It is constructed as follows: draw two lines at any angle you please from D; then take a pair of compasses, with either the drawing pen or pencil, and put one foot in the angle D, and draw a number of segments of a circle at as nearly equal distances as you can guess, and number them in the same manner as is on the figure; then divide the farthest arch from D into three equal divisions, and draw a line from D to the second division, which makes that part on the left only half of what is on the right. To use it, take any length from the large plan, and apply that length in the compasses to the right side of the fan, by moving them down till you bisect the line drawn through the fan. Turn the compasses round, and press in one leg, till the point just reaches the left line on the fan, which gives one half the distance, which apply to the copy that you are to reduce. This method will be found equally expeditious and as correct as the proportional compasses. It can be drawn with a black-lead pencil upon any part of your plan, and can be rubbed out when convenient.

Plate XVIII., (Figs. 6 and 7,)

Is the plan of the mouth of the rivers Esk and Dee, which has a large flat of marshy ground between the river Dee and the sea. On the east are broad sands and a bold shore, with steep rocks; above the rocks is gently rising ground, with a clump of planting; opposite is the village of Esk, with some open fields, a plan of which is made on a reduced scale by means of squares; the proportion is as three to four,—that is to say, three squares on the large plan is to be divided into four squares in the smaller one, or three chains from a scale upon the original plan will measure four on the reduced one. If you make each square on the large plan two inches, each square on the small plan must be an inch and a half. If you use the proportional compasses, let them be set so as two inches betwixt the largest legs may measure only an inch and a half in the short legs. Should you prefer a proportion, let the middle line be in the same proportion as three is to four. (See the fan, fig. 8.)

Plate XVIII., (Fig. 5,)

Represents a pantagraph at work. Of all instruments that have hitherto been invented for reducing, copying, or enlarging plans, the pantagraph is by far the best; not only for being the most expeditious, but the most correct, as every straight and curved line is copied with the greatest exactness. It is as useful to an experienced draughtsman as to those who have had but little practice in drawing.

It saves much time, either in reducing or enlarging plans of estates, and with equal facility may be used for copying figures, sea-charts, maps, profiles, landscapes, &c. This, like all other good instruments, most mathematical instrument-makers lay claim of having made improvements upon.* Those instruments are in general made of brass, from one to four feet in length; it consists of four flat bars, about half an inch broad, and about one-eighth of an inch thick—two long, and two short. The two longest are joined at the end B by a double pivot, which is fixed to one of the bars, and works in two small holes placed at the end of the other. Under the joint is a castor, with an ivory roller, to support this end of the instrument. The two smaller bars are fixed by pivots at E and H, near the middle of the longer bars, and are also joined together at C by a double pivot. By the construction of the pantagraph, the four bars always form a parallelogram or rhombus. There is a slider on one of the larger bars or arms, and another on one of the shorter, which moves upon the arms till they are put in a line, which is known by taking a piece of thread and applying it to the tracing-pin A, the pin in the weight, and the pencil-tube, and are fixed to the arms or bars by means of milled screws. Each of the boxes has a cylindric tube to carry either the pencil-point I or the weight K, which is made of lead, and covered with leather or silk. On this weight the whole instrument moves, and every part is in motion when at work, except the weight, which holds the pantagraph fast to the paper by means of four prongs in the under side of it. The instrument is supported upon castors, with ivory rollers, to facilitate its motions. The long tube, with the pencil, moves easily up and down in a socket, within another tube, to give way to any unevenness in the paper. There is a cap at the top for putting in a piece of lead to increase the strength of the pencil-mark. If the original plan is of large dimensions, and the pantograph cannot take it in all at once, the operation must be done at two, three, or four times, by drawing a line from one point to another upon the large plan with a black-lead pencil; the same line must be taken off from the large plan upon a new sheet, which will correspond with the line drawn upon the copy. The original plan must be removed into such a situation as to allow the reducing or enlarging the remaining part. In this manner, by frequent shift-

* The late Professor Wallace invented an instrument, called an Eidograph, which is also a very useful instrument for similar purposes, and made by Adie & Son, Opticians, Edinburgh.

ing, a pantagraph is made to reduce, enlarge, or copy a drawing of large dimensions, by joining the parts together.

To avoid the trouble of setting the weight, pencil, and tracer each time, two of the bars are divided into ten or twenty of the most common proportions, by which divisions the sliders are to be fixed. When the machine is used, a fine string is fastened in the pencil-case; the other end has a loop, to be fastened to the finger of the operator, by pulling which he can raise the pencil when he does not wish it to mark.

No surveyor should be without this machine, as his plans will have to be reduced to a smaller scale than the rough sketches he makes out for calculating the contents, particularly if the estate is large.

Hitherto I have only given directions for taking surveys, and making out rough plans and sketches. I now come to point out the methods used by practical surveyors for copying and drawing plans with the greatest facility, at the same time with as much ease and exactness as the nature of the work will admit of, and to transfer the rough copies to be drawn on thick paper or vellum. Some surveyors use a pair of compasses with a third leg annexed, which takes off a triangle at once. With the same extent it is applied to the clean paper, and the lines drawn immediately in upon the copy with a black-lead pencil. They go over the whole drawing in the same manner, till such time as it is all taken off from the rough drawings and transferred to the clean paper.

Another method, which is more expeditious than the three-legged compasses, is by laying the rough plan above a sheet of clean paper, and with a needle pricking all the angles and curved lines through the rough plan, and the marks made by the needle are left perceptible upon the clean paper or vellum. It is then drawn, first with a black-lead pencil, to see that nothing has been wrong done or omitted, then inked in with Indian ink. The straight lines are drawn with a ruler and drawing-pen, and the curved lines with the hand.

Another method is by a copying-glass, which is fixed in a frame, and is lifted up from another frame, which it is fixed to with hinges, and is supported at any elevation with two pillars, which rest in niches made in both sides of the under frame. The plan that you have to copy is fixed to a sheet of clean paper with pins, sealing wax, or wafers, at the corners; the plan is laid next the glass, which you see distinctly reflected upon the clean paper. If the drawing-paper is very thick, put a piece of white paper next the under frame, which will make it more transparent, run over the whole with a black-lead pencil, and ink it in.

Another method is, to rub the back part of the rough plan over with black-lead dust, and lay the back part of the rough drawing next the clean drawing-paper; and with a blunt etching needle, trace over all the lines on the rough plan. When done, take it off, and you will have an impression upon the drawing-paper. You then ink it in. If you wish to preserve your rough plans from being damaged by rubbing them with the lead dust, take a sheet of thin paper, and rub it uniformly over with lead dust; lay the rubbed side next the drawing-paper, and the plan above it, and let them be all kept fast with pins or weights to prevent shifting; press the tracing point pretty hard upon the rough draught, and when you have gone all over it, take it off and ink it in.

If you have a plan to copy that has been drawn upon very thick drawing-paper, or a plan that is pasted on linen, or a plan to make a copy from that is highly finished, which the tracing point would damage, get some sheets of cambric or fan paper, both of which are very thin, and rub them over with nut-oil; then lay the oiled paper between sheets of blotting-paper, and it will be fit for use in a few days. Lay the oil-paper, after it is thoroughly dry, upon the original, which must be kept down with weights; then with a pen or a black-lead pencil go over all the lines, till you have copied the whole of the plan upon the oil-paper; then take the oil-paper, and lay it above a sheet of black-lead paper, and with a tracer trace through the oil-paper, which leaves the outline of the plan upon the clean drawing-paper, and you will have the outline of the plan, which afterwards ink carefully in.

ART. II.—OF DELINEATING PLANS.

A proficiency in delineating and making out neat drawings cannot be acquired but by practice. For that purpose, those who are desirous to improve themselves in that art should at every leisure hour be copying or making out drawings, either from drawn plans or copperplates; and should also study the different tints necessary to give drawings effect.

Indian ink, of all colours, is more used by a land-surveyor than any other. A well-finished drawing with Indian ink has a fine effect, and is esteemed by many persons to excel those done in colours. But it requires the hands of an excellent draughtsman to finish a drawing with Indian ink alone; to give it all the different tints necessary, it is more tedious than delineating a plan with colours; and when colours are used, Indian ink is the chief, as all the outlines and deep shading must be done with it.

Verdigris is much used by land-surveyors, as well as all other draughtsmen. It answers better than any other colour for shading lakes, tarns, or lochs, rivers, brooks, and the sea, and when mixed with a little gamboge makes a fine green, either for pasture, meadows, bogs, morasses, trees, &c. &c. Carmine and lake are best for shading buildings; umber, burnt sienna, and bistre, give different shades of brown; Prussian blue is often used for giving dark shades to the sides of rivers, and the sea-shore; gamboge, a fine yellow, and when mixed with lake, is a good colour for shading roads. These are the colours generally made use of by surveyors.

The most expeditious method of delineating a plan, is to draw all the straight lines with a ruler and drawing-pen; the curved lines with the hand; hedges, trees, bushes, and shrubberies, with the pen and Indian ink. Use a hair pencil, and weak Indian ink for hills, for the first tint, and wash the top and bottom off with clear water and a clean brush, before the first coat gets time to dry. Some surveyors mix a little bistre with their Indian ink, which gives them a brownish tint; others mix a little Prussian blue with the Indian ink, which gives them a fine effect. If the hills are very steep, add another coat or two, and shade them according to their steepness; let each coat dry before another is put on, and never neglect to wash off the edges with a clean brush and clear water. The Indian-ink brush should be at one end of the pencil-stick, and the water-brush at the other end of it. If there are a number of hills and rising grounds, do them all in the same way, with one coat, and those that require to be touched up steeper will be dry. Begin again, and give them a second coat. Go from hill to hill till you have finished; and begin again, and give them a third, and so on, till such time as you bring them to the tint required. If the hills are very large, as in the Highlands of Scotland, Yorkshire, Westmoreland, or Wales, use large brushes, and give the rocks a tint resembling the colour of the stone by shades. (As specimens of shading, see various plates given in this work expressly for that purpose, as Plates VIII, IX, XVIII, XIX, XXI, XXII, XXIII, XXV, XXVII, and XXXI, especially for the important method of delineating contours to be afterwards explained.)

MOORS.

Make first the representation of a few scattered hillocks with the pen, or a fine hair pencil, according to the nature of the moor. If the moor is flat, the small hillocks should be omitted. Draw in with the pen a few tufts of furze, if there be any on the moor; then with a hair pencil touch up each hillock on the right side with Indian ink, and wash the edges off with another brush at the other end of the pencil-stick, on the left and bottom part, with clean water; give each furze a touch with Indian ink, and shade them off with the hair pencil towards the right. You may then mix a little Indian ink and bistre, and lay on some broad shades promiscuously, very light, on different parts of the moor; then mix up a little weak Indian ink and Prussian blue, and lay shades on those parts of the moor where no colours have been laid on, washing off the edges to keep them from appearing harsh. Mix Indian ink and a little lake, both very weak, and lay on shades here and there, so as to interfere as little as possible with the tints already laid on; then take a little weak yellow and green, and fill up any vacant spaces that may have been left untouched with the other colours, which will give the moor a variety of shades, after all is gone over. If it is heath, shade it over with Indian ink. If there is pasture and heath mixed, shade it with light green all over, with a large hair pencil. If the moor is of a brownish nature, shade it all over with very weak bistre. In observing the above directions, you will be enabled to give the moor a resemblance very near its natural colour. The furze, whins, or fuzins, as they are termed in different places, look well if you give them a touch of green on the right side, and a touch of yellow on the west side, which should be laid on last, which will give them the appearance of being in blossom.

MORASSES.

Take a fine-pointed hair pencil, with Indian ink of a pale colour, and draw with the hand short horizontal lines, pretty close to one another, some short and some long—which do as quick as you can, till you have gone over the whole; and with the pen, insert rushes, reeds, and herbages, and shade over the whole with a pale green, inclining a little to blue; then touch up the rushes, reeds, and herbage, with a strong green, which shade off to the right, with a tint of lighter green.

MEADOWS.

With the pen, or a fine-pointed hair pencil and light Indian ink, make a few strokes, some long and some short, none to exceed the sixth part of an inch in length, all over the meadow, as they are represented on Plate XVIII, Fig. 6, (plan of Eskmouth,) and wash the whole of the meadow with light green, inclining to yellow.

PASTURE GROUNDS

Are sometimes represented by sloping and upright strokes, very short, none of them to exceed the 50th part of an inch, as in Plate XIX (Military Sketch). But most surveyors content themselves with washing the whole over with green, something darker than the meadows, and running over the whole with horizontal shades of Indian ink, which, if used, should be tinted before the green is laid on.

SANDS UPON THE SEA-SHORE.

Wash them all over with a little weak carmine, and gamboge, mixed neither too strong nor too weak. Some surveyors dot them all over with small dots, as is represented on Plate XVIII. Fig. 6, with a pen; but this is only done on high-finished drawings, particularly if the sands and scale are large, as it is very tedious to do them with dots.

TREES

Ought to be done very neat, upon a plan, as they give a drawing a fine appearance, (see Plate XIX., Military Sketch, or any of the following plates.) They are expressed by a vertical stem with a horizontal shade at bottom, and made broad at the top, and shaded with the pen on the right or east side, or with a touch of Indian ink, and coloured green on the left side, with here and there some with a little brown, others of the trees yellow, for the sake of distinction and variety.

ROCKS

Upon a hill-side are made to appear rugged, as on Plate XIX., and those upon a bold shore as represented on Plate XVIII., Figs. 6 and 7. Houses are often shaded with carmine and lake, and frequently dark with Indian ink, and they are in general shaded darker on the east and south sides. See Plates XVII., XXI., XXXI., and XXXII.

RIVERS, LAKES, AND THE SEA-SHORE,

Are shaded by going round the edges on both sides of a river with a strong liquid blue, which is softened off with a hair pencil, and a weaker blue towards the middle. Some surveyors prefer shading the edges of the rivers and sea-coast with Indian ink, which is softened off with a pencil and clear water, and when dry, shade the whole of the river, lake, or sea, with water blue. This method makes the water look bolder at the edges, than when it is all shaded blue. Surveyors that have time, and wish to excel in drawing, frequently draw in the whole of the river with bold lines near the edge, and fainter towards the middle, in imitation of engraving. The sea-shore is shaded in the same way, but much broader, and washed off with a large pencil and clear water. When the Indian ink is thoroughly dry, a coat of strong blue is laid above the ink, and washed off with a weaker blue, with a large hair pencil and clear water.

CORN FIELDS

Give a plan a fine appearance, if they are neatly shaded. The common way, is to draw parallel lines as near as you can guess, at equal distances, either with the hand, or with a parallel ruler and a drawing pen, and weak Indian ink, or with the colour you intend to shade the field with, to represent the ridges. When that is done with any colour you think proper to make the divisions of the field, suppose yellow, take a little yellow in a hair pencil, and run down the ridges; and before the colour is quite dry, wash the edge of it with the hair pencil on the other end of the stick with clear water. Do the next field in the same way, but a different colour, and vary the colours so that two fields adjoining should be of different colours, till you have gone over the whole plan, some brown, and others blue or red. But most draughtsmen prefer making all the fields at least something betwixt yellow and light brown.

PLATE XVII.

Is a design for a new town, which was proposed to be built in the same way as represented on the Plate, where every corner house has the benefit of a garden. In most towns the corner houses are in general the best, but often deprived of a garden, when the interior houses have the advantage of one. To distinguish the gardens belonging to the corner houses, they are coloured upon the Plate with different colours, merely to show how they are situated from the house they belong to. This plan is introduced principally to give an idea how gardens are generally tinted and finished upon plans. After the outline of the whole is drawn with a drawing-pen and Indian ink, before any colours are used, take a fine-pointed hair pencil or pen, run over with weak Indian ink the lines to represent the different beds; then draw in bushes along the sides of the walks, and here and there some bushes at the divisions of the different beds, and some scattered trees, if there are any in the garden, which should be done in a neat style, and shaded dark with Indian ink on the right cheek, and let the left sides of the trees be left white; they should all have a shade at the bottom, which is commonly done on the right hand from the roots. Houses are sometimes shaded dark in plans with Indian ink, but more frequently with carmine, and on the east and south sides with lake, which is something of a darker red than carmine. The carmine should be diluted with gum water; the best kind is to be bought in powder or in cakes. If Indian ink is used, which some surveyors prefer to carmine or lake, draw a dark line on the east and south side of the buildings, pretty bold, as is done on the Plate, which gives a good effect to the drawing. The beds of the gardens should be tinted with very light and pale colours, such as green, yellow, red, and any other colours you choose, to make a small distinction of one bed from another. The gravel walks are shaded brown, and the grass walks green. The bushes and trees should all have a tint of green, brown, or yellow on the light side, some one colour and some another, but most of them green, which gives a pleasant effect, if they are tastefully laid on. The streets in a town are commonly left white, although some surveyors prefer giving them a tint of very light blue, to represent the causeway; and if the town has a flagged pavement, it is coloured brown in general.

The gravel walks in pleasure grounds, which are frequently numerous, are made of gravel, and laid out very tastefully round beautiful fish-ponds, and the shrubbery, which is extensive, kept in the finest order; the whole sheltered with plantations of considerable extent. A few corn-fields may be added and represented, which gives an idea of the method of laying down fields and hedges adjoining a pleasure ground, and what way to make out a finished drawing. But observe, the dotted lines representing the ridges should be drawn with a drawing-pen with light Indian ink, or with colour on your plan, the same as you intend to shade the different fields with, which should be bolder than the faint colour with which you shade the ground. This gives a fine effect to a drawing, if smoothly laid on.

Many plates are made out to show in what way the drawing of grounds ought to be finished. In order to get an idea of taking the survey, pointing out the numerous distances and angles, recourse must be had to various parts of the treatise; let it suffice to observe, that, by what has been already described, if a pupil has made himself acquainted with the different methods of surveying and making out his protractions, he will find little difficulty either in taking a survey, though ever so intricate, or making a plan of the same.

The drawings of pleasure grounds would require to be plotted upon a pretty large scale, which would give sufficient room to colour all the roads and gravel walks minutely. To avoid confusion, they are left uncoloured on the plates, and are only represented by lines, which can be easily traced. When a pupil improves himself in drawing, he will soon be enabled to make out a neat plan, by following the directions given in delineating, immediately preceding. Those who wish to excel in drawing should provide themselves with good Indian ink, and with a set of the best colours, which can be purchased ready prepared in small cakes, very finely ground, and may be had at any of the colour or print shops. A very fine liquid blue may be made, by mixing three ounces of verdigris and one ounce of cream of tartar, with half a gill of vinegar. Put the mixture into a vial, and shake it two or three times a-day till the verdigris is dissolved, and you will have a fine water blue, which, when mixed with a little yellow, is a beautiful grass green. The above two colours are more used by land-surveyors, with the addition of Indian ink, than any of the other colours.

A land-surveyor who undertakes the survey of a county, ought to study attentively the method of laying down and transferring his rough draughts and sketches with accuracy, and to copy them very minutely and very neat upon his clean drawing.

Before I close this treatise on surveying, I shall offer a few remarks which may prove useful to the surveyor in the delineating of high ground on maps,—viz., mountains, fells, hills, and knolls; which appellations are made use of according to the altitude, which in general is determined from the level of the sea at low water.

Of all the methods that have been invented for drawing high grounds, although many are used, yet none is more generally adopted, or indeed of greater utility, than those represented in Plates XXXI. and XXXII. The former is more particularly adapted to close or complete a country, the latter where it is high and open; yet the principle in both is exactly the same, and simply consists in what is in general termed a *bird's-eye view*, the eye being supposed at a distance from the ground. The greater the altitude of the hill, the deeper the shade. It being impossible to place the eye in that position, recourse must be had to sketching the ground from eminences, commencing first at one side of a mountain or hill, and going all round, so as to introduce as little as possible any perspective view; thus will every part of a hill or mountain have a proper extent, and will be in their proper situations with regard to horizontal distance. If the hill is perfectly flat on the top, it is left white, as on several of the plates. Although many surveyors prefer perspective methods—which, when executed with taste, may please the eye on account of their landscape appearance—and though I have always admired and commended fine drawn plans; yet it is my opinion (as maps are not merely for show) accuracy ought to be preferred. I must here mention, that in no publication on surveying has anything been said of the drawing of maps. This has induced me to introduce several plates as specimens—some for the various parts of the outline, such as towns, rivers, roads, and sea-coast, &c.; the others for high grounds and the heads of rivers; and I may venture to affirm, they will afford more information on the subject than the most lengthened detail. However, I shall show the method I have made use of in sketching the hills. After having done all the outline work on the plan, the best way of introducing the hills with accuracy is to trace on part of the paper the plan of the outline, and fasten it upon a plain table, and sketch in the hills; which again take off the table, and trace the hills upon the outline plan, and shade them according to their steepness; then do all the other smaller hills in the same way. When the work runs off the plain table, use more traced paper, till such time as you have gone over the whole map, taking care to notice all cairns, burrows, or rocks, which are situated on the hills, and shade them as far as they have a declivity towards the rivers or brooks. Great care should also be taken to trace the various ravines and brows of hills, which will give a true representation of the country; and to have it well drawn has a fine effect. The draughtsman should take care not to labour the hills too much; for it frequently happens that greatly-laboured plans lose effect, and drown the most useful part of the plan—the outline.

It will sometimes be necessary, particularly if the scale is large, to determine the falls of hills by levelling, (for which see page 238,

&c.;) but an accurate eye-sketch is sufficient for all scales connected with county surveys.

Some draughtsmen use the hair pencil for hills. It is in my opinion best, as it gives more spirit than the pen; yet I have seen a few drawings with the pen only, in imitation of engraving, similar to Plate XXXI., exquisitely finished. Notwithstanding, from the very few good specimens, it is an art in which very few attain to any degree of perfection, but which, if they do attain, is an excellent specimen to the engraver. In a plan, one of the greatest recommendations that I know is good writing; and nothing tends more to deteriorate a map or plan, even if accurately surveyed and well drawn, than bad writing. The young draughtsman ought therefore to practise the various hands, as represented in the different plates, which will not only be a good specimen to himself, but an excellent one to the engraver.

When a land-surveyor has finished a plan of a nobleman or gentleman's estate, in as elegant a style of drawing as he is capable of—if the scale he has adopted is large, he will upon his plan have several blank corners, one of which should be filled up with a neatly-written title, another with a table of contents. He should also, on any convenient place, insert a scale, and on another blank space a compass; and if there is any other blank corner remaining, it may be filled up with a view of the mansion-house, or an old ruin of a castle, or any particular building, if there are any on the property. If neatly drawn, and like the building, it is a fine embellishment to a plan.

I feel no hesitation in saying, that, with proper attention to the various methods of surveying, and a minute inspection of the plates in this treatise, the young surveyor, with perseverance, may soon become master of his profession.

TRIGONOMETRICAL SURVEYING, &c.

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1. THE figure of the earth is nearly that of a globe, and, for many purposes of surveying, this hypothesis will bring out conclusions sufficiently accurate; but for the nicer and more extended processes, the earth must be considered as a spheroid compressed at the poles. The different measures of arcs of the meridian, &c., concur to prove that the compression is about $\frac{1}{300}$—that is, the polar semiaxis is one three-hundredth part less than the equatorial radius. From the comparison of a number of arcs, I have found the radius of the equator equal to 20922642 feet, and the polar semiaxis 20852900 feet;* and from these, by spherical geometry and the properties of the conic sections, the various formulæ, rules, and auxiliary tables required in Trigonometrical Surveying and Levelling are obtained.

2. Though an extensive trigonometrical survey may be commenced by any of its details, yet it is usual to measure a base, in the first instance, with all possible attention to accuracy. It is generally chosen in as level a position as may be attainable, and it is a good plan to measure it first approximately by a hundred-feet chain, as a trial of its capabilities, and a check on the more accurate methods to be afterwards followed. A good theodolite, or transit instrument, is placed securely on a station at one extremity, and, by the motion of the telescope in a vertical plane, such a number of stakes are intersected throughout the base, by this means placed in a straight line, as are sufficient to guide the subsequent measures. In the course of this process, considerable trouble will be sometimes experienced from the effects of *lateral refraction*, which

* Mr Airy has, from a considerable number of arcs, ancient and modern, deduced 20923713 feet and 20853810 feet, respectively. I have, however, examined all the more recent and unobjectionable measurements of arcs, and the results have induced me to retain my former determinations till I see stronger reasons for a change.

shifts the stakes sometimes to the right and at other times to the left. The same atmospheric irregularities render it necessary to measure the horizontal and vertical angles repeatedly in the subsequent course of the survey, on different days, at the most favourable hours, however powerful the instrument employed may be.

3. Ramsden's steel-chain, made in a peculiar manner, seemed to answer the purpose of lineal measure tolerably well; but it appears that Colonel Colby's compensation bars, constructed by Troughton, and composed of steel and brass, connected on an ingenious plan, possess a decided advantage, because the measurement is not carried on by a contact of the ends, as in Roy's glass rods, or the French metallic rods, with sliding *languettes*, but by ascertaining their coincidence from fine points on platina, with powerful microscopes, having cross wires in their foci, in a manner similar to the coincidence of verniers, or rather to the examination of the divisions of astronomical circles, by powerful reading microscopes. These microscopes are placed on compensating bars also, like the measuring rods; while all these bars themselves have been accurately tested by actual experiment, and found correct. It was in this way that the base line on the shores of Lough Foyle, in Ireland, was measured—the most accurate operation of the kind, perhaps, hitherto performed.*

For purposes of considerable accuracy, 10 feet fir-deal rods will be found very convenient. These ought to be at least three in number, painted different colours, as *white*, *red*, and *blue*, to prevent mistakes in recording—two always lying in position, while the third, or hindmost, is removed to the front. They should be previously baked in a cast-metal tube, whence they are to be, when ready, transferred to another similar tube, full of boiling oil, or varnish, to prevent the effects of the atmosphere from changing their length by heat and moisture. The ends ought to be shod, or covered with a hemispherical steel cap, to a radius of curvature of half the length of the rod, to avoid as much as possible errors in length.†

* A full description of these, accompanied by numerous figures and plates, with an account of experiments, angular measures, &c., connected with the measurement of this base, and the formulæ, rules, and tables used in the subsequent survey, was published in one volume quarto, by the Board of Ordnance, in 1847, of which the author was presented with a copy by order of the Board, and it deserves the consideration of all those engaged in similar operations.

† When the radius of curvature of each cap is equal to half the length of the rod, each circular arc is a part of a circle, which renders it immaterial whether the whole rod be exactly in line or not, if its centre is.

When a difference of elevation in the course of the measurement suddenly takes place, the contact with the vertical line must be effected by a plumb-line, suspended so as to touch the extremities of the adjacent rods, allowance being made for the thickness of the wire or thread which suspends the plummet. For ordinary purposes, a well-constructed steel chain is the most convenient, when carefully used, making allowance for the expansion or contraction of the chain, according as the temperature is above or below 62° Fahrenheit, the standard of British imperial linear measures.

In this last way, the measurement of a base line at Brodick, in Arran, was, by the author, performed during the month of August in 1843, 1844, &c., for a survey of the island. See Plate XXX.

The mean temperature was 54° Fahrenheit nearly, and the mean inclination of the base, 9′ 58″.

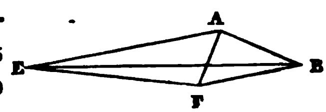
The base, from necessity, was composed of three different portions, connected and reduced to a straight line by means of the angles of inclination of the different parts; though, when practicable, it is better to choose it all in one straight line.

FIGURE OF THE BASE.

A B = 677 feet by direct measurement.

A F = 387 and F A B = 108° 2′ 18″.35

E F = 1409.5 A F E = 121 13 20.00



62° — 54° = 8° below the standard.

Correction for Temperature = 0.00000694 × 8	= 0.0000555	—
" for inclination of base, 9′ 58″ versine	= 0.0000042	—
Tear and wear of chain, . .	= 0.0004800	+
Irregularity of ground, . .	= 0.0019200	—
Total correction, . . .	= 0.0014997	—

Hence,

677 — 677	× 0.0015	=	A B corrected,	=	676.22 feet
387 — 387	× 0.0015	=	A F .	=	387.55
1409.5 — 1409.5	× 0.0015	=	E F .	=	1407.88

The base was only about 15 feet above the mean level of the sea, and therefore requires no reduction to that level.

(1.) In the triangle F A B, there are given the sides A B, A F, and the contained angle F A B, to determine the angle A F B = 47° 10′ 27″.52; the angle A B F = 24° 47′ 14″.13; and B F = 876.68 feet.

(2.) In the triangle A F E, there are given the sides A F and F E, and the contained angle A F E, to find the angle F A E = 47° 9′ 46″.26; F E A = 11° 36′ 53″.74; and the side A E = 1641.87 feet.

(3.) F A E + F A B = E A B = 155° 12′ 4″.61.

(4.) A F B + A F E = E F B = 168° 23′ 47″.52.

(5.) In the triangle B A E, there are given the sides A E and A B, with the contained angle E A B, to find the angle A B E = 17° 37′ 55″.78, and A E B = 7° 9′ 59″.62; and the side B E = 2273.497 feet.

(6.) The same thing may be done from the triangle E F B, by the sides E F, B F, and the contained angle E F B, which give F B E = 7° 9′ 18″.36; F E B = 4° 26′ 54″.12; and E B = 2273.497 feet as before.

4. From the extremities of this accurately measured base, angles are taken with the theodolite to other properly selected points, and thence extended over that portion of the country to be surveyed—the triangles, for the sake of accuracy, being chosen as nearly equilateral as possible. The number of triangles should be as *few* as the nature of the ground and the power of the instrument will admit.

5. The measurement of an arc of the meridian generally either accompanies, or is derived from, the operations connected with the survey. For this purpose, the position of the meridian, passing through one of the extremities of the base, or some of the angular points of the series of triangles, must be determined by a good theodolite, an astronomical circle, or by one of the best transit instruments. Then the angle which some of the sides of the adjacent triangles makes with the meridian must be accurately measured, from which the bearings of all the sides of the connected series of triangles may be found, in order to obtain either an arc of the meridian, or to find the latitudes and longitudes of prominent points in the course of the survey.

The same operations must be repeated for the purpose of verification at the termination of the series, or oftener, if the survey be of great extent. If the survey includes regular observatories of established reputation, advantage may be taken of them to determine the true bearing of the adjacent sides.

6. The latitudes of the extremities of the arc, or of two points adjacent and trigonometrically reduced to them, must be determined by the astronomical circle, or other proper instrument, from

numerous observations on the same stars, at the same time as nearly as possible, so that any small error in the mean places of the stars, or in the necessary reductions, may be thus avoided.*

7. There are three different methods of making the usual calculations of the sides and angles of the triangles—the first, by treating them as spherical triangles; the second, by reducing the angles of the arcs to those of their chords; and the third, the easiest of the three, and sufficiently accurate for every practical purpose, is to deduct one-third part of the *spherical*, or more strictly speaking the *spheroidal excess*, (though the difference is usually insensible;) that is, the excess of the three spherical angles above two right angles, and using the remainders in the calculation, which give the lengths of the opposite sides sensibly the same as that by spherical trigonometry, or by a reduction to the chords, with much less trouble. In this last case, it ought to be recollected that the vertical spherical angles, before deducting one-third of the spherical excess, are equal; but generally, unless the triangles be very small, *after* deducting the spherical excess, they are unequal, if the triangles to which they respectively belong be unequal, since the spherical excess is proportional to the magnitude of the triangle. The angles so deduced are, for the sake of distinction, called *mean angles*.

8. To estimate the corrections to be applied to horizontal angles, measured on the surface of the earth, at any point of observation, let m be the arithmetical mean of the whole, and the seconds of reading s, s', s'', &c., and rejecting from each observation the same quantity, giving the results, if more convenient, a negative sign; then $m - s$, $m - s'$, $m - s''$, &c., are the differences of the individual observations from the mean; and the *weight* of the determination, as it is technically called, or of the average m, is equal to the square of the number of observations, divided by twice the sum of the squares of the errors, as shown in the usual treatises on probabilities, especially by Gauss, according to the following formula:—

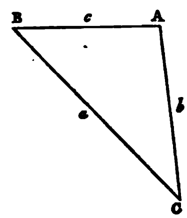
* About the 21st of August 1840, when I observed at Inchkeith the declination of α Aquilæ, as given in the *Connaissance des Temps*, it exceeded that in the *Nautical Almanac* by $2\frac{1}{2}''$, while that of Polaris agreed nearly. Would this have been believed in the present state of practical astronomy? For 1848 the difference is $3''$, apparently increasing with an opposite sign, making a difference of their relative declination in 1840 and 1848 equal to the sum of these, or $5\frac{1}{2}''$! What confidence is to be placed in the annual parallax of the fixed stars, amounting to about a *tenth* of this, when such discordances are to be found in *standard catalogues*? In the *Connaissance des Temps* for 1849, however, a new catalogue of 115 stars is given, from the observations of Greenwich and Paris chiefly, agreeing very closely.

$$x = \frac{\frac{1}{2} n^2}{(m-s)^2 + (m-s')^2 + (m-s'')^2}$$

$$y = \frac{\frac{1}{2} \nu^2}{(\mu-\sigma)^2 + (\mu-\sigma')^2 + (\mu-\sigma'')^2}$$

$$z = \frac{\frac{1}{2} \mathrm{n}^2}{(\mathrm{m}-\mathrm{s})^2 + (\mathrm{m}-\mathrm{s}')^2 + (\mathrm{m}-\mathrm{s}'')^2}$$

The quantities x, y, and z, are called the *weights* of the observations made at each angle of the triangle. The error in the sum of the three angles of the triangle is then divided into three parts, proportional to the reciprocals of the three weights, or to $\frac{1}{x}$, $\frac{1}{y}$ and $\frac{1}{z}$, which must be applied to the three angles of the triangle A B C, respectively, as in Example 2. The reciprocals of the squares of the number of observations would give the same result nearly. In this manner the weight is found for each angle; and the error of the three angles of the triangle is the difference between the sum of the three angles, of which each is the mean of the observed angles, and 180° + ϵ is divided into three parts, proportional to the reciprocal of the weights, which parts form the corrections to be applied, according to their signs, to the angles to which they respectively belong. We have then the three corrected spherical angles, the sum of which is exactly 180° + ϵ, in which ϵ is the small quantity called the spherical excess.



9. *Example* 1.—Let A be East Lomond in Fifeshire, B Bencleuch in the Ochils, and C the Calton Hill at Edinburgh.

Observed Angles.

	°	′	″	
A =	83	55	46.06	by 7 observations.
B =	45	38	55.19	by 6 observations.
C =	50	25	15.83	by 20 observations.
Sum =	179	59	57.08	
180 + ϵ =	180	0	2.79	
Error			5.71	

Though the preceding method (§ 8) be more strictly scientific, yet for ordinary purposes this error may be distributed among the angles simply as the reciprocal of the number of observations, thus:—

$\frac{1}{7} = 0.143$; and $0.360 : 5''.71 :: 0.143$:	correction of A	$+ 2''.27$
$\frac{1}{6} = 0.167$ 0.167 :	correction of B	$+ 2.65$
$\frac{1}{20} = 0.050$ 0.050 :	correction of C	$+ 0.79$
0.360	The whole correction	$+ 5.71$

Hence

	°	′	″	
A =	83	55	48.33	corrected.
B =	45	38	57.84	
C =	50	25	16.62	
	180	0	2.79	

Now, if from each of these one-third of ϵ, or one-third of $2''.79$, be subtracted, there will remain for the *mean angles*

	°	′	″
A =	83	55	47.40
B =	45	38	56.91
C =	50	25	15.69
Sum—ϵ =	180	0	0.00

Also the length of the arc in feet, opposite the angle A, is 146335.0.

1. As	sin A	83° 55′ 47″.40 . . .	9.9975581
Is to	sin B	45 38 56.91 . . .	9.8543501
So is	*a*	146335.0 feet . . .	5.1653483
To	*b*	105230.2 feet . . .	5.0221403

2. As	sin A	83° 55′ 47″.40 . . .	9.9975581
Is to	sin C	50 25 15.69 . . .	9.8869119
So is	*a*	146335.0 feet . . .	5.1653483
To	*c*	113423.3 feet . . .	5.0547021

almost as exact as the more complex method.*

* The logarithmic tables we would recommend for these calculations are Captain Shortrede's first impression in 1844—because, in our opinion, one half of the time arguments have been injudiciously struck out, thus obliging the surveyor to recompute them constantly, while the proportional parts to hundredths of seconds are rendered useless. For the more common calculations, *Galbraith's Mathematical Tables* will be found generally useful.

Example 1.—Corrected by the method of the reciprocal of the squares of the number of observations.

$\frac{1}{44.5} = 0.02245$. $\quad 0.05273 : +\ 5''.71 :: 0.02245 : +\ 2''.43$

$\frac{1}{36} = 0.02778 \qquad\qquad 0.02778 : +\ 3.01$

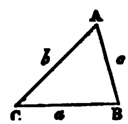
$\frac{1}{400} = 0.00250 \qquad\qquad 0.00250 : +\ 0.27$

$\Sigma = 0.05273$

	Observed Angles.	Cor.		Spherical Angles.	Mean Angles.
	° ′ ″	″		° ′ ″	° ′ ″
A =	83 55 46.06	+ 2.43	=	83 55 48.49	83 55 47.56
B =	45 38 55.19	+ 3.01	=	45 38 58.20	45 38 57.27
C =	50 25 15.83	+ 0.27	=	50 25 16.10	50 25 15.17
	179 59 57.08	5.71		180 0 2.79	180 0 0.00
				s = 2.79	
				$\frac{1}{3}s$ = 0.93	

These would give $b = 105230.29$, and $c = 113423.0$ feet.

Example 2.—Let A be Benlomond in Stirlingshire, B Cairnsmuir upon Deuch in Galloway, and C Knocklayd in Antrim, in Ireland, we shall have, by the more complex method (§ 8) of distributing the errors



Observed Angles.

	° ′ ″	
A =	56 43 28.58	by 3 observations.
B =	79 42 28.69	by 1 observation.
C =	43 34 36.89	by 2 observations.
Sum =	180 0 34.16	

	° ′ ″	
A by 1st obs.	56 43 29.97	s
2d "	27.04	s'
3d "	28.72	s''
m =	56 43 28.58	

Hence

$$\begin{aligned} m-s &= +\ 1''.39, & (m-s)^2 &= 1''.9321 & \quad N &= 3 \\ m-s' &= -\ 1.54, & (m-s')^2 &= 2.3716 & \quad N^2 &= 9 \\ m-s'' &= +\ 0.14, & (m-s'')^2 &= 0.0196 & & \\ S^2 &= \quad . \quad . \quad . \quad . & & 4.3233 & & \end{aligned}$$

$\frac{N^2}{2\,S^2} = \frac{9}{8.6466} = 1.041 =$ weight, and $\frac{2\,S^2}{N^2} = 0.9607 =$ the reciprocal of the weight. In like manner

C 1st obs.	$43°\ 34'\ 38''.36\ s$				
2d „	$35.43\ s'$				
m	$= 43\ 34\ 36.895$				
$m-s$	$= +$	$1.465,$	$(m-s)^2 = 2.1462,$	$N = 2$
$m-s'$	$= -$	$1.465,$	$(m-s')^2 = 2.1462,$	$N^2 = 4$
S^2	$= \ . \ . \ . \ . \ .$		4.2924	

$\frac{N^2}{2\,S^2} = 0.4660 =$ weight. Reciprocal $\frac{2\,S^2}{N^2} = 2.1462$. There being only one observation of the angle B, its weight cannot be computed like those of the other angles. Its weight must either be assumed or estimated by a comparison with those of the other angles. As the reciprocals of the weights in the other two angles are inversely as the squares of the number of observations nearly, this may also be estimated in the same ratio, and $\frac{2\,S^2}{N^2} = 8.6155$ nearly.* Whence the sum $= 0.9607 + 2.1462 + 8.6155 = 11.7224 = \Sigma g$, according to which the error must be applied by distributive proportion as in last example.

The spherical excess must now be computed by the formula

$$s'' = \frac{R''\,a}{r^2} = F^2\,a \sin 1'' \quad . \quad . \quad . \quad (1.)$$

in which a is the area of the triangle in square feet, F the factor from Table XIX. to convert feet into arcs. If the mean radius of curvature of the earth be taken, which, for moderate triangles, will be sufficient, formula (1) becomes

$$s'' = \frac{a}{2122300000} \text{ nearly} \quad . \quad . \quad . \quad (2.)$$

The log of 2122300000 is 9.3268079, and its arithmetical complement is 0.6731921, a constant log, to which the log a, the log of the area of the triangle, being added, will give the log of s'', the spherical excess in seconds, to be applied as formerly indicated. For 2 a const. log $= 0.372162$.†

* $0.9607 \times 3^2 = 0.9607 \times 9 = 8.6463$
$2.1462 \times 2^2 = 2.1462 \times 4 = 8.5848$

Mean to 1^2 or 1 $= 8.6155$

† For a in *toises*, const. log. $\bar{8}.28482$, $2a$ C. L. $= \bar{9}.98380$
a in *mètres*, const. log. $\bar{9}.70518$, $2a$ C. L. $= \bar{9}.40416$

Whence the spherical excess amounts to 1" in about 76 square miles.

From this an easy rule may be derived to find the spherical excess by a simple calculation, or even by the common sliding rule, from a plan of the triangles, to which a scale of miles is adapted for measuring the base and perpendiculars in an approximate manner.

Set 152, on the sliding line of numbers, to the base of the triangle in miles on the fixed line, then opposite to the perpendicular on the slide will be found the spherical excess in seconds and decimals on the fixed line, true to nearly two places of decimals in moderate triangles. Thus, in Example 1, p. 323, set 152 on the slide to the distance of Calton from Bencleuch, 28 miles, measured from the scale to Plate XXIX., then opposite to the perpendicular on this from East Lomond, 15 miles, on the slide will be found 2".77 on the fixed line, nearly the same as stated in the example.

The lines marked A and B on the common carpenter's sliding rule are sufficient for this purpose, when the triangle is not very great.

The triangle now under consideration being large, the more accurate formula (1) will be employed to find ε''.

To mean latitude of the triangle L, about 55½° N.,* and azimuth 45°, there will be found, by the aid of Table XIX., &c.,

Log ½ sin 1″ . . .	4.384545
2 Log F (Table XIX.) to ⅓ $(l + l' + l'')$	5.986630
B = 79° 42′ 28″.7, sin . . .	9.992955
side a = 426794 feet, log . .	5.630402
side c = 352038 feet, log . .	5.546589
ε'' = 34″.763, log . . .	1.541121
⅓ ε'' = 11 .588 . . .	"
By calculation ε is . . .	34.763
By observation it is . . .	34.160
Error of observation . . .	0.603

to be distributed among the observed angles in the ratio of the reciprocals of their respective weights.

* 1. Benlomond, l = 56° 11′ 27″.7 N.
2. Cairnsmuir, l' = 55 15 24.4 N.
3. Knocklayd, l'' = 55 9 46.7 N.

$l + l' + l''$ = 166 36 38.8
L = ⅓ $(l + l' + l'')$ = 55 32 12.9 N.

If extreme precision be required, the spherical excess may be recomputed with the corrected angles and sides.

As 11.7224 :	$-0''.603$: :	0.9607 :	$+0''.050$
	: :	2.1462 :	+ 0.111
	: :	8.6155 :	+ 0.442
Sum =	. . . . . .		0.603

Hence the following spherical angles will be obtained:—

A =	$56°\ 43'\ 28''.58 + 0.05$ =	$56°\ 43'\ 28''.63$
B =	79 42 28.69 + 0.44 =	79 42 29.13
C =	43 34 36.89 + 0.11 =	43 34 37.00
Corrected sum =	$180° + \epsilon$ =	180 0 34.76

It must therefore be remembered that each of these angles is equal to its opposite vertical angle, and not those diminished by the effects of the spherical excess which immediately follow.

If, from each of the spherical angles thus determined, one-third of the spherical excess be deducted, the remainders will be the mean angles, which are to be employed in calculation, as in examples 1 and 2, pages 323 and 325, or combined as follows:—

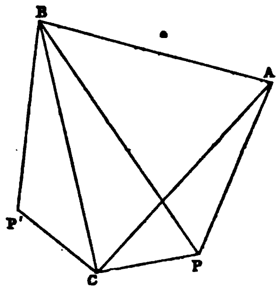
A =	$56°\ 43'\ 28''.63 - 11''.59$ =	$56°\ 43'\ 17''.04$
B =	79 42 29.13 − 11.59 =	79 42 17.54
C =	43 34 37.00 − 11.58 =	43 34 25.42
Sum	. . .	180 0 0.00

Log arc c in feet = log 352037.62	.	5.5465891	α
Log sin A 56° 43′ 17″.04	.	9.9222127	β
Log sin B 79 42 17 .54	.	9.9929511	γ
a . c . log sin C 43 34 25 .42	.	0.1615997	δ
Log arc a 426974.06 feet .	.	5.6304015	$\alpha + \beta + \delta$
Log arc b 502504.47 feet .	.	5.7011399	$\alpha + \gamma + \delta$

In the Trigonometrical Survey, this operation is performed by reducing the spherical angles to those of the corresponding chords, which, in this large triangle, would give precisely the same results. That method requires considerably more labour without almost any corresponding advantage, and is now very generally abandoned. Since the observations by which the angles in Example 1 have been determined were about *six* times more numerous than those in the second, while the error in the former is nearly *ten* times greater than that in the latter, it seems that the laborious calculations depending upon the doctrine of probabilities in such cases may be very well saved, and that the method of distributing the errors proportionally to the reciprocals of the number of observations, as in the first case, is quite sufficient. In the present case, the mean angles would be A=56° 43′ 17″.10, B=79° 42′ 17″. 43, and C=43° 34′ 25″. 47, which would give results not differing much from the preceding.

10. The centre of the instrument should always, when possible, be placed in the vertical line occupied by the axis of the signal. When, however, this cannot be conveniently done, the observed angles must be reduced to it by an appropriate formula.

Let APB be the observed angle to be reduced to ACB, that at the axis of the signal C.



For this purpose it is necessary to measure the distance CP. Let the angle APB=P, BPC=p, the angle of direction reckoned from the observed object on the left to the axis of the signal CP=d, AC=r the distance to the *right*, and BC=l the distance to the *left*,

Then $$C-P=R''\,d\left\{\frac{\sin(P+p)}{r}-\frac{\sin p}{l}\right\} \quad . \quad . \quad . \quad (3.)$$

or $$C-P=R''\,d\sin P\sin(A-p)\div r\sin A \quad . \quad . \quad (4.)$$

in which R″ is 206264″.8, the arc equal to the radius in seconds. When the theodolite cannot be conveniently placed at the same height as the top of the signal observed, the correction of the zenith distance will be

$$d\,\delta=\frac{R''\,d\,h\sin\delta}{D}=\frac{R''\,d\,h}{D}\text{ nearly} \quad . \quad . \quad . \quad (5.)$$

when δ differs little from 90°, in which δ is the observed zenith distance, $d\,h$ the difference between the height of the centre of the circle and the point observed, and D the distance. When the theodolite reads altitudes, and, as is usual in this country, the telescope has a small eccentricity, of which the value is $d\,h$ in the same measure as D, then $d\,a''$ being the correction in altitude, measured by the instrument in seconds

$$d\,a''=\frac{R''\,d\,h\cos a}{D}=\frac{R''\,d\,h}{D}\text{ nearly.} \quad . \quad . \quad . \quad (6.)$$

when a is small and D great, and the height of the instrument above the ground is accounted that of the axis of the vertical circle. When the centre of the station C, the point of observation P, and the signal observed as A, are not all three in the same vertical plane, then as in last formula, designating the angle of direction APC=p, and δ being the zenith-distance,

$$d\,\delta = \frac{R''\,d\,h \cos p \,\cos \delta}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

The same formulæ are applicable when the centre of the station is either before or behind the signal.

In these formulæ the signs of the trigonometrical quantities must be carefully attended to.

EXAMPLE 1. Let P=65° 41′ 6″.5 p=181° 35′ 13″.5, d=155 feet, r=33329.8 feet, and l=74707.5 feet; required the reduction of P to C by formula (3)?

Log R″ . . .	5.314425		
Log d . . .	2.190332		
	+ 7.504757		— 7.504757
P + p=247° 16′ 20″ sin	— 9.964896	p=181° 35′ 13″.5 sin	— 8.442422
$a.c.$ log r . .	+ 5.477167	$a.c.$ log l .	+ 5.126636
1st term — 884″.8 log	— 2.946820	2 term + 11″.8 log	+ 1.073815
2d term + 11 .8			

C ∽ P — 873 .0 = — 14′ 33″.0
P = . . 65° 41 6 .5

C = . . 65 26 33 .5 corrected.

By formula (4.)

Log R″ . . .	5.314425	+
A = + 88° 4′ cosecant	0.000247	+
A — p = 93 31 sine .	9.999181	—
P = + 65 41 sine .	9.959654	+
d = 155 feet log . .	2.190332	+
r = 33329.8 feet, $a.c.$ log	5.477167	+
— 14′ 33″ = — 873″ log	2.941006	—

as before.

Both formulæ may be used as a verification of each other.

Ex. 2. From Allington Knoll the staff on Tenterden Steeple had a depression of 3′ 51″.0, or δ=90° 3′ 51″.0, and the top of the staff was 3.1 feet higher than the axis of the instrument when at that station. On Tenterden Steeple the ground at Allington Knoll was depressed 3′ 35″.0, or δ'=90° 3′ 35″.0, and the axis of the instrument when at this station was 5.5 feet above the ground;

required the corrections of the observed zenith-distances, the lineal distance between the stations being 61781.8 feet?

By formula (5.)

Log R″ .	5.314425	. . .	5.314425
$a \,.\, c \,.$ log D .	5.209139	. . .	5.209169
$dh = +\ 3.1$ feet log	+ 0.491362,	$dh = -5.5$ feet log	— 0.740363
$d = +\ 10''.35$ log	+ 1.014926	$d\delta = -18''.36$ log	— 1.263957

the corrections of the zenith distances sufficiently accurate by the more simple formula, since δ and δ' are so near 90°.

When these corrections are to be applied to the angles between the verticals of two given points, they may be combined as follows:—

Log R″	5.314425
$dh + dh' = +\ 3.1 - 5.5 = 2.4$ feet log	— 0.380211
$a \,.\, c \,.$ log D	5.209139
$\delta + \delta' = -8''.02$ log . . .	—0.903775

the same as $d\delta + d\delta' = +10''.35 - 18''.36 = -8''.01$, of which the application will be subsequently shown or required in reducing observations to the axis of the instrument, or from the signal to the ground.

Ex. 3. A pole 12 feet high, was placed on the top of Bencleuch, at the distance of 146335 feet from the Calton Hill, Edinburgh; What angle would the pole subtend at the Calton?

By formula (5) we have $d\,\delta = \frac{12\ \text{R}''}{146335} = 16.''3914$

Ex. 4. At the Calton station, the angle between the East Lomond and Kellie Law was observed to be 41° 33′ 23″.04 from pile to pile. A pole was placed to the eastward of the pile 6 feet perpendicularly to the line joining Calton and East Lomond; what was the change of the angle at Calton produced by this change of position, the distance from Calton to East Lomond being 89704.5 feet?

By formula (5)	$d\,\delta = \frac{6\ \text{R}''}{89704.5} =$	—0° 0′ 13″.80
		41 33 23 .04
Reduced angle, . . .		41 33 9 .24

Ex. 5. At the Calton station, what angle will a pole 1 foot in diameter subtend on Aller Muir, at the distance of 28667.93 feet?

By formula (5) $d\,\delta = \frac{\text{R}''}{28667.93} = 7.''195$

Ex. 6. Suppose the smallest angle at which two lights can, by the naked eye, be seen distinctly separate is 3′ 20″=200″, then $\frac{R''}{200''}=\frac{206265''}{200''}$ nearly=1031. Hence, the nautical mile being about 6086 feet, then 6086÷1031, or even 1000, gives 6.086 feet, the direct distance between two vertical or horizontal lights when they can be seen distinctly separate at the distance of one nautical or geographical mile. This doubled will be that at two miles, &c. From this it appears two lights will appear separate when the distance between them is one-thousandth part of the observer's distance from them.

Ex. 7. The distance of Brown Carrick Hill from Ayr Spire is 26556 feet—the new spire is 20 feet to the eastward of the old; what would be the change of the angle at Brown Carrick Hill, referred successively to the new and old spires?

$$\text{Ans. } \frac{R'' \times d}{D} = \frac{206265'' \times 20}{26556} = 155''.4 = 2'\ 35''.4$$

11. In measuring horizontal or vertical angles in reference to terrestrial objects, if the atmosphere is not sufficiently clear, it is difficult to intersect the signals with the necessary accuracy. In this case, an instrument called a heliotrope is generally used to reflect the sun's image in the direction of the observer. I have found the usual reflecting horizon of coloured glass set in a frame, turning on a horizontal and on a vertical axis to obtain any requisite inclination, very convenient for this purpose. The proper direction of the sun's image may be given by a circular piece of polished block tin or brass, with a circular hole of three or four inches in diameter in it, stuck in the groove of the usual offset-staff, through which hole the station of the observer must be seen, while, by reflection from the glass, the ring of the perforated disc must be illuminated.

12. To conduct a series of observations, either on land or at sea, for the purposes of surveying, &c., the following general remarks will be found useful.

1°, To record the state of the barometer and thermometer three or four times a-day, more especially when making observations.

2°, To take altitudes carefully on objects useful for time, &c., from three to six hours distant from the meridian.

3°, To find the error of chronometer as often as possible, to be able to compute the correct time of transit of the sun, stars, &c., by it, for latitude by circum-meridian altitudes, &c.

4°, To observe objects having equal altitudes nearly, to the north and south of the zenith, to destroy the effects of errors in the instruments employed, such as bias of axis, errors of division, glasses, artificial horizon, &c. The same method should be pursued for the accurate determination of time by selecting objects to the east and west.

5°, In the case of marine surveys, to observe on land as often as safe and convenient, with the best instruments for time, latitude, and longitude, by lunars, moon-culminating stars, occulations, &c.

6°, To choose a station somewhat elevated, free from woods, jungle, &c., so that, with ordinary care, surprise by the natives will be impossible or difficult.

7°, To take magnetic bearings of well-defined and conspicuous objects whenever practicable, from points well determined in latitude and longitude. If convenient, angular observations with the theodolite and other instruments would be better.

8°, To repeat your observations if possible at least *three* times, to guard against mistakes, which, even with the greatest care and experience, will sometimes happen. To make one or more assistants take observations along with you, and to receive their reports without communicating your own. If there be such a difference as to indicate a decided fault somewhere, the observations ought to be repeated till the cause of the discrepancy be removed.

9°, To make such calculations only as may be absolutely necessary to carry on a connected series of useful observations.

10°, To keep regular and clearly written note-books, on a systematic plan, in which everything is recorded; so that you or any mathematician or astronomer may be enabled to deduce fair conclusions at any future period. These books must be all properly ruled, titled, and numbered, for future reference. Marks and abbreviations should be all carefully recorded and explained.

13. These general views being premised, it will now be necessary to enter into the practical details. It is hardly possible, in these operations, to divest the formulæ entirely of an algebraical character in some cases, though it will be done as often as possible. One of the processes in trigonometrical surveying is the determination of the latitude. This operation is most simply performed by a meridian altitude or a zenith distance; and if a circumpolar star be selected, the result will be independent of the exact position of the star, because the latitude, in that case, is equal to half the sum of the altitudes of the star above and below the pole, corrected for the effects of refraction by Table V. When a zenith sector like that belonging to the Board of Ordnance is used, the stars must be

selected near the zenith, and consequently little error is to be feared from the effects of refraction, while the great power of its telescope, and the general accuracy of its construction, render a single observation by it a close approximation to the truth. When, however, the smaller classes of instruments are employed, it then becomes necessary to repeat the observations near the meridian, reducing those taken at a short distance, such as about ten or fifteen minutes, to what they would have been on it, from a knowledge of its distance from that circle in time, the approximate latitude and declination of the object observed. In this way the results from smaller instruments become nearly equivalent to those of the greater, since as many observations may be taken by the former in one day as by the latter in ten.

ON FINDING THE LATITUDE.

14. The most easy and ready way of finding the latitude is by a meridian altitude of a celestial body whose declination is known. Should the object have a sensible diameter, like the sun or moon, the altitude or zenith-distance of the lower or upper limb, or, what is superior, both are alternately observed, and, by the application of several corrections, that of the centre is obtained.

When reflecting instruments, such as the sextant, repeating circle, &c. with an artificial horizon, are employed, the arc read off must, from the nature of the instruments, be halved before the corrections are applied. At sea, since the lower limb of the sun, moon, or the centre of a planet or a star, is generally brought to the visible horizon, the dip from Table I. must be subtracted before the corrections from Table II. &c. are taken. At land, a meridian altitude of the sun, moon, or a planet, must be corrected for refraction, parallax, and semidiameter, but not for dip. At sea, the same corrections are applied after the dip has been subtracted. All these may be found by the following tables and the Nautical Almanac. The refraction constitutes the whole correction of a fixed star at land. At sea, the dip must be previously subtracted.* If the instrument does not give the zenith-distance, it may be found by taking the complement of the altitude to 90°, denominated *north* or *south*, according as the observer is north or south of the object.

Now, when on the upper meridian, if the zenith-distance and declination be of the *same name*, their *sum* is the latitude; but if

* The method of applying all these corrections is given in the explanation of the tables, and illustrated by the following examples.

of *contrary names*, their *difference* is the latitude, of the same name as the greater. When on the meridian under the pole, the sum of the altitude and the polar distance will be the latitude of the same name as the elevated pole.

EXAMPLE 1. At Pladda Light, in longitude 20ᵐ 30ˢ W., 20 feet above the level of the sea, on the 15th of August 1836, the following observations of the sun's lower limb referred to the sea-horizon, were made with a pocket-sextant, within two or three minutes of the meridian, and both sides of it in succession; required the latitude, the index error being + 2′ 0″?

1st observation, sun's lower limb . .	48° 20′ 0″
2d	21 0
3d	21 0
4th	20 0
Mean of the four	48 20 30
Index error	+ 2 0
Dip to 20 feet (Table I.) . . .	— 4 24
	48 18 6
Correction to alt. 48° (Table II.) . .	— 0 46
True alt. of sun's lower limb . .	48 17 20
Sun's semi-diameter by Naut. Almanac .	+ 15 49
True alt. of sun's centre . . .	48 33 9
	90 0 0
True zenith-distance . . .	41 26 51 N.
Sun's declination by Naut. Almanac* . .	13 59 5 N.
True observed latitude	55 25 56 N.

2. At Edinburgh, on the 13th of March 1841, the following observations were made with a Dollond's sextant and an artificial horizon, one-half of which was made by a contact of the lower limbs, and the other by a contact of the upper, alternately, while the artificial horizon was reversed at the middle of the observations.

	h.	m.	s.
Time of apparent noon . . .	12	0	0
Equation of time at Edinburgh . .	+	9	41
Error of watch	±	0	0
Time of transit by watch . . .	12	9	41

In regard to reading, when the zenith-distance does not exceed 90°, I have caused on a pocket-sextant to be engraved numbers on

* See explanation of Table XXX.

the arc, commencing with 0° at 90°, in an order the reverse of that usually adopted, and likewise on the vernier, so that I read zenith-distances in place of altitudes, even with the sextant, in such cases as it may appear more convenient; or I may read alternately, in different series of observations, both ways, as a check upon each other, to avoid mistakes in the reading. In the present instance, however, the zenith-distance, when doubled, exceeded the limits of the instrument, as with the artificial horizon must be the case, and therefore double altitudes were necessarily taken.

Barometer 30.3 *inches.* *Thermometer* 53° *Fahr.*

	Times of Observation.		Double Altitudes.
	h. m. s.		° ′ ″
1st observation .	11 58 35	. .	61 42 50 l. l.
2d	12 1 53	. .	62 59 0 u. l.
3d	12 7 12	. .	62 53 30 u. l.*
4th	12 9 58	. .	61 56 50 l. l.
5th	12 15 16	. .	62 52 30 u. l.
6th	12 19 20	. .	61 52 40 l. l.
Means . .	12 8 42.3	. .	62 22 53.3
Long. W. .	+ 12 43.5,	Half .	31 11 26.7
Error of watch .	0 0.0	. .	90 0 0.0
Greenwich M. T.	12 21 25.8	Zenith-dist.	58 48 33.3 N.

The refraction must now be computed by Table V.:

Zenith-distance observed 58° 48′.6. log $\delta\ \theta$ . .	1.9832
Barometer $b = 30^{in}.3$, log (Table VI.) . .	0.0043
Thermometer $\tau = 53°$, log (Table VII.) . .	9.9999
Thermometer $t = 53°$, log (Table VIII.) . .	9.9973
Refraction . . $r = 96''.5$ log. .	1.9847

Sun's parallax (Table XII.) = — 7 .4 = π

$r - \pi$ = cor . .	89 .1=0° 1′ 29″ .1
Mean zenith-distance . .	58 48 33 .3 N.
Corrected zenith-distance . .	58 50 2 .4 N.

It is now necessary to apply the reduction of the different particular observations by Table XVII., to reduce each to what it would have been had it been made precisely on the meridian, which is most concisely done by grouping the whole together. For this purpose let Δ be the required zenith-distance upon the meridian,

* Here it was inconvenient to change the limbs, on account of clouds, but the same number of observations was made on each limb.

and δ that obtained as above, then, in order to reduce δ to Δ, we have the following formula:—*

$$\Delta = \delta - 2\sin^2 \tfrac{1}{2} t \cos l \cos d \operatorname{cosec}(l-d) + 2\sin^4 \tfrac{1}{2} t \{\cos l \cos d \operatorname{cosec}(l-d)\}^2 \cot(l-d) \quad (6.)$$

in which t is the time from the meridian either before or after transit, in mean solar time if the sun be observed, but in sidereal if a star, l the latitude, and d the declination, reckoned *minus* if of a contrary name to l. This distinction may be avoided by substituting the zenith-distance for $l-d$ or $d-l$. Between the zenith and the elevated pole, $l-d$ becomes $d-l$. Between the elevated pole and the horizon, the sign of d must be changed from — to +, and the sum $d + l$ subtracted from 180°. Or let x be the colatitude, and p the polar distance; δ the zenith-distance on the upper meridian, and δ' that under the pole, $\delta = x-p$, and $\delta' = x + p$. It is clear that $2 \sin^2 \frac{1}{2} t$ is the versine of t, and that $2 \sin^4 \frac{1}{2} t$ is half the square of the former, which are designated V and v in the table. To express the reduction in seconds of arc, each of these must be multiplied by R″, an arc equal to the radius in seconds. This is accomplished by the logarithms for V and v at the termination of the table, which include also the division of the sum of the versines by the number of observations, thus simplifying the operation considerably. The computation is performed in the following manner:—

	h. m. s.		m. s.	V	v
Transit by watch	12 9 41				
1st observation	11 58 35,	t =	11 6	11726	1375
2d ...	12 1 53,	t_1 =	7 48	5791	335
3d ...	12 7 12,	t_2 =	2 29	587	4
4th ...	12 9 58,	t_3 =	0 17	8	0
5th ...	12 15 16,	t_4 =	5 35	2967	88
6th ...	12 19 20,	t_5 =	9 39	8863	786
				29942	2588

Now by the formula,

* This formula is easily deduced from elementary investigations, but we are restricted to practice here.

Estimated lat.	55° 56′ 58″ N.	cos	9.748129		
Sun's declinat.	2 51 54 S.	cos	9.999457		
Zenith-dist.	58 48 52 N.	cosec	0.067783	cot	9.781954 } log *f*.
Log F			9.815369 × 2 =		9.630738 }
To 6 observations log for V			7.536274	for *v*	5.235244
V=29942 log *			4.476281	log *v*	3.412964
1st term . .	—67″.293	log	1.827924	log 2d	8.060900
2d term .	+ 0.012				
Reduction . =	— 67.281 =	. . .	—0° 1′ 7″.3		
Corrected zenith-distance			58 50 2 .4 N.		
True meridian zenith-distance . .			58 48 55 .1 N.		
Sun's declination			2 51 53 .6 S.		
True latitude			55 57 1 .5 N.		

By repeating the observations on stars both to the north and south of the zenith, the latitude will be accurately determined.

If the observations for latitude are taken by the mural or transit circle placed truly in the meridian, these are made when the celestial body is in or very near the centre of the field of view, at the intersection of the horizontal and vertical wires; but when the observations are *repeated* near the meridian, an exact knowledge of the time or error of the watch becomes indispensable, in order to find the time of transit by that watch with which the observations are recorded. For this purpose an approximate value of the latitude may be found as shown in the first example, from which, and the following rule, the error of the watch within a few seconds may be obtained. With this error and a good sextant, a nearer approximation to the true latitude may be found, as in Example 2, whence a new determination of the time may be found sufficiently exact to obtain the latitude correctly, if a series of observations, at nearly equal distances from the meridian before and after transit, be employed. The time may also be found by the method of equal altitudes, as shown in the explanation of Table XVIII., whenever the weather is steady, especially in fine climates. In our unsteady climate, absolute altitudes taken at nearly equal distances from the meridian east and west, and as close upon the prime vertical as possible, will prove very satisfactory, and then corresponding observations nearly will not easily be lost.

* Instead of versed sines, arcs in seconds may be taken from Table XXVIII. of *Galbraith's Tables*, or from those of G. H. L. Warnstorff, published at Altona in 1845.

TO FIND THE TIME.

15. Set down under each other the true altitude, polar distance, and latitude. Find half the sum of these three, and the difference between that half sum and the altitude. Then to the log cosecant of the polar distance add the log secant of the latitude, the log cosine of the half sum and the log sine of the difference—half the sum of these four logarithms will be the log sine of half the hour angle from the meridian. In case of determining the time in this manner, it would be convenient to estimate it according to the astronomical method of reckoning, namely, from noon to noon throughout the twenty-four hours. Hence in the forenoon of the civil day, the hour angle thus found must be deducted from 24 hours, and the remainder will be the time past noon of the preceding day, when the sun is the object.

In using a table of reduced versines, such as that given in my collection of mathematical and astronomical tables,* the sum of the four logarithms mentioned above, rejecting tens in the index, will be the hour angle to be taken from the top of the page when the observation is made in the afternoon of the given day, but from the bottom if in the forenoon, to give the time past the preceding noon. This result will be the apparent solar time, to which the equation of time reduced, for the approximate time and longitude, to the corresponding Greenwich mean time (G. M. T.,) according to the directions given in the Nautical Almanac (N. A.,) page 1 of each month, will give the mean solar time (M. T.) at the place of observation.

If a star be the object, the horary angle must be taken from the top of the page, if the star be west of the meridian, but from the bottom if east, to be *always* reckoned west (W.) To this meridian distance add the star's right ascension reduced to the given time, and the complement to 24 hours of the sidereal time at mean noon (S. T. M. N.), reduced by Table XXVI. to the time and place of observation: the sum, rejecting 24 hours as often as possible, will be the mean solar time. If two stars be chosen, one to the east and another to the west, having the same altitudes nearly, any error from a faulty method of observing, or a bias in the instrument,

* Dr Inman has given a more extensive table to every second of time in 24 hours. At the bottom of the page, I have in my copy supplied by the pen proportional parts to every tenth of a second, rendering the calculation of time both easy and accurate.

will be avoided, and they should not have more than 30° of declination, because, from their slow motion even on the prime vertical, stars having great declinations are in this case to be avoided.

The method of observing with a sextant having been already shown, that by the smaller classes of astronomical circles will now be exemplified. That which I generally use is six inches in diameter, having three verniers, each reading 10", and the scale of its level, a fixed one, indicates 2" for each division, and reads from a central zero. The general formula to correct for the readings of the level, when applied to the zenith-distance, is

$$l = \frac{(e-o)\,a''}{2n} \quad . \quad . \quad . \quad . \quad (7.)$$

in which l is the resulting effect, e the sum of the readings at the eye-end of the telescope, o the sum of those at the object-end, a'' the value of one division of the scale of the level, and n the number of observations.

In making $a''=2''$ the preceding formula becomes

$$l = \frac{e-o}{n} \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

by the scale of my circle, the most convenient.

In all cases care must be taken of the sign, according to the rules of algebra. The signs must be changed when the instrument shows altitudes. There are three parallel horizontal wires in the focus of the telescope, at each of which the contact of the sun's limb may be observed. I generally observe the contact of the upper limb only at all the three when the sun is ascending, and then, on reversing the circle, the lower limb. I reverse this order when descending, taking care of the apparent change of position, by an astronomical telescope, which shows objects inverted—that is, I observe the *apparent lower limb* FIRST when the object is *ascending*, the apparent upper first when descending, consequently the contacts are observed at nearly the same altitude, and have the same refraction.

EXAMPLES.—1. On the 11th of August 1836, at Lamlash, in the Island of Arran, in latitude, by estimation, 55° 31′ 56″ N., longitude 20ᵐ 32ˢ W., the following observations on the sun were made to determine the time. The assistant-watch, by which the observations were made, was 28ˢ fast of the chronometer, while the barometer stood at 30 inches, and Fahrenheit's thermometer at 50°.

NOTE.—In the following observations, the scale of the level read to 3″ at Lamlash, and formula (7) was employed to find l, the effects of the level; but at Inchkeith it read to 2″, and formula (8) was employed. See pages 341 and 354.

	Times by Watch, A. M.	Ver.	Z. D.	Level +	Level —
	h. m. s.		° ′ ″	″	″
1.	9 1 49	A	54 42 50	8.5	21.5
		B	43 0		
		C	43 20		
2.	9 7 7	A	53 35 30	22.0	7.5
		B	36 0		
		C	35 30		
3.	9 13 52	A	52 46 30	30.0	1.5 —
		B	46 0		
		C	45 20		
4.	9 17 46	A	52 45 0	15.0	14.0
Mean .	9 10 8.5	B	45 10	75.5	41.5
Watch fast	— 9 43.5	C	45 40	41.5	
Long.	+ 20 32.0 W.				
			53 27 29.2	34.0	
E. G. M. T.	9 20 57.0, l	=	+ 12.8	3	
			53 27 42.0	8)102	
	r	=	+ 1 18.7		
	π	=	— 0 6.7	12.75	

True zenith-distance	53 28 54.0		
	90		
True altitude A . .	36 31 6		
Polar distance . .	74 45 9	cosec	0.015563
Latitude	55 31 56	sec	0.247228
Sum	166 48 11		
Half H	83 24 5.5	cos	9.060360
Difference H — A =	46 52 59.5	sin	9.863300
	20 55 23.7	v. s.	9.186451
Equation of time . .	+ 4 54.3		
Mean time	21 0 18.0		

	h. m. s.
Mean time	21 0 18.0
Time by watch + 12h = ...	21 10 8.5
Watch fast of M. T.	9 50.5
Watch fast of chronometer	— 28.0
Chronometer fast	9 22.5*

* A proportional part of the daily rate, 3s gaining, should properly be applied to the error of the chronometer, here determined at 9h 10m A. M., to reduce it to 10h 29m P. M.; but from more numerous observations, made in the afternoon, it appeared that, after allowing for rate, the exact error did not differ more than a fraction of a second from that stated above.

On the evening of the same day, by a watch 10ˢ slow of the same chronometer, and which was gaining 3ˢ a-day, the following observations were made on *α* Aquilæ.

	Times by Watch.			V.	Z. D.			Level. +	Level. —
	h.	m.	s.		°	′	″	″	″
1.	10	18	10	A	47	9	50	22	10
				B		9	20		
				C		9	40		
2.	10	25	50	A	47	3	10	15	16
				B		3	10		
				C		3	30		
3.	10	33	0	A	47	6	20	18	13.5
				B		5	50		
				C		6	20		
4.	10	39	50	A	47	5	0	12	19.0
Mean .	10	29	12.5	B		5	10	67	58.5
Watch fast	—	9	12.5	C		5	10	58.5	
Long. .	+	20	32.0 W.						
					47	6	2.5	8.5	
G. M. T. .	10	40	32	$l=$	+		3.2	3	
					47	6	5.7	8)25.5	
				$r=$	+	1	2.8		
								+3.2	

Corrected zenith-distance = 47 7 8.5 N.

To find the mean time of transit by Tables XXVI. and XXVII., we have, by the Nautical Almanac, the

		h.	m.	s.
Sidereal time at Greenwich mean noon	$\sigma=$	9	19	55.09
Reduction for long. 20ᵐ 32ˢ W. (Table XXVI.)		+		3.37
Sidereal time at Lamlash M. N. . .		9	19	58.46
Star's right ascension . . .	$s=$	19	42	49.27
Difference, or $s-\sigma$		10	22	50.81
Reduction to $s-\sigma$ (Table XXVII.) .		—	1	42.04
Mean time of transit		10	21	8.77
Error of watch, fast		+	9	12.50
Time of transit by watch . . .		10	30	21.27

	h. m. s.		m. s.	
Transit by watch .	10 30 21			
1st observation .	10 18 10,	$t=$	12 11	$d=8°\ 26'\ 29''.95$ N.
2d	10 25 50,	$t_1=$	4 31	
3d	10 33 0,	$t_2=$	2 39	
4th	10 39 50,	$t_3=$	9 29	

	m. s.		V.		v.	For Rate,	
t =	12 11	.	14126	.	1995	M. So. to sid. T. log	+ 0.002375
t_1 =	4 31	.	1942	.	38	Daily Rate,	
t_2 =	2 39	.	668	.	5	Gaining 3ˢ cor. .	— 30
t_3 =	9 29	.	8560	.	733		
	Sums,		25296		2771	Log for rate,*	+ 0.002345

Latitude	55° 31′ 56″ N. cos 9.752772		
Declination	8 26 30 N. cos 9.995269		
Zenith-dist. δ = 47 5 26	cosec 0.135233	cot	9.968280 } log f
Log F	9.883274	× 2 =	9.766548 } log f
Log V′ for 4	·7.712365	log v_1 =	5.411335
V = 25296 log	4.403052	log v_2 =	3.442636
Log for rate	0.002345		
		log	8.588790
1st term — 100″.240 log . .	2.001036		
2d term + 0 .039			

— 100 .201 =	—0° 1′ 40″.20
Corrected zenith-distance	47 7 8 .50
True meridian zenith-distance . .	47 5 28 .30 N.
Star's declination	8 26 29 .95 N.
True latitude	55 31 58 .25 N.

In this manner the observations may be repeated a sufficient number of times to insure, from a mean of the whole, the requisite accuracy.

When the observations on *stars* are continued for a length of time, the logs of F and f remain nearly constant for the same star, and consequently these may be computed for such a number of stars as may be selected for observation. Indeed, special tables may be drawn up to every ten seconds, and these may be interpolated to every second in t, as was done by myself for α Aquilæ, when I observed at Inchkeith, from which the reduction to the meridian may be made at sight. For this computation special tables are sometimes given, but it may be easily effected by a table of reduced versines employed in the computation of time in last example, or by a table entitled *Rising* in the usual books of navigation.

* This logarithm may be readily taken, by inspection, from *Shortrede's Tables*, No. XXX., first impression in 1844.

Let $F'' = 2\,R'' \cos l \cos d \operatorname{cosec} \delta$ and . . . . . . (9.)

$f'' = 2\,R'' (\cos l \cos d \operatorname{cosec} \delta)^2 \cot \delta$ . . . . . (10.)

be computed for the given star.

For α Aquilæ, at Inchkeith, in latitude 56° 2′ N., in August 1840, when the star's declination was 8° 27′ 7″ N., will be found for a table of log R. V. S.

1. Log F″ = ... 5.489705, log f . 5.324769
For t = 15^m log R. V. S. . 7.029602 × 2 = 4.059204

m'' = 1st term = — 330″.60 log 2.519307 9.383973
n'' = 2d term = + 0 .24

$m'' + n''$ = Red. = — 330 .36

2. For a table of log rising to find m and n in *Galbraith's Table*, XXVIII.
Log m = const. log 0.314425 + log rising.
Log n = const. log 5.013395 + 2 log rising.

Time from meridian, or τ = 0 h. 30 m. 0 s. . 1st C. L. = 0.314425—, 2 C. L. 5.013395 +
log rising . 2.932227 2 log R. 5.864454

m = — 1764″.623 . . log m = 3.246652—, log n = 0.877849 +
n = + 7 .548 . . . . n = + 7″.5483

Continuing this process for every 10ˢ of t, the reduction may easily be found for single seconds by interpolation, which renders this method very easy; and then the smaller classes of circles become in effect nearly equal to the larger, on account of the facility with which observations may be very numerously repeated.

16. In mean latitudes, such as in Britain, observations on the pole-star are very advantageous and convenient for the determination of both latitudes and azimuths at the same time, which may be computed by the following formulæ :—

1. $\tan u = \cos t \tan p = \cos t \cot d$
2. $\sin \lambda = \cos u \cos \delta \sec p = \cos u \cos \delta \operatorname{cosec} d$
3. $l = \lambda \pm u$
4. $\sin r = \sin t \sin p = \sin t \cos d$
5. $\tan m = \tan r \sec \lambda$, or
6. $\tan m = \sin t \tan p \sec \lambda = \sin t \cot d \sec \lambda$

nearly, and more simply than by (4) and (5) combined.

In these formulæ, t is the sidereal time after transit, p the star's polar distance, d the declination, λ the latitude of the foot of the perpendicular arc from the star upon the meridian, and δ the zenith-distance. Also l is the true latitude, in determining which

u is *minus* in the first and fourth quadrants of t, and *plus* in the second and third. In like manner r is the perpendicular from the star upon the meridian, and m the azimuth.

17. If the latitude be previously well known, the azimuth may be found by Napier's Analogies, or from formulæ or rules derived from them. For this purpose let c be the complement of the latitude, and p the polar distance.

$$1.\ \tan \tfrac{1}{2}(m+e)=\cot \tfrac{1}{2} t \cos \tfrac{1}{2}(c \backsim p) \sec \tfrac{1}{2}(c+p),$$
$$2.\ \tan \tfrac{1}{2}(m-e)=\cot \tfrac{1}{2} t \sin \tfrac{1}{2}(c \backsim p) \operatorname{cosec} \tfrac{1}{2}(c+p).$$

Hence $\frac{1}{2}(m+e)+\frac{1}{2}(m-e)=m$, the azimuth of the pole-star from the meridian referred to the horizon. Or let d be the declination of Polaris, &c., and l the latitude of the place of observation,

$$3.\ \tan \tfrac{1}{2}(m+e)=\cot \tfrac{1}{2} t \cos \tfrac{1}{2}(d \backsim l) \operatorname{cosec} \tfrac{1}{2}(d+l),$$
$$4.\ \tan \tfrac{1}{2}(m-e)=\cot \tfrac{1}{2} t \sin \tfrac{1}{2}(d \backsim l) \sec \tfrac{1}{2}(d+l).$$

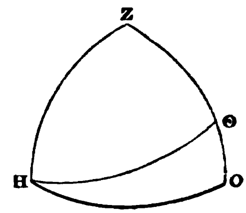
Since d and l remain constant during a series of observations made in one day, while t varies, the logs of the two last factors are constant, and this renders the computation of an azimuth by the pole-star remarkably easy.

$$5.\ \tan m=\frac{\sec l \cot d \sin t}{1-\tan l \cot d \cos t}.$$

A formula, though not well adapted to logarithmic calculation, sometimes employed by the French engineers.

18. In many cases of nautical surveying, the true bearing of any well-defined object at a considerable distance, and on, or nearly on, the same level with the eye of the observer, is required to be determined with a reflecting instrument. To perform this operation, bring the image of the sun to the object, and make its nearest limb accurately to touch the object, while at the same time with another instrument let the sun's altitude be taken. Correct the observed distance for index error, if necessary, and add the sun's semidiameter; the result will be the apparent distance of the sun's centre from the object. In like manner correct the sun's altitude for index-error, dip, and semidiameter, the result will be the sun's apparent altitude. Now, to compute the azimuthal angle between the sun and the object, there will be formed, when the object is on the same level with the eye, a quadrantal spherical triangle H Z ☉, of which the sides are the zenith-distance Z H=90°, the sun's zenith-distance Z☉, and the oblique-observed distance H☉—to find the angle Z at the zenith, which is the difference between the bearings of the object and the sun. Compute the sun's

true azimuth from the altitude in the usual manner, take the sum or difference of these, according to circumstances, as indicated by their relative positions with respect to the meridian, and the true bearing of the object will be determined. If the bearing of the same object be taken with the azimuth compass, the variation of the compass will likewise be obtained. To determine the true bearing in this manner, it must be remarked that the sun's vertical motion should be as great as possible, or his position ought to be near the prime vertical, and that the object to which the sun is referred should be about 90° from the point of the horizon to which the sun is vertical. When this is impossible, the object should be chosen so that the angle which the observed arc or distance makes with the horizon, may not by estimation exceed 45°. When the object is elevated above the level of the eye, it is necessary to observe its altitude, and compute the angle at the zenith from the three sides of an oblique-angled spherical triangle formed by the observed distance, and the zenith-distances of the sun and the object whose azimuth is required; and this is in fact the first part of the method of reducing, by spherical trigonometry, the apparent distance to the true in lunar observations—that is, from the two apparent altitudes and apparent distance, to find the angle at the zenith.



The azimuth of a point or signal by means of the sun or a star, may be found readily when the time is accurately known. In this case there are given the polar distance and the hour angle, or that contained at the pole, to determine the angle at the zenith by the Analogies of Napier.

If the sun be the object, the angle at the pole is the complement of the time to 12^h in the forenoon, but the apparent time itself if in the afternoon. If a star be observed, the angle at the pole is equal to the sidereal time, *minus* the right ascension of the star, or equal to the apparent time *plus* the right ascension of the sun, *minus* the right ascension of the star. The polar angle is *minus* when the star is east of the meridian, *plus* when west.

When extreme accuracy is required in determining the azimuth of a signal, the observations of the angular distance, by the reflecting circle or by Borda's repeating circle, between the star and the signal should be made at the same instant with the zenith-distance of the star and the signal, if convenient, though that of the latter may be made at any time either preceding or following the observations, since, with the exception of refraction, it remains stationary. These distances are the apparent distances as affected by parallax and refraction. If the zenith-distance of the star cannot be conveniently observed at the same time when the angular distance between the star and signal are taken, it may be calculated by spherical trigonometry, as will be now shown, taking care to apply the effects of refraction and parallax to find the apparent zenith-distance with a contrary sign to that used in finding the true.

To find the true and apparent zenith-distance.

Rule.—To twice the log cosine of half the horary angle, add the log cosine of the latitude and the log cosine of the declination; half the sum of these three logarithms will be the log cosine of an arc (2.) If the latitude and declination be of the same name, take half their sum, but if of contrary names, take half their difference; the result will be arc (1.) Find the sum and difference of arcs (1) and (2.) To the log sine of the sum add the log sine of the difference; half the sum of these two logarithms will be the log sine of half the true zenith-distance. To the true zenith-distance apply the corrections with a contrary sign, and the apparent zenith-distance will be found. If the object be near the horizon, a repetition of the last operation may be required, using the last found apparent distance to obtain the necessary corrections, when a table for calculating the refractions to the true zenith-distance, as in Puissant's *Geodesy*, is not at hand.

Example 1.—Brodick, island of Arran, August 26th, 1841, in latitude 55° 35′ 20″ N., longitude 20^m 40^s W., at 4^h 9^m 44. P.M. mean time; what was the true and apparent zenith-distance of the sun, the barometer being at 30.15, and Fahrenheit's thermometer at 62°.6?

	h. m. s.
Mean time by chronometer, ...	4 9 44 P.M.
Equation of time, subtract, ...	0 1 34
Apparent time = t, ...	4 8 10
$\frac{1}{2}t$, ...	2 4 5
Sun's declination, ...	10° 20′ 28″ N.

	h. m. s.			
$\frac{1}{2}t$, . .	2 4 5	cos × 2, . .		19.865942
Latitude,	{ 55 35 20 N.	cosine, . .		9.752146
Declination,	{ 10 20 28 N.	cosine, . .		9.992888
Sum, . .	65 55 48			
Half (1) .	32 57 54			19.610976
Arc (2) .	50 17 2	cosine, . .		9.805488
(2+1) = .	83 14 56	sine, . .		9.996978 }
(2—1) = .	17 19 8	sine, . .		9.473764 }
				19.470742
	° ′ ″			
$\frac{1}{2}z$ . .	= 32 56 13	sine, . .		9.735371
	2			
			° ′ ″	
True Z.D. .	65 52 26	of centre. True alt. of C.	24 7 34	
Correction .	— 1 59			
App. Z.D.C. .	65 50 27	App. cent. alt.	24 9 33	
Sun's S.D. .	± 15 51			
App. Z.D, *l.l*, .	66 6 18	App. alt. *l.l*,	23 53 42	
App. Z.D, *u.l*, .	65 34 36	App. alt. *u.l*,	24 25 24	

Example 2.—Ayr, August 12th, 1841, in latitude 55° 27′ 55″ N., longitude 18^m 31^s W., at 8^h 58^m 5^s mean time; required the apparent zenith-distance of *α* Aquilæ, the barometer being at 29^{in}.86, and the thermometer at 61° Fahrenheit.

		h. m. s.
α Aquilæ, R A, ...		19 43 5.30
Sidereal time at Ayr mean noon, . .		9 23 6.30
Difference, ...		10 19 59.00
Reduction, Table xxvii., ...	—	1 41.57
Mean time of star's transit, ...		10 18 17.43
Mean time of observation, subt., . . .		8 58 5.00
Interval in mean time before transit, . .		1 20 12.43
Reduction of this interval to sidereal time, Table xxvi.,	+	13.17
Interval in sidereal time, or *t*, . . .	=	1 20 25.60
$\frac{1}{2}t$,	=	0 40 12.80

	h. m. s.		
½ t, .	= 0 40 12.8	cosine × 2= .	19.986560
Latitude, .	55 27 55 }	N. cosine, . .	9.753512
Declination,	8 27 19 }	N. cosine, . .	9.995254
Sum, .	63 55 14		19.735326
Half (1), .	31 57 37 }		
Arc (2), .	42 29 43 }	cosine, . .	9.867663
Sum, .	74 27 20	sine, . .	9.983817 }
Diff., .	10 32 6	sine, . .	9.262062 }
			19.245879
	° ′ ″		
½ z= .	24 48 57	sine, . .	9.622940
	2		
	49 37 54	True altitude, .	° ′ ″ 40 22 6
Correction,	— 0 44	. . . .	+ 44
App. Z.D, .	49 37 10	Apparent altitude,	40 22 50

When the altitudes are very low, as within two or three degrees of the horizon, the correction must be taken out to the last found apparent zenith-distance, and again applied to the true, to get the apparent zenith-distance correctly.

When the azimuth is found by observations on the pole-star, or similar methods, the horizontal circle must be read at the same time with the vertical, in order to compare the azimuth of the star with a referring lamp, and from this, at any convenient opportunity, other conspicuous points selected as stations in the general survey.

To find the time of the pole-star's greatest eastern and western elongation.

BRODICK, August 10, 1848.

		h. m. s.
Sidereal time of Greenwich mean noon, . .		9 16 21.22
Reduction to longitude, 20^m 37^s W., Table xxvi., .	+	3.39
Sidereal time at Brodick mean noon, σ . .		9 16 24.61
*'s right ascension, August 10, 1848, or s, .		1 5 23.51
Difference, or s—σ, ...		15 48 58.90

	h. m. s.
Brought forward, $s-\sigma$, ...	15 48 58.90
a to $s-\sigma$, Table xxvi., ...	— 2 35.47
Mean time of *'s transit over the meridian, .	15 46 23.43
6^h of sidereal time in mean time, . . .	+ 5 59 1.02
Mean time of *'s greatest western elongation, .	21 45 24.45
12^h of sidereal time in mean time, . .	+11 58 2.05
Mean time of *'s greatest eastern elongation, .	9 43 26.50

Otherwise,

	h. m. s.
Transit of Aries, ...	14 41 14.02
Reduced for longitude, 20^m 37^s W. . . .	— 3.38
Reduced time of transit, ...	14 41 10.64
Stars, R A + 6^h, ...	7 5 23.51
Sum, ...	21 46 34.15
Reduction from Table xxvii. to 7^h 5^m $23^s.5$, .	— 1 9.69
Mean time of western elongation, . .	21 45 24.46
This increased by 12^h sidereal in mean time, .	11 58 2.05
Mean time of eastern elongation, as before, .	9 43 26.51

To these times of elongation, taken each as a transit, any number of observations may be reduced by Table XVII., in the same manner as is shown in page 337, formula (6), which enables the observer to repeat his observations for a considerable time before and after the greatest elongation, thereby diminishing the errors of reading, level, &c.

1. To find the latitude by circumpolar stars.

When the observer possesses an instrument capable of observing in daylight, the pole-star or any other circumpolar star that does not pass the under meridian too near the horizon to obtain the refraction with sufficient accuracy, he may obtain the latitude very accurately by observing the greatest and least altitude or zenith-distance on the same day, or on different days, and then correcting them for the effects of refraction. Half the sum of the true altitudes thus corrected will be the latitude; or half the sum of the zenith-distances will be the co-latitude. The method of proceeding is exactly similar to the methods already explained.

Example 1.—At Barcelona, on the 17th of December 1793, Mechain observed the zenith-distance of *Polaris* near its upper culmination to be 46° 49′ 3″.08, by the repeating circle of Borda; the reduction to the meridian was —10″.02, the refraction +1′ 1″.10, consequently, the true meridian zenith-distance was 46° 49′ 54″.16.

Again, on the 27th of December 1793, he observed the zenith-distance of Polaris near its lower passage of the meridian to be 50° 23′ 10″.48; the reduction to the meridian was +6″.90, the refraction +1′ 10″.82: whence the true meridian zenith-distance was 50° 24′ 28″.18.

	° ′ ″
Hence, first meridian zenith-distance was .	46 49 54.16
Second, ...	50 24 28.18
Sum, ...	97 14 22.34
Half sum or co-latitude, . .	48 37 11.17
And latitude, ...	41 22 48.83

Example 2. To find the azimuth by circumpolar stars. With a good instrument observe the greatest and least angular distance of the pole-star from the vertical plane in which the given line is situated; half the sum of these two measures will be the angle required.

By the *Ordnance Survey*, vol. i. pages 245, 291, in the year 1793, the direction of the meridian at Dunnose was determined by the polar star from corresponding observations at the western and eastern greatest elongations referred to Brading Staff.

		° ′ ″
1. April 28.	Afternoon, at western elongation,	24 4 23.00
,, 29.	Morning at eastern elongation, .	18 24 0.00
Mean angle between the meridian and staff, .		21 14 11.50
		° ′ ″
2. May 12.	Afternoon at western elongation,	24 4 29.50
,, 13.	Morning at eastern elongation, .	18 23 53.25
Mean angle between the meridian and staff, .		21 14 11·375
		° ′ ″
The mean of 1 and 2, is . .		21 14 11.4375

3. By calculations from independent observations. Taking the latitude of Dunnose at 50° 37′ 8″ N., which is near the truth, (since, by a subsequent recomputation, it has been found to be 50° 37′ 7″

N.,) and the apparent polar distance of the pole-star, on different days were obtained,

April 21, 1793.	P. D.	1° 47′ 57″.2	azimuth, .	2° 50′ 11″.2
,, 22,		1 47 57.4	. .	11.5
May 5,		1 48 0.7	. .	16.8

which, applied to the observed angles, give 3, 4, and 5.

4. General results.—1.	. . . . .	21° 14′ 11″.500
2.	. . . . .	11.375
3.	. . . . .	10.050
4.	. . . . .	10.500
5.	. . . . .	10.450
Mean of the whole, or bearing of staff,	.	21 14 10.775

5. Deanhill, Dunnose, Brading Staff.—1.	.	55° 58′ 38″.500
2.	.	38.750
Mean,	. . . . .	55 58 38.625
Bearing of Brading Staff,	. .	N. 21 14 10.775
From Dunnose, Deanhill bears,	.	N. 34 44 27.850 W.

6. Bearing of Brading staff,	. . .	N. 21° 14′ 10″.775 E.
Brading Staff, Dunnose, Butserhill,	.	N. 0 15 31.500 W.
From Dunnose, Butserhill bears,	.	N. 20 58 39.275 E.

In the application of those principles to practice, as has been already shown, the time of transit of the star must be ascertained in terms of the clock or chronometer by which the observations are made. Then, six hours after transit, by a sidereal clock, the star will be at its greatest western elongation; and twelve hours after this last, at its greatest eastern, or $5^h\ 59^m\ 1^s.02$, and $11^h\ 58^m\ 2^s.05$, in succession, or $17^h\ 57^m\ 3^s.07$ after transit of mean time, allowing for rate during the interval, as previously given by calculation.

In the example for the latitude, the observations were not made on the same day, though, to avoid any inaccuracy in the change of position of the star, it would have been better if they had: but certainly the corrections of the mean places of the stars for aberration and nutation, can now affect their true position very slightly.

The direction of the meridian may also be very accurately determined by the transit instrument, for the truly placing of which in the meridian, see the description and use of that instrument in a following part of this work. It may also be determined nearly by the transit, as with Ramsden's theodolite, by observing the greatest western and eastern deviations referred to marks, such as staffs by day or lamps by night, placed after a few trials nearly in the proper position. When the results prove satisfactory, the angular distance between the western and eastern marks, being bisected, will give the direction of the meridian required.

If the horizontal screws have a motion of about 45′ on each side of the meridian, this operation would be readily performed in one position of the instrument without shifting its feet screws.

19. I shall now proceed to illustrate these rules and formulæ by practical applications. Having determined the error and rate of my chronometer, as previously exemplified, the following observations were made at Inchkeith Lighthouse, to determine the latitude and direction of the meridian by the pole-star. For this purpose, I resided on the island a few days, during which I made several observations on the heights in the vicinity of Edinburgh, as well as some on the latitude by the sun and α Aquilæ, for which a special table was drawn up in the manner already explained, by which the reduction to the meridian for the distance t was made by inspection. I chiefly trusted those made on the 21st of August upon the pole-star, which I continued to observe from about 10 o'clock in the evening to 1 o'clock next morning. During this period I completed eight series of double observations, reversing the circle each time, or sixteen single observations, comprehending forty-eight readings of the verniers on each circle, accompanied by the times of observations, and the readings of the level. The circles used were six inches in diameter, having each three verniers reading to 10″, and a level whose divisions each indicate 2″. Having made these preliminary remarks, so that everything relative to my operations may be fully understood, I shall record the first series of observations, and perform the computations at full length, so as to render the whole operation clear and distinct to every one having a very ordinary knowledge of such subjects. In this record, b signifies the height of the English barometer, τ the temperature by its attached or interior thermometer, t the temperature of the air by the exterior thermometer, in degrees of Fahrenheit; Ver., the different verniers of the respective circles marked A, B, C: Z. D. the observed zenith-distance; H. D. the horizontal angular distance to the referring lamp; I. M. T. Inchkeith mean time; G. M. T. Greenwich mean time; S. T. G. M. N. sidereal time at Greenwich mean noon, &c.*

* The calculations will be most readily performed by Shortrede's *Logarithmic Tables*, first impression of 1844. The later impressions, having half the time arguments struck out are thereby far less convenient.

POLARIS.

Inchkeith, August 21, 1840, b = 29in..70, τ = 64°, t = 64°. Error of chronometer at $10\frac{1}{4}$h P.M. fast 1m 58s.4, rate 19s.7 gaining.

Obs.	Times. h. m. s.	Ver.	Z. D. ° ′ ″	Ver.	H. D. ° ′ ″	Level. + e	Level. — o
1.	10 9 5	A	33 34 40	A	61 54 55	24	22
		B	34 25	B	54 50		
		C	34 30	C	54 55		
2.	10 19 35	A	33 27 20	A	54 60	22	23
		B	27 20	B	54 45		
		C	27 30	C	54 55		
Means,	10 14 20		33 30 55.8		61 54 53.3	46	45
Error Cr.	— 1 58.4	l =	+ 0.5			45	
I.M.T.	10 12 21.6		33 30 56.3			2) 1	
Long. I.	+ 12 32.0	r =	+ 37.2				
G.M.T.	10 24 53.6	T.Z.D.	33 31 33.5 = δ			+ 0″.5 = l	

Refraction.

Z. D.	33° 31′	log δ θ.	1.5876
b =	29.70	log .	9.9956
τ =	64°	log .	9.9994
t =	64	log .	9.9875
r =	37″.2	. log	1.5701

Sidereal Time, t.

	h.	m.	s.
S. T. G. M. N.	9	59	29.28
I. M. T. . . .	10	12	21.60
Red. to G.M.T.	+ 1		42.65
Sid. time obs.	20	13	33.53
Star's R. A. .	1	2	36.78
t . . . . . =	19	10	56.75

p	1° 32′ 33″.8	tan	8.4302701	sec	0.0001575	tan	8.4302701
t	19h 10m 56s.75	cos	9.4837861		. . . .	sin	9.9788502
u —	0° 28′ 12″.28	tan	7.9140562	cos	9.9999854		
	δ =		33° 31′ 33″.5	cos	9.9209762		
λ	56 30 9 .00		. . . .	sin	9.9211191	sec	0.2581391
l	56 1 56 .72	m*	=		N. 2° 39′ 40″.15 E	tan	8.6672594

In the same manner the remaining parts of the series were computed by the formulæ in § 16.

But since the star moves in a circle, the mean zenith-distance and horizontal angle is not that at the middle of the arc described during the interval between the observations, as it ought to be,

* Formula (5), page 345, would give m = N 2° 39′ 40″.04 E. nearly the same.

and, by investigation, the following corrections must be applied to the latitude and azimuth.

$$d\,l = (p'' \sin 1'' \cos t + p''^2 \sin^2 1'' \cos 2\,t \cot \delta)\,f \quad . \quad . \quad (11)$$
$$= p'' \sin 1'' \cos t\,f\text{; in this case nearly}$$
$$d\,m = p'' \sin 1''\, t \sec l\,f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

in which f is the factor, from Table XVII.

$p'' = 1°\ 32'\ 33''.8 = 5553''.8$	log	. . . .	3.744590
$\sin 1''$	log	. . . .	4.685575
$p'' \sin 1''$	log	. . . .	8.430165
$l = 56°\ 1'\ 59''.6$ . .	log secant	. . . . . .	0.252812
$p'' \sin 1'' \sec l$ . .	log	. . . .	8.682977

	h. m. s.
1st Observation	10 9 5
2d Observation	10 19 35
Mean . .	10 14 20

Mean — 1st = 5 15 V = 2623 log 3.418798
Log V′ for one observation 8.314425

Log f	1.733223	. . . .	1.733223
$t = 19^h\ 10^m\ 56^s.75$ cos	9.483786	sin t	9.978850
$p'' \sin 1''$ log . .	8.430165	$p'' \sin 1'' \sec l$. log . . .	8.682977
$d\,l = +\ 0''.44$ log .	9.657174	$d\,m = +\ 2''.48$ log . .	0.395050

In this way the corrections were computed for the whole series, and the final results are as follows:—

No.	l'		dl'		l		Successive values.
	° ′ ″		″		° ′ ″		° ′ ″
1	56 1 56.72	+	0.44	=	56 1 57.16	. .	56 1 57.16 N
2	2 1.14	+	0.33	=	2 1.47	. .	1 59.31
3	2 2.04	+	0.69	=	2 2.73	. .	2 0.44
4	2 2.00	+	0.74	=	2 2.74	. .	2 1.03
5	1 56.95	+	0.30	=	1 57.25	. .	2 0.26
6	1 58.30	+	0.21	=	1 58.51	. .	1 59.98
7	1 59.70	+	0.29	=	1 59.99	. .	2 0.12
8	2 0.10	+	0.49	=	2 0.59	. .	2 0.06
	Reduction to centre of tower for 25 feet						— 0.24
	True latitude						56 1 59.82

Azimuth of Light.

No.	H. D.		m′		d m		m
	° ′ ″		° ′ ″		″		° ′ ″
1	61 54 53.3	+	2 39 40.15	+	2.48	=	N 64 34 35.93 E
2	62 0 40.5	+	2 33 49.35	+	1.24		31.09
3	62 8 3.7	+	2 26 35.20	+	2.16		41.06
4	62 14 58.0	+	2 19 31.10	+	1.92		31.02
5	62 21 15.0	+	2 13 24.80	+	0.69		40.49
6	62 27 7.4	+	2 7 27.60	+	0.43		35.43
7	62 33 0.0	+	2 1 37.20	+	0.52		37.72
8	62 40 14.5	+	1 54 17.20	+	0.66		32.36

Mean azimuth, ...	N 64 34 35.64 E
Reduction to centre of tower, ...	— 35.70
Azimuth at centre of tower, . . .	N 64 33 59.94 E
Angle to Observatory, ...	134 9 7.30
Observatory bears from lighthouse, . .	198 43 7.24
	180 0 0.00
Observatory bears from Inchkeith, . .	S 18 43 7.24 W
Convergence of the meridians c'' . .	— 2 21.50
Inchkeith bears from Observatory, . .	N 18 40 45.74 E

The method of computing c' will be subsequently given.

The Trigonometrical Survey station is S. 27° 27′ W., distant 6.855 feet from the centre of the dome or pillar, therefore,

To the bearing of Inchkeith above, . . .	N 18° 40′ 45″.74 E
Add reduction, ...	+ 6.74
Inchkeith bears from Trigonomet. Survey station,	N 18 40 52.48 E
By Trigonometrical Survey, ...	18 40 53.50
Mean, ...	18 40 52.99

20. Having shown the method of preparing the observed horizontal angles for computation, of fixing the latitude of any selected point, and the bearing of another from it, I shall now give a few rules and formulæ for deducing from these, and an extended triangulation, the latitude, longitude, and azimuth of the principal points of the series, reserving the computation of heights to a succeeding part of this work.

In deducing latitudes, longitudes, azimuths, and heights geodetically, it is necessary to be enabled to convert readily any distance measured in feet on the earth's surface into arcs; and hence the radius of curvature of the measured arc, in any given position on the terrestrial spheroid, is required by the principles of the conic sections.

Now, the radius of curvature is to an arc R″, equal to the radius in seconds, as the distance in the same measure with the radius of curvature is to the corresponding arc in seconds. Let A″ be the required arc in seconds, corresponding to A, any measured arc on the earth's surface in feet, to which r is the radius of curvature,

$$\text{I.} \quad r : A :: R'' : A'' \text{ or } A'' = \frac{R''}{r} \times A \quad . \quad . \quad . \quad . \quad (13)$$

Wherefore, if M be the factor to convert a curvilineal distance on the meridian into seconds of arc; P that on the perpendicular to it; and O that on any oblique arc, making an angle α with the miredian; then, if a denote the radius of the equator, b the polar semiaxis, e the eccentricity, and l the latitude:

$$\text{II.} \quad M = \frac{R''}{a^2 b^2}(a^2 \cos^2 l + b^2 \sin^2 l)^{\frac{3}{2}} = \frac{R''}{a(1-e^2)}(1 - e^2 \sin^2 l)^{\frac{3}{2}} \quad (14)$$

$$\text{III.} \quad P = \frac{R''}{a^2}(a^2 \cos^2 l + b^2 \sin^2 l)^{\frac{1}{2}} = \frac{R''}{a}(1 - e^2 \sin^2 l)^{\frac{1}{2}} \quad . \quad (15)$$

$$\text{IV.} \quad O = M \cos^2\alpha + P \sin^2\alpha = P\frac{1 - e^2(1 - \cos^2 l \cos^2\alpha)}{1 - e^2} \quad . \quad (16)$$

From these formulæ Tables XIX., XX., and XXI. have been computed, the coefficient for terrestrial refraction n, in the two last, having been taken equal to 0.08 or $\frac{1}{12.5}$ of the intercepted arc, which is a sufficient approximation to the truth in ordinary atmospheric circumstances.

21. Previously to the determination of heights trigonometrically, the curvilineal distance, or its chord at the level of the sea, ought to be augmented for the height of the lower station, since the radii from the centre through their summits diverge proportionally to that height. This correction may be obtained from the following formula, or the results derived from it arranged in a table. Let K be the chord of the augmented arc A at the height h, derived from the arc a at the level of the sea, then

$$\text{V.} \quad \text{Log } K = \log a + \frac{M h}{\varrho} - \frac{M a^2}{24 \varrho^2} = \log a + m h - p a^2 \quad . \quad . \quad (17)$$

From this formula Table XXIV. was computed. The number S is the difference of the log secant of half the angle v between the verticals and log $p\,a^2$, which contributes to greater accuracy in considerable heights. I shall now give the necessary formulæ and rules to find latitudes, longitudes, and heights geodetically. In this formula the *second term*, in the right-hand side of the equation, gives the reduction from the level of the sea to the height h, and the *third term*, the reduction of the arc to the chord, as may be readily seen.

Explanation of Symbols, with their Values.

A = the measured arc in feet on the surface of the Terrestrial Spheroid.	
R'' = an arc equal to the radius in seconds, . .	log 5.3144251
a = the radius of the equator in feet, . . .	log 7.3206165
b = the polar semiaxis in feet, ...	log 7.3191664
$e=\left(\frac{a^2-b^2}{a^2}\right)^{\frac{1}{2}}=\frac{\{(a+b)(a-b)\}^{\frac{1}{2}}}{a}=0.0815815$ . . .	log $\bar{2}$.9115918
$\epsilon=\frac{1}{2}e^2+\frac{1}{8}e^4+$ &c. $=\frac{1}{300}$ = ellipticity, . . .	log $\bar{3}$.5228787
$f=a\,e=1706900$ feet, ...	log 6.2322083
$c=a-b=a\,\epsilon=69742$ feet, ...	log 4.8434944

l = the given latitude farthest from the equator.
l' = the required latitude nearest the equator.
λ = the latitude of the foot of the perpendicular from the required point, upon the meridian passing through the given point.
z = the given azimuth.
z' = the required azimuth.
Δl = the difference of latitude.
Δp = the difference of longitude.
Δz = the difference of azimuth or convergence of the meridians passing through the given and required points.

Making $\frac{A\,R''}{a}=a''$ we shall have, from an investigation that cannot be conveniently given here,

(1.) $\Delta l=-a''(1+2\epsilon-3\epsilon\sin^2 l)\cos z+a''^2\,\frac{1}{2}\sin 1''\tan l\sin^2 z$ (18)*

(2.) $\Delta p=a''(1-\epsilon\sin^2 l)\sin z\sec l-a''^2\sin 1''\sin z\cos z\tan l\sec l$ (19)

(3.) $\Delta z=a''(1-\epsilon\sin^2 l)\sin z\tan l'+a''^2\,\frac{1}{2}\sin 1''\sin z\cos z$. (20)

These are the principal formulæ generally required. In addition to these, that for determining an oblique arc o may be added,

(4.) $\Delta o=a''(1-\epsilon\sin^2 l+2\epsilon\cos^2 l\cos^2\alpha)$ . . . (21)

Log sin 1″ = 4.685575, log ½ sin 1″ = 4.384545

* Formula (18) above is sufficiently correct for moderate distances not exceeding 30 or 40 miles. For greater, one term ς'' at least more is required, $\varsigma''=-a''^3\sin^2 1''\tan^2 l\sin^2 z\cos z$; or, taking advantage of the preceding part of the computation, $\varsigma''=m'\,r''\sin 1''\tan l$ nearly, in which m'' and r'' are the results of the first and second terms of the formula.

Introducing the values of M, P, and O, of which the logarithms are given in Tables XIX., XX., and XXI., according to the directions given along with them, making first r'' equal to the reduction of λ to l, derived from the last part of formula (18,) given in Tables XXII. and XXIII., and solving the spherical triangle after determining the required latitude, these formulæ become—

$$(5.)\ \Delta l = -\mathrm{A\,M}\cos z + r'' - \varrho'' = \mathrm{A\,M}\cos m - r'' + \varrho'' \quad . \quad (22)$$

$$(6.)\ \Delta p = \mathrm{A\,P}\sin z \sec l' = \mathrm{A\,P}\sin m \sec l' \quad . \quad . \quad . \quad (23)$$

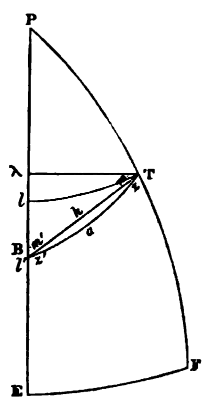
$$(7.)\ \Delta z = \Delta p \sin \tfrac{1}{2}(l + l') \sec \tfrac{1}{2}(l - l') \quad . \quad . \quad . \quad . \quad (24)$$

$$(8.)\ r'' = p''^{2}\, \tfrac{1}{2} \sin l'' \tan l \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

In north latitudes, the azimuth z is generally reckoned from the south towards the west or east, and is the supplement of m, or that reckoned from the north, in the application of which attention must be paid to the signs. Indeed, in some operations, the azimuth is reckoned from the south westwards round the whole circle, in accordance with which the arguments to Tables XIX., XX., and XXI., have been so given.*

PRACTICAL RULES.

22. To illustrate the method of employing these formulæ and tables in calculation, let P be the north pole in this instance, E a point in the equator, B a point of which the latitude and longitude are known, T another place whose bearing and distance from B are given, and from these the latitude and longitude of T and the azimuth of B from T are required. Also, let P B E be the meridian passing through B, P T F the meridian passing through T, P B T the azimuth denoted by a in the formulæ, or m' or z' in the tables, B T the distance or curvilineal arc a in feet, of which the chord is k, T λ a perpendicular from T, the required point upon the meridian passing through the given point B, the distance from the foot of which to the equator,



* It would perhaps be better to reckon the azimuth *always* from the elevated pole, whether in north or south latitude, because the *sign* of the cosine would give the sign of application to m'' in every case, while r'' must be considered always negative.

measured by E λ, is the latitude of λ; l' the latitude of the place nearest the equator, l that of the more distant, and T l the parallel of latitude passing through T, making E l the latitude of T, or that required. The very small arc λ l, called the reduction of λ to l in Tables XXII. and XXIII., must always be subtracted from λ to give l'.

If this small arc exceeds the limits of the tables, it may be computed. For this purpose, it may be observed that if p'' be the perpendicular arc, then p''=A P sin z, the argument to find r'' from the tables. But, independent of the tables,

$$r'' = A^2 P^2 \sin^2 z \tfrac{1}{2} \sin 1'' \tan l = p''^2 \tfrac{1}{2} \sin 1'' \tan l \quad . \quad . \quad (26)$$

the formula from which the tables were constructed, and may supply their place in cases beyond their limits.

It must likewise be observed that B λ is a small arc of the meridian to be added to the given latitude in proceeding towards the pole, or subtracted when receding from it, to give the latitude of the foot of the perpendicular λ, the argument for taking the log P from the tables. The argument to obtain log M is half the sum of the latitudes approximately, or $\frac{1}{2}$ $(l+l',)$ to be derived from a provisory calculation, in order to get the mean latitude between the given stations. The number of *minutes* to be added to the smaller latitude l', or subtracted from the greater l, to get $\frac{1}{2}$ $(l+l',)$ may be computed by the following rule.

To the constant log 5.914630, add the log of the meridian-distance in feet, the sum will be the log of half the difference of latitude in minutes, or $\frac{1}{2}$ $(l—l')$ to be added to l', or subtracted from l, to give $\frac{1}{2}$ $(l+l')$ the middle latitude sufficiently near the truth for taking log M from the tables.

1. By a provisory calculation from the rule just given, or by a repetition of the more accurate method now to be shown, if thought necessary, find the middle latitude, or $\frac{1}{2}$ $(l+l'.)$

2. To the logarithm of the curvilineal distance, or arc a, add the log cosine of the azimuth, or m, and the log M from Table XIX., answering to the mean latitude, or $\frac{1}{2}$ $(l+l',)$ the sum will be the logarithm of an arc of the meridian in seconds m'', to be added to the latitude l' if approaching the pole, but subtracted from l if receding from it, the sum or difference will give λ, the latitude of the foot of the perpendicular upon the given meridian from the point in that required.

3. To the log of a add the log sine m, the azimuth, the log P

answering to λ, the sum will be the log p'', the perpendicular arc in seconds.

4. To the constant log 4.384545 (the log $\frac{1}{2}$ sin 1″) add log tan λ and twice the log p'', the sum will be log r'', the reduction of λ to l *always subtractive.* This may be also taken from Tables XXII. or XXIII., if within the limits of the tables. It may be observed, that four times r'', answering to $\frac{1}{2}$ p'', will be the reduction to p'' nearly, which will extend the table, and the results will not differ much from the truth. This, at least, will be a check to calculation.

5. To the log p'' add the log secant l', the sum will be the log $\Delta\, p$, the difference of longitude, which, properly applied to the longitude of the place of observation, will give the longitude of the point required.

6. To log $\Delta\, p$ add log sine $\frac{1}{2}$ $(l+l')$ and the log secant $\frac{1}{2}$ $(l-l',)$ the sum will be the log $\Delta\, z$, the convergence of the meridians of the given and required points, which, added to the azimuth m', at the latitude nearest the equator, will give m, or rather z, the azimuth at the latitude farthest from it, and *vice versa.*

7. To the log O, answering to the middle latitude and given azimuth α, from Table XIX., add the log of the given distance a, the sum will be the log of the intercepted arc in seconds, which measures the angle between the verticals of the given points. If the log O be taken from Table XX., the result will be angles of the verticals diminished by the effect of refraction, taken at 0.08, of the intercepted arc. The log. O from Table XXI. is the log of $\frac{1}{2}$ ϱ $(1+n^2,)$ employed in the computation of heights by the depression of the horizon of the sea, the mean value of n being 0.08 as before. By these rules, the position of any number of points may be fixed; but in practice a different arrangement is frequently followed.

Suppose a parallel to the meridian of Edinburgh and to its perpendicular to be drawn through each station, we have the bearings and distances of the other stations from such parallels, calculated by means of a right-angled plane triangle, of which the distance or hypotenuse, and the bearings or one angle, are given, to find the other two sides. Thus, let k be the distance, m the azimuth, and ι the spherical excess, we have strictly a triangle deviating slightly from a right-angled triangle, when the spherical excess is applied, but in all ordinary cases of practice the latter may be safely omitted. Now, if x be the distance from the parallel to the perpendicular on the meridian in feet, y the distance from the parallel to the meridian also in feet, introducing ι, we have—

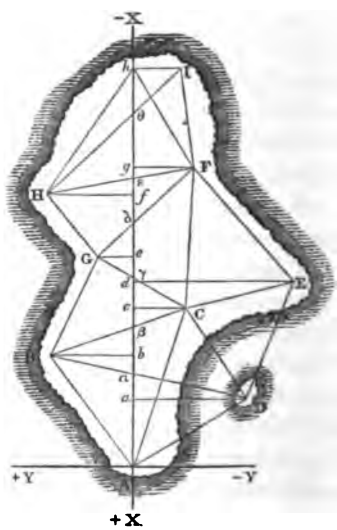
1. $x = k \cos (m - \frac{2}{3} \epsilon)$. 2. $y = k \sin (m - \frac{1}{3} \epsilon)$

Omitting ϵ, as is the general practice, and

3. $x = k \cos m$. 4. $y = k \sin m$.

This may be permitted, because each determination of a point is an independent operation, and is not affected by an accumulation of errors.

23. I shall now give a general outline of the method of conducting the survey of a country or of an island on the preceding principles. In this case, it is necessary to determine the latitude, longitude, and direction of the meridian of any convenient point A, as has already been shown, with reference to a side of one or more of the triangles, such as *b* A B, or *c* A C, &c. It will then be necessary to throw a series of judiciously chosen triangles over the surface of the island and adjacent islets as may be near its coasts, such as A B C, C B G, &c., so as to embrace the chief features of the whole island. These points must next be referred to the principal meridian by means of perpendiculars let fall from each point upon it, thus forming the abscissæ +X,—X, &c. to the south and north of the point A, and ordinates parallel to the perpendicular to it +Y,—Y, &c. to the west and east of the same meridian. These are represented by A *a*, A *b*, &c., and D *a*, B *b*, &c., by drawing temporary parallels to +X,—X, &c., +Y,—Y, &c., throughout the whole compass of the survey; those abscissæ to the south of A being conventionally reckoned positive, those to the north negative; while those ordinates to the east of A are considered negative, and those to the west positive. If a distance as A B cannot be deduced from an adjacent survey with sufficient precision, then a fundamental base in some convenient situation must be measured with great care, and connected with some of the sides trigonometrically, from which the sides of the whole series of triangles must be deduced by calculation as formerly shown. This is, for the sake of distinction, called the *primary* triangulation, in which the sides of



the triangles extend from about 30 to 50, or even occasionally to 100 miles. These larger triangles are next broken down into a smaller class, called the *secondary*, whose sides are limited to about 10 or 15 miles, in which the angles may be measured with somewhat inferior instruments. The intermediate points are subdivided by ten or twelve inch circles into triangles of 3 or 4 miles a side, and then filled in by the five-inch theodolite, the surveying compass, and the chain, which may be called the tertiary triangulation, and concluding process.

TABLE I.

THE FOLLOWING IS THE TRIANGULATION OF THE PRECEDING FIGURE, EXPRESSED IN NUMBERS, WHICH IS CHIEFLY HYPOTHETICAL, BUT CONVENIENT FOR ILLUSTRATION.

Bearing of the two Primary Sides in reference to the Meridian :—
—XAB = N. 36° 37′ 14″.5 W.—XAC = N. 17° 35′ 34″.0 E.

No.	Tri.	Mean Angles.	Log. sines.	Log. opp. sides.	Opposite sides in feet.
		° ′ ″			
	A	54 12 48.5	9.9091286	4.4617086	28094
1	B	73 43 0.3	9.9822202	4.5348000	34261
	C	52 4 11.2	9.8969450	4.4495247	28153
		180 0 0.0			
		° ′ ″			
	A	40 40 0.8	9.8140212	4.3494135	22357
2	D	87 0 31.6	9.9994079	4.5348000	34261
	C	52 19 27.6	9.8984415	4.4338338	27154
		180 0 0.0			
		° ′ ″			
	B	43 59 45.6	9.8417399	4.3043397	20153
3	G	86 19 49.6	9.9991087	4.4617086	28954
	C	49 40 24.8	9.8821656	4.3447655	22119
		180 0 0.0			
		° ′ ″			
	D	53 47 55.5	9.9068452	4.3445495	22108
4	C	71 30 40.5	9.9769851	4.4146893	25983
	E	54 41 24.0	9.9117097	4.3494135	22357
		180 0 0.0			
		° ′ ″			
	C	69 30 8.0	9.9715939	4.4738080	29772
5	E	66 25 34.0	9.9621538	4.4643703	29132
	F	44 4 18.0	9.8423331	4.3445494	22108
		180 0 0.0			
		° ′ ″			
	C	64 55 8.0	9.9569885	4.4395695	27515
6	G	73 31 24.0	9.9817893	4.4643703	29132
	F	41 33 28.0	9.8217591	4.3043397	20153
		180 0 0.0			
		° ′ ″			
	G	81 14 8.2	9.9948991	4.4789415	30126
7	H	64 30 38.6	9.9555267	4.4395695	27515
	F	34 15 13.2	9.7503987	4.2344414	17157
		180 0 0.0			
		° ′ ″			
	H	34 37 40.0	9.7545339	4.3253720	21153
8	I	54 1 35.6	9.9081038	4.4789415	30126
	F	91 20 44.4	9.9998802	4.5707180	37215
		180 0 0.0			
		° ′ ″			
	H	46 45 33.85	9.8624197	4.3869745	34375.2
9	h	64 12 17.85	9.9544138	4.4789415	30126.0
	F	69 2 8.30	9.9702553	4.4947829	31245.2
		180 0 0.0			

TABLE II.

BEARINGS AND DISTANCES, WITH THE CORRESPONDING NORTHINGS AND SOUTHINGS, AND EASTINGS AND WESTINGS.

No.	Tri.	Mean angles.	Dist.	Mer. Arc.	Perp. Arc.	
			Feet.	Feet.	Feet.	
	AC	N 17° 35′ 34″.0 E	34261.0	N 32658.56	E 10355.38	—
1	CF	N 4 15 18.0 E	29132.0	N 29051.71	E 2161.46	—
	FI	N 8 35 16.4 W	21153.0	N 20915.83	W 3158.70	+
				N 82626.10	E 9358.14	—

MIDDLE SERIES.

No.	Tri.	Mean angles.	Dist.	Mer. Arc.	Perp. Arc.	
	AB	N 36° 35′ 14″.5 W	28153.0	N 22595.66	W 16793.68	+
2	BG	N 25 39 59.6 E	22119.0	N 19936.52	E 9580.46	—
	GH	N 35 25 22.2 W	17157.0	N 13981.19	W 9944.30	+
	HI	N 45 26 19.2 E	37215.0	N 26112.73	E 26515.67	—
				N 82626.10	E 9358.15	—

WESTERN SERIES.

No.	Tri.	Mean angles.	Dist.	Mer. Arc.	Perp. Arc.	
	AD	N 58° 15′ 34″.8 E	27154.0	N 14284.91	E 23092.87	—
3	DE	N 19 4 1.9 E	25983.0	N 24557.47	E 8488.04	—
	EF	N 39 49 0.1 W	29772.0	N 22867.78	W 19064.01	+
	FI	N 8 35 16.9 W	21153.0	N 20915.83	W 3158.70	+
				N 82625.99	E 9358.20	—

FIRST EASTERN SERIES.

No.	Tri.	Mean angles.	Dist.	Mer. Arc.	Perp. Arc.	
	AD	N 58° 15′ 34″.8 E	27154.0	N 14284.91	E 23092.87	—
	DC	N 34 43 53.6 W	22357.0	N 18373.68	W 12737.51	+
4	CE	N 73 45 25.9 E	22108.0	N 6183.79	E 21225.49	—
	EF	N 39 49 0.1 W	29772.0	N 22867.78	W 19064.01	+
	FI	N 8 35 16.4 W	21153.0	N 20915.83	W 3158.70	+
				N 82625.99	E 9358.16	—

SECOND EASTERN SERIES.

FINAL RESULTS.

		North.		East.
	1.	82626.10		9388.14
	2.	82626.10		9358.15
	3.	82625.99		9358.20
	4.	82625.99		9358.16
Means,	N	82626.04	E	9358.16

TABLE III.

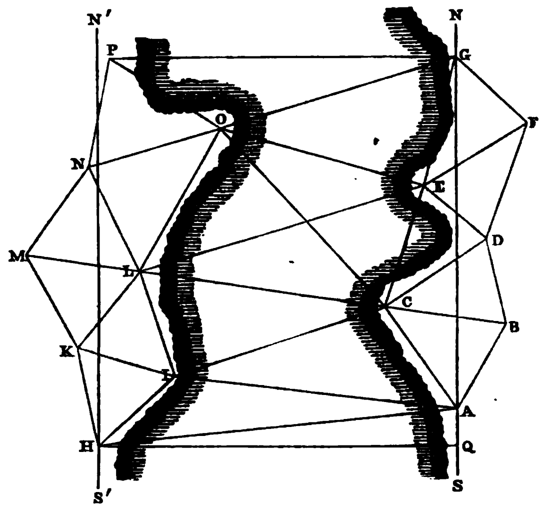
OF THE POINT A., THE ASTRONOMICAL LATITUDE IS 55° 26′ 40″ N., LONGITUDE 5° 10′ 20″ W.; THE FIXED POSITION FROM WHICH THE OTHERS ARE DERIVED GEODETICALLY BY MEANS OF THE PRECEDING RESULTS, AS EXPLAINED IN THE SUCCEEDING PAGES.

No.	Tri.	Latitudes N.		Longitudes W.	
		° ′ ″		° ′ ″	
1	A	55 26 40.00	N	5 10 20.00	W
2	B	55 30 22.42	N	5 15 11.83	W
3	C	55 32 1.77	N	5 7 20.23	W
4	D	55 29 0.63	N	5 3 38.89	W
5	E	55 33 2.52	N	5 1 11.11	W
6	F	55 36 48.02	N	5 6 42.70	W
7	G	55 33 39.04	N	5 12 26.23	W
8	H	55 35 56.79	N	5 15 19.41	W
9	I	55 40 14.25	N	5 7 37.21	W

The signals, to be well observed, should be about $\frac{1}{10000}$ of the distance in height, and the diameter of the bottom $\frac{1}{4}$ of the height. To be readily seen, the angle, subtended by the height, should not be less than 20″. Now, tan 20″ = 0.0001, whence the rule is derived.

24. In a similar manner may a survey of the adjacent coasts of a strait, firth, or river be completed, and the bearings and distances of corresponding points on opposite sides be laid down, whether they be visible from each other or not. This may be readily done in various ways, one of which is, to run two parallels or two meridians of known distance from each other, as may be most convenient under given circumstances, and, by finding the position of each station on its own meridian—that is, its distance on the meridian from a given point in it—and the perpendicular from it upon that meridian, then these will afford the means, by the solution of a triangle, to find the bearings and distances of all or any of them, in such directions as it may be thought necessary or convenient to lay down soundings, leading marks, dangers, &c.; by which means the nautical surveyor will be enabled to complete his chart in a satisfactory manner.

Let NS, N′S′ be the two conventionally chosen meridians by one or more surveyors, whose operations embrace the opposite shores of a river or strait, where it is possible and safe to have the necessary piles and staffs erected on shore, then the perpendicular distance HQ being found by observations taken on purpose, the points A, C, E, G; H, I, L, O, P, may be referred to their respective meridians NS, N′S′, as in the following figure. By the solution of the right-angled plane triangle HQA, right-angled at Q, having AQ, QH given, the angles QAH, AHQ may be found, together with the side AH. Hence the angle IHA may be found, consequently with the sides IH, HA, and the contained angle IHA, the side IA may be found whether the point A be visible from H and I or not. Now the angle CAQ being known, and IAQ having been found by



computation, the angle CAI becomes known. Whence, with the given sides IA, AC, and the contained angle IAC, the side IC may be determined.

It is clear that this method, combined with others easily deduced, may be followed through the whole series, as in the preceding example; from which the form and contour of the shores and distances on which soundings, dangers, &c., should be placed or laid down on a chart are readily inferred. Should the survey be carried on in a foreign country, or barbarous shores, where, from danger, the necessary marks cannot be safely erected on shore, the masts of lighters, boats, and barges, properly secured, may be used as signals, especially if they have polished frusta of cones of zinc-plates, or sheets of block-tin fixed to the mast-head. These may have the greater diameter about nine inches, the less six, and the height twelve, or in these proportions nearly, greater or less according to the distance. These will reflect the sun's image readily to the observer, even in thick weather, whence the angles will be obtained in an easy and satisfactory manner, when the observer and the objects are in a proper position, the time of which must be estimated and carefully watched.

If the lines of reference assumed are parallels of latitude, they will continue equidistant, but if meridians, they will converge towards the pole, and diverge towards the equator, and the distance between them will vary as the radius of the parallel. In nice operations of considerable extent, this variation cannot be neglected, though in those of smaller magnitude, it will be so inconsiderable, as, in ordinary circumstances, to be of little consequence. To take this into account when thought necessary, let R be the radius of curvature of any parallel whose latitude is l,

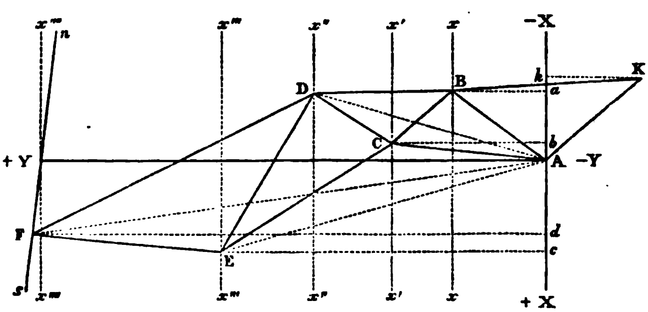
$$R=\frac{a \cos l}{(1-e^2 \sin^2 l)^{\frac{1}{2}}}=a \cos l\,(1+\epsilon \sin^2 l) \text{ nearly} \qquad (26)$$

Whence, by computing R for the parallels of l and l', the distance at l may be reduced to that at l'.

In surveying large rivers, for considerable distances, this method must always be followed when any approach to accuracy is desired; the method of surveying by the common theodolite and *plotting* being quite inadequate to secure the requisite precision. Prony's map of the river Po is a good example of this method of surveying.

25. If, however, a parallel to the primary meridian be assumed, the operation will be more simple, as it will be unnecessary to compute the convergence in feet, or their distance at different latitudes, while the latitudes, longitudes, and azimuths may still be readily found by the preceding rules; and this is the method generally adopted.

Let XX be the meridian passing through the Observatory of Edinburgh, A, YY a perpendicular to it, $x\,x$, $x'x'$, &c., parallels to XX, passing through the stations, B Bencleugh, C Bencampsie, D Benlomond, E Goatfell, F Cairn Aird in Islay, and K Kellylaw



in Fife. Hence there are formed the triangles ABC, BCD, DCE, and EFD, which are treated as already directed, pages 324, &c.

Having determined the bearing of Bencleugh, or angle BAX = a, and the distance AB = k, there may be found A $a = -x$ and B $a = +y$, and so on to the last triangle AFd; from which the absciss Ad = $+x$, and Fd= $+y$ are obtained from a combination of all the intermediate triangles computed in a similar manner, and the results are stated in a table.

Though the signs in the following examples are those employed by many engineers, especially on the Continent, yet, in my opinion, it would probably be better to make those of *x positive* when they increase the latitude, and negative when they diminish it. The lines $x\,x$, &c., being all parallel to XX, are therefore, it must be recollected, not meridians. The latter meet at the poles, and consequently are inclined to one another at certain angles. Thus the meridian $n\ s$ is inclined to XX at an angle n Y x''' of 2° 34′ 17″, which is called the convergence of the meridians, Δz, and varies with the latitudes and difference of longitude as computed by formula (6) page 359, and recorded for each station in the table, if thought necessary. It must be properly applied to the bearings, such as BAa, so as to get the bearing of A from B, called technically, especially by marine surveyors, *the back bearing*.

Having established these general principles, we shall now illustrate the whole by practical examples. By our operations at Inchkeith, in latitude 56° 1′ 59″.82 N., longitude 12ᵐ 32ˢ W., we have found that Edinburgh Observatory bears S. 18° 43′ 7″.24 W., distant 30,272 feet; it is therefore required to find the latitude and longitude of Edinburgh Observatory, and thence the position of the Trigonometrical Survey Station, in order to connect these triangles with it, so that the results in our examples may be comparable with those in the survey.

	° ′ ″
Inchkeith Light, latitude, l . . .	56 2 0 N.
Edinburgh Observatory, latitude, l' . .	55 57 16 N.
Sum, or $l + l'$	111 59 16
Half, or $\frac{1}{2}(l + l')$	55 59 38
Difference, or $l - l'$	4 44
Half, or $\frac{1}{2}(l - l')$	2 22
	° ′ ″
Edinburgh Observatory, longitude, . .	3 10 46 W.
Inchkeith Light, longitude, . . .	3 7 56 W.
Difference, or Δp	2 50

These being points pretty well known, their latitudes and longitudes will therefore turn out, by a geodetical computation, the same nearly as stated above.

This operation is performed by the formulæ given at page 359, or the subsequent practical rules, by the aid of the Tables XIX., XX., XXI., XXII., and XXIII.; for the method of using which tables their explanation must be consulted.

$\frac{1}{2}(l+l')=55° 59' 38''$	log M = 7.9937224	λ gives log P =	7.9928141
$z = 18\ 43\ 7.24$	cos 9.9763985	sine	9.5063992
A = 30272 feet	log 4.4810411		4.4810411
$m'' = -0°\ 4'\ 42''.59$	log 2.4511620, $p''=1'35''.6$ log		1.9802544
$l = 56\ 1\ 59.82$			
$\lambda = 55\ 57\ 17.23$			
$r'' = -\ 0.03$*			
$l' = 55\ 57\ 17.20$	secant		0.2519306
$\Delta p = 0\ 2\ 50.68$	log		2.2321850
L′ 3 7 56.00	$\frac{1}{2}(l+l')=55° 59' 38''$	sine	9.9185430
L = 3 10 46.68	$\Delta z = 0°\ 2'\ 21''.49$ log		2.1507280
	$z = 18\ 43\ 7.24$		
	$m' =$ N 18 40 45.75 E.		

Hence l', the latitude of Edinburgh Observatory, is 55° 57′ 17″.20 N.,† longitude L = 3° 10′ 46″.68 W., and the bearing of Inchkeith

* This correction, r'', may be readily taken from Table XXIII. in general. It may also be computed by the formula (8), page 359.

Log $p'' \times 2 =$	3.96051
Log $\frac{1}{2}$ sin 1″	4.38455
Tan λ	0.17028
$r'' = 0''.033$	8.51534

which is always small, when the difference of longitude is not great. Here r'', being very small, ϵ'' is insensible. It may be remarked, that ϵ'' is *additive* when the latitude is *increasing*, subtractive when decreasing, as shown at page 372, when our method of performing the calculations is employed. If, however, the distance does not exceed 30 or 40 miles, it is almost insensible, and may be generally neglected. A like remark may be made relative to longitudes and azimuths.

† The latitude observed with the mural circle is 55° 57′ 23″.2, exceeding that above by 6″; which, in a paper published in *Jameson's Edinburgh Journal*, in 1841, the Author endeavoured to show, arose from local attraction. This cause has been assigned, in many similar cases, to account for analogous irregularities, though, so far as he is aware, was here for the first time attempted to be proved directly from observation, by shifting the position of the circle. The same course has lately been pursued by Colonel Colby at Dunnose, in the Isle of Wight, and at Balta, in Shetland,

Light from Edinburgh Observatory, or m', is N. 18° 40′ 45″.75 E., agreeing with the result in page 356. Hence, as is stated there, from the Trigonometrical Survey Station, near the pillar in the Observatory,—

Inchkeith Light bears . . .	N. 18° 40′ 53″.00 E.
Angle, Inchkeith, Calton, Bencleugh, .	73 16 29.28
Bencleugh bears from Calton Station .	N. 54 35 36.28 W.
Angle, Bencleugh, Calton, Bencampsie, .	28 43 26.76
Bencampsie bears from Calton Station .	N. 83 19 3.04 W.
Bencleugh bears from Calton . .	N. 54° 35′ 36″.28 W.
Benlomond, Calton, Bencleugh, . .	18 36 57.00
Benlomond bears from Calton . .	N. 73 12 33.28 W.
Also, by observation, Kellie Law bears from Calton Station . . .	N. 37° 23′ 4″.00 E.

With the distances in feet from the Calton to these different points, their latitudes and longitudes may be found in the manner just shown.

From the Trigonometrical Survey Station on the Calton Hill, then, there will be obtained

1. Kellie Law bears	N. 37° 23′ 4″.00 E.	distant	135083.5 feet.
2. Bencleugh bears	N. 54 35 36.28 W.	distant	146334.6 feet.
3. Bencampsie bears	N. 83 19 3.04 W.	distant	196909.0 feet.
4. Benlomond bears	N. 73 12 33.28 W.	distant	308307.6 feet.

Though the preceding *data* are sufficient to fix the positions of the respective points recorded, yet we shall treat the whole in a systematic manner as a small arc of a parallel across the country, in order to exemplify the method of conducting such operations, and deducing the results successively from each other.

Commencing at the Trigonometrical Survey Station on the Calton Hill, fixed by our previous deductions, we shall now determine the positions of the places recorded above, beginning with that of Kellie Law.

1. Constant logarithm, p. 360	5.914630
α=N. 37° 23′ E. cosine	9.900144
A=135083.5 feet	5.130602
$\frac{1}{2}(l-l')$ . . . . + 0 8′.8 N. m' log	0.945376

the extremities of the British arc of the meridian, amounting to 3″ or 4″, as noticed in the *Astronomical Society's Monthly Papers* in 1847. The azimuths at our observatories will be affected in a similar manner. For these sources of inaccuracy, there is no other remedy than shifting the position of the instrument to different places, and taking a mean of the results.

½ (l—l′)	Brought over	+ 0° 8′.8
l′	. . . . .	+ 55 57 .3 N.
½ (l+l′)	. . . . . .	56 6.1 N.

This preliminary step is only an approximation to the middle latitude, or ½ (l+l′), in order to get the argument to take the logarithm of the factor M from Table XIX. for converting feet on the surface of the earth into seconds of arc, to determine l accurately when l′ is known.

½ (l+ l′)=	56° 6′ 6″ log M	=7.9937149	λ gives log P,	7.9928071
m=	37 23 4 cos .	9.9001374	sine . . .	9.7833033
A=	135083.5 log .	5.1306020	. . . . .	5.1306020
m″=+0° 17′ 37″.92 log .		3.0244543	p″ =13′ 26″.7 log	2.9067124
l′= 55 57 17.20				
λ= 56 14 55.12				
r″= — 2.36*				
l= 56 14 52.76 secant			. . . .	0.2552382
Δp=—0° 24′ 11″.95 E. log			. . . .	3.1619506
L= 3 10 46·68 W.	½ (l+l′)=56° 6′ 4″.98		. sin	9.9190916
L′= 2 46 34.73 W.	Δz=+ 0° 20′ 5″.15		log	3.0810422
	m=N. 37 23 4.00 E.			

Bearing of Calton, or z=S. 37 43 9.15 W. from Kellie Law.

2. Constant logarithm	5.91463
α= 54° 36′ cosine	9.76289
A=146334.6 log	5.16535
½ (l—l′) + 0° 7′.0	log 0.84287
l′ 55 57 .3	
½ (l+l′)=56 4 .3	

* Log p″ × 2 =	. . . .	—5.81342
Log ½ sin 1″ =	. . . .	4.38455
λ = 56° 15′ tan	. . . .	0.17511
r″ = —2″.36 log	. . . .	0.37308
	m″ log	3.02445
	log sin 1″	4.68557
	log tan λ	0.17511
ε″ = + 0″.02	log	8.25821

r″ + ε″ = — 2 .34 = correction of λ to reduce it to l.

The last term, ε″, is generally so small that it may be neglected.

$\frac{1}{2}(l+l')=56°\ 4'.3$ log M=7.9937170 . . log P 7.9928086
$m=54°\ 35'\ 36''.28$ cos . 9.7629597 . . sine 9.9111902
A=146334.6 log . 5.1653471 . . . 5.1653471

$m''=+0°\ 13'\ 55''.65$ log . 2.9220238, $p''=19'\ 33''$ log 3.0693459
$l'=$ 55 57 17.20

$\lambda=$ 56 11 12.85
$r''=-$ 4.98

$l'=$ 56 11 7.87 secant 0.2545430

$\Delta p=+0°\ 35'\ 8''.09$ log 3.3238889
L′= 3 10 46.68, $\frac{1}{2}(l+l')=56°\ 4'\ 12''.53$ . . sine 9.9189323

L= 3 45 54.77, $\Delta z=$ 0° 29′ 9″.13 . . log 3.2428212
$m=$ 54 35 36.28

Bearing of Calton, or $z=$S. 55 4 45.41 E. from Bencleugh.
=S.304 55 14.59 W.

In the same manner may the computations of the positions of the other points be performed.

We shall, however, here determine the position of Bencampsie from Bencleugh, and then that of Benlomond from Bencampsie, whence, in a similar manner, may any number of points be fixed in succession.

Angle, Calton, Bencleugh, Bencampsie . . . 105° 33′ 43″.68
Calton bears from Bencleugh . . . S. 55 4 45.41 E.

Bencampsie from Bencleugh bears . . . S. 50 28 58.27 W.
distant 98240.3 feet.

3. Constant logarithm 5.91463
$\alpha=50°\ 29'$ cosine 9.80366
A 98240.3 feet log 4.99229

$\frac{1}{2}(l-l')=-\ 5'.1$ S. log 0.71058
$l=$ 56 11 .1 N.

$\frac{1}{2}(l+l')=56°\ 6'.0$ log M 7.9937150 log P . 7.9928126
Z= 50° 28′ 58″.27 cos 9.8036681 sine . . 9.8872989
A= 98240.3 log 4.9922897 log . . 4.9922897

$m''=-0°\ 10'\ 16''.13$ log 2.7896728 $p''=12'\ 25''.4$ log 2.8724012
$l=$ 56 11 7 .87

$\lambda=$ 56 0 51 .74
$r''=-$ 2 .00

$l'=$ 56 0 49 .74 secant 0.2525937

$\Delta p''=+0$ 22 13 .51 log 3.1249949

$\Delta p''=+$ 0° 22′ 13″ .51, brought over, log . . 3.1249949
L′= 3 45 54 .77 $\frac{1}{2}(l+l')$=56° 5′ 58″.8 sine 9.9190828

L= 4 8 8 .28 Δz=−0° 18′ 26″.82 log . 3.0440777
= 50 28 58 .27

Bearing of Bencleugh, z= N. 50 10 31 .45 E. from Bencampsie.
= S. 230 10 41 .45 W.

Angle, Benlomond, Bencampsie, Bencleugh, by observation, is . . . . . . 107° 22′ 41″.33
Bencleugh from Bencampsie bears . . . N. 50 10 31.45 E.

Benlomond from Bencampsie bears . . . N. 57 12 9.88 W.
4. Constant logarithm 5.91463
α=57° 12′ cosine 9.73377
A=119569 feet log 5.07762

$\frac{1}{2}(l-l')$= 0° 5′.3 0.72602
l'=56 0.8

$\frac{1}{2}(l+l')$=56 6.1 log M=7.9937150 log P . =7.9928085
m=57° 12′ 9″.88 cos 9.7337331 sine . 9.9245855
A=119569 feet log 5.0776186 log . 5.0776186

m''=+0° 10′ 38″.36 log 2.8050667 p''=16′ 28″.6 l 2.9950126
l'= 5 0 49 .74

λ= 56 11 28 .10
r''= — 3 .54

l= 56 11 24 .56 secant 0.2545829

Δp=+0° 29′ 36″.62 log 3.2495955
L′= 4 8 8 .28 $\frac{1}{2}(l+l')$=56° 6′ 7″.15 sine 9.9190946

L= 4 37 44 .90 N. Δz= 0° 24′ 34″.65 log 3.1686901
m=57 12 9 .88

Bencampsie bears, or z=S. 57 36 44 .53 E. from Benlomond.
=S.302 23 15 .47 W.

The following are the azimuths reckoned from the south throughout the circle, and the latitudes and longitudes of the preceding stations in pairs, the longitude west being marked +, east —.*

Names of Stations.	Azimuth from S.	Latitude N.	Longitude W.
1. Calton Station, .	217° 23′ 4″.00	55° 57′ 17″.20	+ 3° 10′ 46″.68
2. Kellie Law, . .	37 43 9.15	56 14 52.76	+ 2 46 34.73
3. Calton Station, .	125° 24′ 28″.72	55° 57′ 17″.20	+ 3° 10′ 46″.68
4. Bencleugh, . .	304 55 14.59	56 11 7.87	+ 3 45 54.77
5. Bencleugh, . .	50° 28′ 58″.27	56° 11′ 7″.87	+ 3° 45′ 54″.77
6. Bencampsie, . .	230 10 31.45	56 0 49.74	+ 4 8 8.28
7. Bencampsie, . .	122° 47′ 50″.12	56° 0′ 49″.74	+ 4° 8′ 8″.28
8. Benlomond, . .	302 23 15.47	56 11 24.56	+ 4 37 44.90

* In like manner, latitudes north may be marked +, south —.

In the preceding table, the azimuth opposite Calton signifies that Kellie Law bears 217° 23′ 4″.00 from the south towards the west round the circle, and conversely from Kellie Law the Calton bears S. 37° 43′ 9″.15 W.; and so on of the rest.

26. In some cases, the eastings and westings, and northings and southings, are put down, as already remarked, as co-ordinates, and from these the latitudes, longitudes, and azimuths are determined. This gives some advantages and some disadvantages, and therefore may or may not be practised at the option of computers. They are tabulated in the following manner, the azimuths being *all* referred to the meridian of Edinburgh.

No.	a	x	Log x.	y	Log y.
1	N. 37° 23′ 4″.00 E.	— 107334.5	5.0307394	+ 82017.3	4.9139053
2	N. 54 35 36.28 W.	— 84782.6	4.9283068	+ 119271.7	5.0765373
3	N. 83 19 3.04 W.	— 22914.0	4.3601014	+ 195571.3	5.2913050
4	N. 73 12 33.28 W.	— 89063.0	4.9496976	+ 295163.3	5.4700623

From the co-ordinates in this table the positions may be fixed as before, and all referred to the same meridian. For an exemplification of this, see my Mathematical Tables. To extend right-angled triangles in this manner by parallels to the primary meridian, however, should not be carried too far. To avoid this, a new meridian may be assumed at the distance of every two or three degrees of longitude.

The method of measuring an arc of the meridian may be readily understood from the figure, page 362. Suppose the latitude of the point A to be accurately determined, and the azimuths of the sides of the triangle A B D in reference to the meridian X X′, then by the perpendiculars D a, B b, C c, &c., the parts A a, $a b$, $b c$, &c., may be found, the sum of which will be the total arc A h; or by the intersections α, β, γ, δ, &c., the portions A α, $\alpha \beta$, $\beta \gamma$, &c., may be found, from the sum of which arises A h, the whole arc. Now, the latitude of the point h being likewise determined, and the azimuths of the sides h H, h F being also obtained as a verification of those derived from the other extremity at A, the length of the whole arc A h in feet with its corresponding arc in the heavens, the difference of latitude become known, from which the length of a degree at the middle latitude will be readily found by dividing the extent of the arc in feet by that of its corresponding arc in degrees.

In measuring an arc of the meridian, if the perpendicular I h, fig., page 362, be small, the points I and h may be considered as nearly on the same parallel of latitude; but if it be somewhat considerable, h is not upon the same parallel with I, the difference between which is the small arc of the meridian $\lambda\, l$, fig., page 359, computed from the formula

$$r'' = p''^2 \tfrac{1}{2} \sin 1'' \tan l \quad . \quad . \quad . \quad . \quad (27.)$$

in seconds of arc. This formula may be transformed into another, giving the reduction in feet instead of seconds; and it then becomes

$$r = \tfrac{1}{2} \sin 1'' \text{ M} \tan l \sin^2 z \text{ A}^2 \quad . \quad . \quad . \quad (28.)$$

when z and A are given, or when p''^2 is given,

$$r = \tfrac{1}{2} \sin 1'' p''^2 \mu \tan l \quad . \quad . \quad . \quad . \quad (29.)$$

in which μ is the reciprocal of M, readily obtained by using the arithmetical complement of log M in the computation, while p''^2 is the square of the perpendicular arc in seconds, found by a previous part of the computation, and l the latitude.

Constant logarithm $= \log \tfrac{1}{2} \sin 1'' = 4.384545$.

Since the point I is always farther from the equator than the point h, the foot of the perpendicular from it upon the meridian, this correction must be applied to reduce h to the same parallel as I, and must be *added* to the arc of the meridian, when the point I is at the end *nearest* the equator, but subtracted, as in this case, when it is farthest from it.

EXAMPLES.—1. By the *Trigonometrical Survey*, vol. ii. p. 56, giving an account of the measurement of an arc of the meridian between Dunnose and Clifton, the distance, I h, of the station at Clifton is 4770 feet from the meridian of Dunnose, X X′, on an arc perpendicular to it, in the latitude of 53° 27′ 30″ N. nearly; required the correction $\lambda\, l$, fig., p. 359, of the meridian arc A h, fig., p. 362, in feet? I have found, from a new computation, 4737.59 feet, instead of 4770 feet.

Here, in reference to formula (28), $l = 53° \, 27' \, 30''$, and A $\sin z = 4737.59$ feet.

Whence $\tfrac{1}{2} \sin 1''$, log	4.384545
Log M (Table XIX.) to $l = 53° \, 27'.5$.	7.993904
$l = 53° \, 27' \, 30''$ log tangent	0.130131
Log $\text{A}^2 \sin^2 z = 2$ log of 4737.59 feet .	7.351112
$r = -0.724$ foot, log	9.859692

This reduction being required at the end of the arc farthest from the equator, must be subtracted from the arc between the *perpendiculars*, to reduce it to that between the *parallels*.

From one computation, the distance between the perpendiculars, page 55 of Trig. Survey, vol. ii., is . . 1036334.40 feet.

By another, page 57, it is . . .	1036333.90
Mean of these two	1036334.15

But the zenith-sector was placed 6.5 feet south of the theodolite station at Dunnose, and 3.5 feet south of the station at Clifton, which increases the arc by three feet, their difference; whence, by applying these corrections, we have

Mean value from the triangles . . .	1036334.15 feet.
Correction for the positions of the zenith-sector	+ 3.00
Reduction to the parallels	— 0.73
True length of the arc	1036336.42 feet.

in Ramsden's scale, or 1036408.03 feet of the imperial standard, supposing the trigonometrical computations in the survey to have been accurately performed.

The length stated in the survey, in which this reduction is neglected, is 1036337 feet, that does not differ materially from the preceding result, on account of the smallness of the perpendicular arc, and for that reason was probably omitted.

New determination by the author.

	Feet.	Feet.
1. By Roy's base on Hounslow Heath	1036361.06	$E^1 = -27.73$
2. By Mudge's base on Hounslow Heath	1036418.83	$E^2 = +30.04$
3. By Mudge's base on Salisbury Plain	1036408.78	$E^3 = +19.99$
4. By Mudge's base on Misterton Carr	1036366.48	$E^4 = -22.31$
Mean of these four . . .	1036388.79	

which, divided by the intercepted celestial arc 2°.8398389, gives 364946.34 feet for the length of one degree at the middle latitude 52° 2′ 19″ N., which in metres is 111233.63.

2. The same omission in the measurement of the French arc of the meridian between Montjouy and Formentera would have produced an error of 169.88 French toises, on account of the magnitude of the perpendicular arc from Formentera on the meridian of Paris, had it not been counteracted partly by an opposite error, arising from the insufficiency of a formula of Delambre, as there employed, which is given in the third volume of the *Base du Sys-*

tème Métrique, page 4, illustrated by a numerical example, page 190, producing an error of 100.07 toises, with a *contrary sign*. The difference of these two make on the whole an error of 69.81 toises in the results of the original commission, published in the *Connaissance des Tems* for 1810. This error was first detected by M. Puissant, and has been finally verified by a new commission lately appointed for that express purpose. This correction ought, therefore, hardly in any case to be neglected. M. Puissant, however, shows that Delambre's formula, giving—

$$1.\ d\,l = -\,A \cos z - \frac{1}{2\,r} A^2 \sin^2 z \tan l + \frac{1}{6\,r^2} A^3 \sin^2 z \cos z\,(1 + 3 \tan^2 l) \text{ in feet, } (\alpha)$$

the azimuth z being reckoned from the south, is quite accurate when the convergence of the meridians, c'', is taken into account, or if, instead of z, the azimuth simply, ($z + c''$,) be employed. It is certain that Delambre did, in some instances, understand the formula in this sense; but it appears probable that he had not attended to it in some manuscript instructions communicated by him to the commission of 1808. This last term is the same as ϱ'' in page 358 (*note*), whose sign evidently depends on that of cos z.

$$2.\ d\,l = -a'' \cos z - a''^2\, \tfrac{1}{2} \sin 1'' \tan l \sin^2 z - a''^3\, \tfrac{1}{6} \sin^2 1'' \tan^2 l \sin^2 z \cos z\ (\beta)$$

calling the last term ϱ'', then $\varrho'' = m'' r''$ sin 1″ tan l, by my investigations. The term ϱ'' will be nearly insensible when the distance does not exceed 30 or 40 miles.

To avoid any difficulty from this cause, it may be recommended, in general, to trace the meridian arc through a continued series of triangles, so that the extremities of that arc may commence and terminate in the vertices of the first and last triangle, as nearly as may be convenient. If not, the small correction derived from this formula must not be neglected.

It has often been alleged that the repeating circles of Mechain and Delambre could not separate the double star ζ Ursæ Majoris, or rather show the smaller star about 15″ distant from it separate from the larger. I cannot think this assertion credible, because, in my small altitude and azimuth circle, by Robinson, with a power of twenty-five, they appear distinctly separate, and, since the telescopes of these astronomers possessed greater power than this, those stars must undoubtedly have appeared distinctly separate. This could not therefore, as has been sometimes asserted, be the cause of the irregularities in the French arc of the meridian.

It would extend this paper too much to enter at length into this subject, which may be seen more fully developed in the introduction to my Mathematical Tables.

In the computation of an arc of any parallel whose latitude is λ, to obtain that portion of it dP, corresponding to any oblique arc A, of which the azimuth is z, the latitude of one extremity is l, that of the other l', and the normal corresponding to λ is ϱ, and that l' is ϱ', then

$$d\,P = \frac{\varrho \cos \lambda}{\varrho' \cos l'} \left\{ A \sin z - \frac{1}{6\varrho'^2} A^3 \sin z + \frac{1}{6\varrho'^2} \cdot \frac{A^3 \sin^3 z}{\cos^2 l'} \right\} \quad . \quad . \quad (\gamma)$$

Example 1.—The distance of Benlomond from the Calton Hill station is 308307.6 feet, bearing N. 73° 12′ 33″.28 W.; the latitude of the Calton Hill is 55° 57′ 17″.20 N., longitude 3° 10′ 46″.68 W.; the latitude of Benlomond is 56° 11′ 24″.56 N.; required the distance between their parallels, and the arc of the parallel between their meridians at the latitude of 56° N., together with the longitude of Benlomond.

$$\text{Log} \frac{1}{2\varrho} = \log \tfrac{1}{2} \sin 1'' + \log f; \ \log \frac{1}{6\varrho^2} = \log \tfrac{1}{6} \sin^2 1'' + 2 \log f,$$

f being the factor from Table XIX.

1. Let the difference between the parallels be found by formula (α), and then the amplitude of the arc between the parallels by formula (γ).

1. Z	= N. 73° 12′ 33″.28 W.	cos	9.4607134	Sin^2	9.962156	. .	9.96216
A	= 308307.6 ft.	log	5.4889842	A^2	0.977968	A^3	6.46695
1 t	= + 89063.06		4.9496979	l' tan	0.170273	cos z	9.46071
				$\frac{1}{2\varrho}$	2.377359	$\frac{1}{6\varrho^2}$	4.57863
2 t	= — 3074.37	.	2d term, or	2 t	3.487756	3 t	0.46845
3 t	= — 2.94	.	. . .	.	. .	3 log	0.47712
						$\tan^2 l'$	0.34055
						4 t	1.28612
4 t	= — 19.32		4th term.				
dP	= + 85966.43	log	4.9343289 the distance between the parallels.				
$\frac{1}{2} l+l'$	= 56° 4′.4	log M	7.9937669				
dl	= + 0° 14′ 7″.42	log	2.9280958				
l'	= 55 57 17 .20						
l	= 56 11 24 .62 N.						

2. Let the difference between their meridians be now found by formula (γ) at the latitude of 56° N.

2. λ	=	56° 0′ 0″	log ϛ	7.3216121	$\frac{1}{6\varsigma^2}$	4.57862 —		
λ	=	56 0 0	cos	9.7475617	A²	6.46695		
			x	7.0691738		1.04557 —	.	1.04557
l′	=	55° 57′ 17″.20	log ϛ′	7.3216109	sin z′	9.98372	sin² z′	9.96745
l′	=	55 57 17 .20	cos	9.7480694		1.02929	sec² l′	0.50386
			y	7.0696803	log F′	9.99949	log F	9.99949
F′ log		. . .	x — y	9.9994935	2, t	1.02878 —	3, t	1.51637 +
A	=	308307.6 N.	log	5.4889842				
z′	=	S. 74° 24′ 42″.41 E.	sin	9.9837246				
1, t		+ 296621.28	log	5.4722023				
2, t		— 10.69						
3, t		+ 31.63						

d P + 296642.22 feet west from Calton meridian to that of Benlomond, in latitude 56° N.

Otherwise thus :—

3. λ	=	56° 0′ 0″	log ϛ	7.3216121	$\frac{1}{6\varsigma^2}$	4.57862 —		
λ	=	56 0 0	cos	9.7475617	A²	6.46695		
			x	7.0691738		1.04557 —	.	1.04557
l	=	56° 11′ 24″.62	log ϛ	7.3216166	sin z	9.98108	sin² z	9.96216
l	=	56 11 24 .62	cos	9.7454168		1.02665	sec² l	0.50917
F		. . .	x — y	0.0021404	log F	0.00214	log F	0.00214
A	=	308307.6	log	5.4889842	2 t	1.02879 —	3 t	1.51904 +
z	=	N. 73° 12′ 33″.28 W.	sin	9.9810781				
1, t	=	+ 296621.54	log	5.4722027				
2, t	=	— 10.69						
3, t	=	+ 33.11						
d P	=	+ 296643.96 (2)						
		+ 296642.22 (1)						
Mean	=	296643.09						

By combining formula (13), page 357, with formula (26), page 368, we get $A'' = \frac{R''}{r} A = \frac{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}{a \cos l} R'' A$ . . . . (γ)

by which *d* P may be converted into an arc of the parallel in seconds, or the difference of longitude.

e² log			$\bar{3}$.8231836	Calton longitude	3° 10′ 46″.68 W.
λ, 56° sin² . . .			9.8371484	*d p* now found	1 26 57 .78 W.
	+ 1.0				
	— 0.00457438	log	$\bar{3}$.6603320	Long. of Benlomond	4 37 44 .46 W.
	0.99542562	log	9.9980089	agreeing with the result in page 374 very nearly.	

Brought forward,

$\log (1 - e^2 \sin^2 l)^{\frac{1}{2}} =$	. .	9.9990044	
log R″	. . . . .	5.3144251	
a, c, log a	. . . . .	2.6793835	
λ = 56° secant	. . .	0.2524383	*Nota.*—$\log (1 - e^2 \sin^2 l)^{\frac{1}{2}}$ may be taken from my *Mathematical Tables*, XXIII.
d P = 296643.09 log	. .	5.4722342	
d p = 1° 26′ 57″.78	. .	3.7174855	

In *Marine Surveying*, a table of meridional parts is generally required to one or two places of decimals, and in cases of great accuracy they should be used for a spheroid of about $\frac{1}{300}$ of compression. If the reduction of the latitude from Table XVI. be subtracted from the apparent or observed latitude, the meridional parts answering to the remainder or geocentric latitude, will be those on the spheroid. I generally, however, prefer the following formula, in which the first term gives the meridian parts on the sphere, and the remaining terms give the corrections to reduce the meridian parts on the sphere to those on the spheroid of $\frac{1}{300}$ of compression. Let P = the meridian parts on the spheroid to l, the observed latitude, then

$$P = 7915'.705 \log \{\log \tan (45° + \tfrac{1}{2} l) - 10\}$$
$$- 22'.9182 \sin l + 0'.0127 \sin 3 l - \&c. \quad . \quad . \quad . \quad (30)$$

Log 7915′.705 = 3.8984896, log 22′.9182 = 1.360181, and log 0′.0127 = 2.10380.

Example 1. Required the meridian parts to latitude 55° 30′, both on the sphere and the spheroid?

1. Constant log .	3.8984896	2. Constant log .	— 1.360181
45° × ½ l = 72° 45′ .	9.7058012	sin l = . .	+ 9.915994
1st term = + 4020′.60 log	3.6042908	2d term — 18′.89 log.	— 1.276175
2d term = — 18 .89			
3d term = + 0 .00		3. Constant log . .	+ 8.10380
		Sin 3 l	+ 9.36818
P′ . = 4001 .71			
		3d term + 0.003 log	+ 7.47198

Hence the meridian parts for latitude 55° 30′ are 4020′.60 (the first term) on the sphere, and 4001.71 on the spheroid of $\frac{1}{300}$ of compression.

Example 2. Required the meridian parts for latitude 56° 30′?

Ans. On the sphere P 4127.90
On the spheroid, or P′ = . . 4108.80

In this way the meridian parts may be computed to every degree and minute throughout the extent of the survey. Now, when the proper scale is chosen for a degree of longitude, the differences of the meridian parts for each degree, &c., throughout the extent of the survey from the same scale, will give the graduation of the scale of latitude. Thus, $P - P' = 4108'.80 - 4001'.71 = 107'.09 = 1° 47'.09$ to be taken from the scale selected for longitude, to give the extent of a degree of latitude, between the latitudes 55° 30′ and 56° 30′, on the terrestrial spheroid, as shown in page 292.

TABLE

OF MERIDIONAL PARTS TO EVERY DEGREE OF THE QUADRANT, IN THE SPHERE AND SPHEROID OF ONE-THREE-HUNDREDTH OF COMPRESSION.

Deg.	Sphere.	Spheroid.	Difference.	Deg.	Sphere.	Spheroid.	Difference.	Deg.	Sphere.	Spheroid.	Difference.
0	0.00	0.00	59.60	30	1888.38	1876.92	69.29	60	4527.37	4507.52	121.67
1	60.00	59.60	59.62	31	1958.01	1946.21	70.03	61	4649.23	4629.19	125.56
2	120.02	119.22	59.66	32	2028.38	2016.24	70.81	62	4774.98	4754.75	129.77
3	180.08	178.88	59.71	33	2099.53	2087.05	71.61	63	4904 94	4884.52	134.30
4	240.19	238.59	59.79	34	2171.48	2158.66	72.49	64	5039.42	5018.82	139.22
5	300.38	298.38	59.88	35	2244.29	2231.15	73.37	65	5178.81	5158.04	144.53
6	360.66	358.26	60.00	36	2317.99	2304.52	74.32	66	5323.51	5302.57	150.44
7	421.05	418.26	60.12	37	2392.63	2378.84	75.31	67	5474.01	5453.01	156.56
8	.481.57	478.38	60.26	38	2468.26	2454.15	76.36	68	5630.82	5609.57	163.59
9	542.23	538.64	60.45	39	2544.93	2530.51	77.45	69	5794.56	5773.16	171.22
10	603.07	599.09	60.63	40	2622.69	2607.96	78.61	70	5965.92	5944.38	179.65
11	664.09	659.72	60.83	41	2701.60	2686.57	79.81	71	6145.70	6124.03	189.01
12	725.32	720.55	61.07	42	2781.71	2766.38	81.09	72	6334.84	6313.04	199.46
13	786.78	781.62	61.33	43	2863.10	2847.47	82.42	73	6534.42	6512.50	211.21
14	848.49	842.95	61.58	44	2945.81	2929.89	83.85	74	6745.74	6723.71	224.49
15	910.46	904.53	61.88	45	3029.94	3013.74	85.33	75	6970.34	6948.20	239.63
16	972.73	966.41	62.19	46	3155.55	3099.07	86.88	76	7210.07	7187.83	257.05
17	1035.30	1028.60	62 54	47	3202.71	3185.95	88.55	77	7467.21	7444.88	277.27
18	1098.22	1091.14	62.89	48	3291.53	3274.50	90.19	78	7744.57	7722.15	301.06
19	1161.49	1154.03	63.27	49	3382.08	3364.69	92.22	79	8045.71	8023.21	329.42
20	1225.14	1217.30	63.69	50	3474.47	3456.91	94.09	80	8375.20	8352.63	363.80
21	1289.20	1280.99	64.11	51	3568.81	3551.00	96.13	81	8739.06	8716.43	406.34
22	1353.69	1345.10	64.58	52	3665.19	3647.13	98.33	82	9145.46	9122.77	460.31
23	1418.63	1409.68	65.06	53	3763.76	3745.46	100.64	83	9605.82	9583.08	531.02
24	1484.06	1474.74	65.56	54	3864.64	3846.10	103.10	84	10136.89	10114.10	627.69
25	1549.99	1540.30	66.12	55	3967.97	3949.20	105.70	85	10764.62	10741.79	767.87
26	1616.47	1606.42	66.69	56	4073.90	4054.90	108.50	86	11532.52	11509.66	989.57
27	1683.52	1673.11	67.29	57	4182.62	4613.40	111.46	87	12522.11	12499.23	1394.30
28	1751.16	1740.40	67.93	58	4294.30	4274.86	114.63	88	13916.43	13893.53	3383.11
29	1819.44	1808.33	68.59	59	4409.14	4389.49	118.03	89	16299.56	16276.54	
30	1888.38	1876.92		60	4527.37	4507.52		90	Infinite	Infinite	
Deg.	Sphere.	Spheroid.	Difference.	Deg.	Sphere.	Spheroid.	Difference.	Deg.	Sphere.	Spheroid.	Difference.

In Nautical Surveying it is sometimes convenient or necessary to find the distance of a point near the horizon of the sea by its observed depression from a given height. An imperfect solution of this problem is given in Horsburgh's edition of *Mackenzie's Marine Surveying*, section iv., by considering the plane triangle *right-angled*, and omitting the effects of curvature and terrestrial refraction.* For this purpose let H be the *obtuse angle* near the horizon, formed by the line from the eye of the observer at A, a given height above B at the level of the sea, to that point, and another line from the centre of the earth to the same point, D the observed depression, H—90° = d, c'' the angle at the earth's centre, subtended by the cord H B, M the logarithmic modulus, r the earth's radius, and h the height A B on which the depression is taken, then making log $\frac{M}{r} = 8.317198$, when r is the mean radius of the earth, the following formulæ may be readily investigated.

1. $\text{Log} \sin H = \log \cos D + \frac{M\,h}{r} = \log \cos d$
2. $D - d = c''$; also $a'' = 0.42\, c''$ and $\alpha'' = 0.08\, a''$ †
3. $\text{Log}\, K = \log \text{cosec}\,(D - a'') + \log \cos (D + \alpha'') + \log h$
4. $\text{Sin} \frac{1}{2} H = \{ \frac{1}{2} \left(1 \pm \sec D_{,} \, [\sin (D + D_{,}) \sin (D - D_{,})]^{\frac{1}{2}} \right) \}^{\frac{1}{2}}$.

By reflecting on the steps of the investigation, it appears that

5. $\text{Sin}\, d = \sec D_{,} \, \{\sin (D + D_{,}) \sin (D - D_{,}) \}^{\frac{1}{2}}$.

Because $D_{,}$ is always a small arc, its secant differs little from radius, therefore its effect will be nearly insensible.

In the (4.) formula D—$D_{,}$ is the difference of the depression of the given point D and that of the horizon $D_{,}$, which difference may be measured with a sextant, while the depression of the horizon may be computed by the usual formulæ, or observed with the dip sector when possible, and then $D_{,}$ and D—$D_{,}$ become known without the employment of an altitude-circle on shore, and in this case the upper sign must be used. If the under sign be used, the result-

* There appears to be an error committed in the operation or solution of the example, making the distance 7¾ miles, instead of 4741 feet, or about ¾ of a mile only!

† In general terms—

$$a'' = (0.5 - n) \frac{R''}{\epsilon} K = (0.5 - n) f K = (0.5 - n)(D - d)$$

$$\alpha'' = n \frac{R''}{\epsilon} K = n f K = n (D - d)$$

in which n is the coefficient of refraction, computed from the state of the barometer and thermometer by Table XI., &c., and f the factor from Table XIX., to convert feet on the earth's surface into seconds of arc in the given direction of the object.

ing value of H will be the supplement of that by the upper in (4.) Formula (5), however, is preferable, by giving d the arc here required.

Example. Let the observed depression of a given point from the top of Goatfell, in the island of Arran, be D = 2° 18′ 8″.4 by an astronomical circle, and the height h of the circle above the level of the sea be 2861.5 feet; required K, the cord measuring the distance of the point observed from the station on which the observation was taken?

Constant log .	$\bar{8}$.317198		
h = 2861.5 feet, log	3.456594		
Sum . .	$\bar{5}$.773792	Natural number	0.0000595
D = 2° 18′ 8″.4	cosine . .		9.9996493
d = 2 5 52 .4	cosine . .		9.9997088

$c'' =$ D − d = 12 16 .0 = 736″ and 736″ × 0.42 = 5′ 9″.12 = α″
736″ × 0.08 = 58″.88 = α″

D = . .	2° 18′ 8″.4	. . .	2° 18′ 8″.4
α″ = . .	− 5 9.1, α″ =	. . .	+ 58.9
D − α″ = . .	2 12 59.3, D + α″ = .		2 19 7.3

D − α″ = 2° 12′ 59″.3	cosecant . .	1.4125687
D + α″ = 2 19 7 .3	cosine . . .	9.9996443
h = 2861.5 feet,	log . . .	3.4565938
K = 73927.6 feet,	log . . .	4.8688068

the first approximation, which, in moderate distances, will in general be sufficient.

For a second approximation, the following method may be employed, in which log K is that previously found.

5. Log α″ = Const. log 7.617058 + log K.
6. Log α″ = Const. log 6.896900 + log K.

Const. logs, 1st	7.617058,	2d	6.896900
Log K .	4.868807	. . . . .	4.868807
α″ = 5′ 6″.1 log .	2.485865,	α″ = 58″.3 log .	1.765707

$D = 2^{\circ}\ 18'\ 8''.4$ . .	$2^{\circ}\ 18'\ 8''.4$,	$D - \alpha''$ cosecant	1.412406
$\alpha'' = -\ 5\ \ 6.1$ $\alpha'' =$	$+\ \ 58.3$,	$D + \alpha''$ cosine	9.999644
$D - \alpha'' = 2\ 13\ \ 2.3$, $D + \alpha'' =$	$2\ 19\ \ 6.7$	h log . .	3.456594
$K = 73900$ feet		log . .	4.868644

Another repetition, using this last value of K, would not produce any sensible change in its value, and it may therefore be reckoned correct.

In making use of formula (4) or (5), we suppose $D - D_{\prime}$, the angle between the point whose distance is required and the visible horizon, to be measured with a sextant, and found to be $1^{\circ}\ 25'\ 29''.7$.

To compute the dip we have

Constant logarithm		. .	1.771208
h	$= 2861.5$ feet, $\frac{1}{2}$ log	. .	1.728297
$D_{\prime}$	$= 0^{\circ}\ 52'\ 38''.7$	log. .	3.499505
$D - D_{\prime}$	$= 1\ 25\ 29.7$	by observation, sine	8.395623
D	$= 2\ 18\ \ 8.4 =$ sum		
$D + D_{\prime}$	$= 3\ 10\ 47.1$	sine . .	8.744047
			17.139670
d	$= 2\ \ 7\ 43.3$	sine . .	8.569835*
D	$= 2\ 18\ \ 8.4$		
$D - d$	$= 0\ 10\ 25.1$		
$\frac{1}{2}(D - d)$	$= 0\ \ 5\ 12.5 = a''$		
$\frac{1}{10}(D - d)$	$= 0\ \ 1\ \ 2.5 = \alpha''$		
$D - a''$	$= 2\ 12\ 55.9$ cosec	. .	1.412754
$D + \alpha''$	$= 2\ 19\ 10.9$ cos	. .	9.999644
h	$= 28\ \ 6\ \ 1.5$ log	. .	3.456594
K	$= 73959$ feet log	. .	4.868992

Another repetition, as shown in last example, gives K = 73900 feet, the true value, as before.

The former method is recommended when the observer has a good altitude and azimuth circle; the latter, when he has a sextant or reflecting circle only, as is frequently the case with nautical surveyors.

* NOTE.—Secant $D_{\prime} = 0.000051$ should strictly be added to this log sin d, but, being small, may generally be omitted.

TRIGONOMETRICAL LEVELLING.

1. Trigonometrical Levelling is an operation which generally accompanies Trigonometrical Surveying, because the exact situation of a given point on the earth's surface is accurately fixed by the three co-ordinates, latitude, longitude, and elevation above the mean level of the sea. The triangle formed in a vertical plane above the earth's surface, in this operation, is called a *hypsometrical triangle.** It is formed by the chord of the terrestrial arc comprised between the verticals of the two stations where the reciprocal zenith-distances have been observed, by the straight line which joins the two points of observation and the difference of level $d\,h$. When reciprocal and simultaneous observations are made—that is, when observations are made from two different points on one another at the same time—the results are esteemed the most accurate, as the effects of refraction at the two places is determined by observation. In the case of reciprocal and simultaneous observations, let δ be the observed zenith-distance at the one place, and δ' that at the other, less than the former, while C is the angle contained by two vertical lines drawn from the surface of the earth to its centre; then the difference of altitude will be found by the following formulæ.

$$d\,h = \mathrm{K} \sin \tfrac{1}{2} (\delta - \delta') \sec \tfrac{1}{2} (\delta - \delta' + \mathrm{C}) \quad . \quad . \quad . \quad . \quad (1)$$

$$= \mathrm{K} \tan \tfrac{1}{2} (\delta - \delta') \text{ very nearly.} \quad . \quad . \quad . \quad . \quad (2)$$

But it frequently happens that reciprocal and simultaneous observations cannot be observed; then, in that case, if n be the coefficient of terrestrial refraction,

$$d\,h = \mathrm{K} \sec \tfrac{1}{2}\, \mathrm{C} \cot \{\delta + (n - 0.5)\, \mathrm{C}\} \quad . \quad . \quad . \quad . \quad (3)$$

$$= \mathrm{K} \cot \{\delta + (n - 0.5)\, \mathrm{C}\} \text{ very nearly.} \quad . \quad . \quad . \quad (4)$$

The same thing may be done by the following formula, in which the first part is the solution of a right-angled plane triangle, and

* From the investigations of Laplace, the most appropriate distance, when the *minimum* errors of observation and refraction are combined, is about 40,000 English feet, or seven and a half English miles nearly; but this distance is too small for primary triangulation.

the second contains the effects of the curvature of the earth combined with the refraction.

$$d\,h = \mathrm{K} \cot \delta + \frac{\mathrm{K}^2}{\varrho}(0.5-n) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which ϱ is the radius of curvature equal to the mean radius of the earth nearly, or exactly $\frac{1}{\varrho} = f \sin 1''$, in which f is the factor, from Table XIX., to convert feet into seconds of arc.

To determine the height of the point of observation by the observed depression of the horizon of the sea,

$$d\,h = \tfrac{1}{2}\,\rho\,(1+n)^2 \tan^2 (\delta-90^\circ) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which δ–90° may be replaced by D, the observed depression of the horizon of the sea,

$$d\,h = \tfrac{1}{2}\,\rho\,(1+n)^2 \tan^2 \mathrm{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $\frac{1}{2}\,\varrho = \frac{\frac{1}{2}\,\mathrm{R}''}{f}$, as before.

For many practical purposes the mean value of n=0.08 of the intercepted arc, will be sufficient, and

$$d\,h = 0.5832\,\rho \tan^2 \mathrm{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or, when the depression D does not exceed a few minutes,

$$d\,h = 0.5832\,\rho \sin^2 1''\,\mathrm{D}''^2 \text{ nearly.} \quad . \quad . \quad . \quad . \quad (9)$$

Constant log. of $0.5832\,\rho \sin^2 1'' = 6.457582$.

The same value of n might be introduced into formulæ (4) and (5), and the former would become

$$d\,h = \mathrm{K} \cot (\delta - 0.42\ \mathrm{C}) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

the latter becomes

$$d\,h = \mathrm{K} \cot \delta + \mathrm{K}^2 . \frac{0.42}{\varrho} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

To K in feet, $\log \frac{0.42}{\varrho} = \bar{8}.302632$. for correction in feet.

To K in imperial chains, $\log = \bar{5}.941720$ for correction in feet.

These mean coefficients are, however, combined properly in Tables XXI. and XXII., corresponding to formulæ (4) and (9).

By a paper in the *Edinburgh New Philosophical Journal* for April 1841, I have given the following formula to compute the value of n in given circumstances, and the computation may be readily performed by the aid of Table XI. and the auxiliary refraction tables.

$$n = \frac{a\,r}{2\,\mathrm{B}\,l}\,b\left(\frac{1}{1+\beta(\tau-50^\circ)}\right)^2 \cdot \frac{1}{1+\beta'(\tau-50^\circ)}\left(0.75-\frac{f}{3b}\right) \quad . \quad . \quad (12)$$

$\mathrm{Log}\frac{a\,r}{2\,\mathrm{B}\,l}$=Constant logarithm 7.57877.

This const. log. is combined with the factor $\left(0.75-\frac{f}{3b}\right)$ in Table XI., for the use of which see the explanation of the tables.

Example 1.—To exemplify these formulæ, we find, from observations made by the French engineers, that at

	° ′ ″
Clermont Ferrand, δ'=	83 33 33.37
Puy de Dôme, δ = ...	96 30 38.67
$\delta-\delta'$	12 57 5.30
$\frac{1}{2}(\delta-\delta')$=	6 28 32.65

Also b'=28.839 in., τ'=52.7° F. t'=45.14° F.
b =25.383, τ =51.1 F t =48.92 F.

The base K in English feet log, . . .	4.4875708
To $\frac{1}{2}(l+l')$=45° 47′ and a=85°.7 log o, . .	7.9930826
C= 5′ 2.45″ . . log, .	2.4806534
$\delta-\delta'$=12° 57 5.30	
$\delta-\delta'$ +C=13 2 7.75	
$\frac{1}{2}(\delta-\delta'+\mathrm{C})$= 6 31 3.87 secant, . . .	0.0028161
$\frac{1}{2}(\delta-\delta')$= 6 28 32.65 sine	9.0522415
K . . log, ...	4.4875708
$d\,h$ = 3488.42 feet log, ...	3.5426284

the result by reciprocal and simultaneous observations, which is reckoned the most correct method; but as we have the state of the barometer and thermometer recorded, we shall compute the same difference of level by formula (3), requiring the calculation of the value of n by Table XI.

2. Log to $b=25.383$ in. and $t=49°$ nearly	7.45114
$\tau=51°.1$ log × 2 (Table VII.,)	9.99990
$t=48°.92$ log (Table VIII.,)	0.00097
$b=25^{in}.383$ log, ...	1.40454
$n=$ 0.07187 log, ...	8.85655
Log C as above, ...	2.4806534 }
$n-0.5=-0.42813$ log, ...	$\bar{1}$.6315757 }
$v'=-0°\ 2'\ 9''.48$ log, ...	2.1122291
$\delta=$ 96 30 38 .67	
$\delta_{,}=$ 96 28 29 .19 cotangent, ...	—9.0549565
$\frac{1}{2}$ C= 0 2 31 .22 secant, ...	+0.0000001
Log K, ...	+4.4875708
$d\,h=-3487.61$ feet, log, ...	—3.5425274

From the sign —, this shows that Clermont Ferrand is 3487.6 feet under the summit of Puy de Dôme. We shall also determine the elevation of Puy de Dôme above Clermont Ferrand.

3. Log from Table XI. to $b'=28.839$ in. and $t'=45°$.	7.45178
$\tau'=$ 52°.7 F. log × 2 (Table VII.) . .	9.99976
$t'=$ 45°.14 F. log (Table VIII.,) . .	0.00442
$b'=$ $28^{in}.839$ log, ...	1.45998
$n'=$ 0.082402 log, ...	8.91594
$-0.5=-0.5$	
$n'-0.5=-0.417598$ log, ...	— $\bar{1}$.6207584
Log C, as before, ...	. 2.4806534
$v'=-0°\ 2'\ 6''.31$ log, ...	— 2.1014118
$\delta'=$ 83 33 33 ·37	
$\delta_{,}=$ 83 31 27 .06 cotangent, . . .	+9.0550268
$\frac{1}{2}$ C= 0 2 31 .22 secant, . . .	. 0.0000001
Log K, ...	. 4.4875708
$d\,h=3488.17$ feet, log, ...	. 3.5425977

Nat. No n =0.07187		
n' =0.08241		
$\frac{1}{2}(n+n')$=0.07714	by computation=$\frac{1}{13}$ nearly.	
page 392=0.08334	by observation =$\frac{1}{12}$ nearly.	
Difference, 0.00620	or, .	$\frac{1}{161}$ only.

This last height agrees almost exactly with the first solution by reciprocal and simultaneous observations, while, by the value of n computed from the formula, the results differ by about half a foot only,—a strong proof of the accuracy of the principle which I employed in its investigation.

4. To determine the coefficient of refraction by observation, we have $r=n$ C, $r'=n'$ C, C being the intercepted arc, and n the effect of refraction, a part of that arc.

But $\frac{1}{2}(r+r')=\frac{1}{2}\text{C}-\frac{1}{2}(\delta+\delta'-180°)$.

Introducing this into the preceding equation, and it becomes by reduction,

$$\frac{1}{2}(n+n')=\frac{180°+\text{C}-(\delta+\delta')}{2\text{C}} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The mean of the refractions at the two stations.

		°	′	″
Now at Puy de Dôme,	δ = . .	96	30	38.67
„ Clermont Ferrand,	δ'= . .	83	33	33.37
$\delta+\delta'$=		180	4	12.04
180°+C=		180	5	2.45
$180°+\text{C}-(\delta+\delta')=\theta''_1+\theta''_2$ = . . .		0	0	50.41*

* Log C, page 389, . .	2.48065 . . .		2.48065
Log n, page 390, . .	8.85655	log n', . .	8.91595
θ_1''=21″.74 log, . .	1.33720,	θ_2''=24″.92 log,	1.39660
θ_2''=24 .92			

$\theta_1''+\theta_2''$=46 .66 computed refractions
50 .41 observed refractions

Difference= 3 .75 a small quantity of error.

* Reciprocal and simultaneous observations, though the best method attainable, is also certainly liable to error.

0′ 50″.41	log, ...	1.7025167
2 C=10 4.90	log, ...	2.7816836
n=0.083336	log, ...	$\bar{2}$.9208331
—0.5		
—0.416664	log, ...	—$\bar{1}$.6197860
Log C, as before, ...		2.4806578
v'=—0° 2′ 6″.02	log, ...	—2.1004438
δ_1= 96 30 38.67		
δ_i= 96 28 32.65	cotangent, . .	—9.0550213
K . . .	log, . . .	4.4875708
$d\,h$ 3488.13 feet,	log, . . .	—3.5425921

Here the result is the same as before, and v' found by observation, agrees almost exactly with that previously determined from the computed value of n, which is a verification of the formula for computing n. In fact, the distance is too small for the refraction to produce a very marked effect, though this example has been selected by both Puissant and Biot to test their formulæ. From this, too, it appears that the refraction determined by observation is not that at either point, but a mean between them.

From Tenterden Steeple, Allington Knoll bore N.85° 47′ 25″ E., distant 61781.8 feet of the imperial standard, the latitude of Tenterden steeple being 51° 4′ 8″ N., and that of Allington knoll 51° 4′ 46″ N.; required n the coefficient of terrestrial refraction from the formula $r=n$ C, r being the refraction, and C the intercepted arc?

From Table XIX. we obtain to—

½ (l+l')=51° 4½′ and a=85° 47½′ log f, . .		=7.9929486
D=61781.8 feet, .	log, . .	4.7908606
Intercepted arc=10′ 7″.87 .	log, . .	2.7838092 (a)

δ=	90° 3′ 51″.00	
δ'=	90 3 35.00	
$\delta+\delta'-180°$=	0 7 26.00	observed intercepted arc.

$\delta+\delta'-180$ =	0° 7′ 26″.00	log (a) . .	2.7838092 (a)
$d\delta+d\delta'$ =	− 8.01	computed correction, p. 331	
=	0 7 17.99	corrected observed intercepted arc	
	0 10 7.87	computed intercepted arc	
Difference,	0 2 49.88		
Half,	0 1 24.99	log, . . .	1.9293678 (b)
n=	0.13982	. . . .	$\bar{1}$.1455586 (b—a)
or n=	$\frac{1}{7.2}$	of the intercepted arc	

It is preferable to retain n in the decimal form, in order to obtain the mean of numerous observations readily.

I shall now proceed, from Inchkeith, to determine the heights of the points mentioned in the preceding part of this work, above the sea.

5. At Inchkeith, in August 1840, I found the zenith-distance of the summit of the dome of Edinburgh Observatory to be 89° 40′ 24″.45, at the distance of 30117 feet, bearing S., 18° 43′ 7″.24 W., when the barometer b stood at 29.675 in., and Fahrenheit's thermometer at 63°.8; what was the height of the summit of the dome above the place of observation, and above the mean level of the sea?

Log from Table XI. to b and t, ...	7.44999
τ=63°.8 log ×2 from Table VII., . .	9.99882
t=63 .8 log from Table VIII., . . .	9.98769
b=29 .675 log, ...	1.47239
n= 0.081076 log, ...	8.90889
—0.5	
n—0.5=—0.418924 log, ...	9.6221352—
Log O for lat 56° and z=19° . . .	7.9936237
A=30117 feet, log, ...	4.4788117
v'=—0° 2′ 4″.33 log, ...	2.0945706—
δ= 89 40 24.45	
$\delta_{\prime}$= 89 38 20.12 cotangent, . . .	7.7994839
A=30117 feet, log, ...	4.4788117
$d\,h$=189.80 feet, log, ...	2.2782956

dh = 189.80 feet.
h = 4.00 = height of circle above ground.
h' = 175.00 = height of ground above high water.
h'' = 8.50 half rise of tide.

H′ = 377.30 feet, the height of the summit of the dome above mean tide.
Station 26.90 feet under the dome.

H = 350.40 feet, the height of the axis of the circle on the cylindrical stone south-west of the Observatory, from which I took the following observations on Bencleugh.

6. From this point the summit of Bencleugh was observed to have a zenith-distance of 89° 22′ 41″.85, when the barometer stood at 29.68 in. and Fahrenheit's thermometer at 62°, the middle latitude being 56° 4′ 12″ N., bearing N. 54° 36′ W., distant 146334.6 feet; required the height of Bencleugh above the place of observation, and also above the mean level of the sea?

To b = 29.68 and t =	62° log (Table XI.)	7.45022
τ = 62°	log × 2	9.99896
t = 62	log	9.98927
b = 29.68	log	1.47246
n = 0.081454	log	8.91091
− 0.5		
	log A	5.1653471
n − 0.5 = − 0.418546	log	9.6217432 −
Log O to 56° and 54°.6		7.9931205
v' = −0° 10′ 2″.85	log	2.7802108 −
δ = 89 22 41 .85		
$\delta_{\prime}$ = 89 12 39 .00	cotangent	8.1390736
A = 146334.6 log		5.1653471
mh, from Table XXIV.		+ 73
S, from Table XXIV.		+ 19
dh = 2015.72 feet, log		3.3044299
h = 350.40 = height of Calton Station.		
H = 2366.12 = height of Bencleugh.		

In this manner the elevation of any number of points may be determined successively.

The difference of level may be found approximately with sufficient precision for many purposes by reciprocal zenith-distances, independent of triangulation. This method is not so accurate as the preceding, but will frequently be useful where tolerable accuracy only is necessary.

From a simple investigation, when $n = 0.08$, it will be found that the sum of the refractions, or

$$r + r' = \tfrac{1}{5}\,(\delta + \delta' - 180^\circ) \text{ nearly.} \quad . \qquad (14)$$

where δ and δ' are the apparent zenith-distances. If C be the true angle at the centre, and c the apparent,

$$C = c + r + r' \quad . \quad . \quad . \quad . \qquad (15)$$

in which $r + r'$ is got from formula (14,) and $c = \delta + \delta' - 180^\circ$.

If, however, n and n' be computed from the state of the barometer and thermometer, as has been already shown, then

$$C = \frac{c}{1 - (n + n')} \quad . \quad . \qquad (16)$$

This value of C will generally be more accurate than the preceding. The difference of level will then be computed by the following formula,

$$dh = 2\varrho \tan\tfrac{1}{2} C \tan\tfrac{1}{2}(\delta - \delta' + r - r') + 2\varrho \tan^2\tfrac{1}{2} C \tan^2\tfrac{1}{2}(\delta - \delta' + r - r') \quad (17)$$

But since $r = n\,C$ and $r' = n'\,C$, the preceding expression becomes by substitution,

$$dh = 2\varrho \tan\tfrac{1}{2} C \tan\tfrac{1}{2}\{\delta - \delta' + (n - n')C\} + 2\varrho \tan^2\tfrac{1}{2} C \tan^2\tfrac{1}{2}\{\delta - \delta' + (n - n')C\} \quad (18)$$

$$dh = 2\varrho \tan\tfrac{1}{2} C \tan\tfrac{1}{2}(\delta - \delta') + 2\varrho \tan^2\tfrac{1}{2} C \tan^2\tfrac{1}{2}(\delta - \delta') \text{ nearly} \qquad (19)$$

in which the first term will generally be sufficient.

7. Let $\delta = 90^\circ\ 25'\ 43''.11$, $b = 28.355$ in., $r = 44.4^\circ$, $t = 44.4^\circ$ F.
$\delta' = 89\ 54\ 41.22$, $b' = 29.339$, $r' = 55.4$, $t' = 52.7$ F.

$\delta + \delta' - 180^\circ = c =$	20 24.33	From these, $n =$	0.08130
$\frac{1}{5}\,c =$	4 4.87	$n' =$	0.08236
$C = c + \frac{1}{5}\,c =$	24 29.20	$n + n' =$	0.16366
			1.0
$\delta - \delta' =$	31 1.89		
$\frac{1}{2}(\delta - \delta') =$	15 30.94	$1 - (n + n') =$	0.83634

Log $c'' = \log 20'\,24''.33$	. . . . .		3.08790
Log $1 - (n + n') =$ 0.83634	. .	(subt.)	$\bar{1}.92238$
C $= 24'\,23''.90$	. . .	log	3.16552

Formula (18) would give dh = 668.77 feet, scarcely different from that by (19.)

Logarithm of $2 R''$	5.6154551
$a . c .$ log O to $\frac{1}{2}(l + l') = 38° 52'$ and $\alpha = 19° 47'$	2.0052373
$\frac{1}{2}$ C = 0° 12′ 14″.06 tangent	7.5500681
$\frac{1}{2}(\delta - \delta')$ = 0 15 30 .94 tangent	7.6544995
$d h$ = 668.63 feet, log	2.8252500

From other data the value of $d h$ = 675.5, exceeding the preceding by 6.87 feet only.

The arc measuring the distance is $A = \frac{C}{O}$ . . . . . (20)

in which O is the radius of curvature in the given direction.

The approximate distance may therefore be obtained by adding to

Log C	3.16677
$a . c .$ log O	2.00520
A = 148580 feet, log	5.17197

8. Application of formula (19) to Example 1, page 389.

By calculation n' = 0.08241
n = 0.07187

	°	′	″	$n' - n$ = 0.01054 log . .		$\bar{2}$.02284
C =	0	5	2.45	log		2.48066
$C(n' - n)$ =	0	0	3.20	. . log . . .		0.50350
$\delta - \delta'$ =	12	57	5.30			
2)	12	57	8.50	log 2ϱ = log $2R''$ − log f Table XIX.		7.6223681
$\frac{1}{2}\{\delta - \delta' + (n' - n)\}$ C =	6	28	34.25	tangent		9.0550514
$\frac{1}{2}$ C =	0	2	31.22	tangent		6.8651841
$d h$ =	3488.23			feet log		3.5426036
Log C						2.48066
$a . c . l . \varrho$						2.00691
A = 30730.7 feet nearly log						4.48757

Hence, if the horizontal angles between distant objects be roughly and rapidly measured at the same time with the zenith-distances, the results would be sufficiently accurate for geological and military purposes, as well as for the exploration of new countries, and such other objects as require a moderate degree of accuracy only.

9. By an astronomical circle, the axis of which was 3.5 feet above the rock, the depression of the horizon of the sea from the summit of Dunii, in the island of Iona, was 17′ 52″, bearing about

S. 70° W.; required the height of Dunii, the highest hill in Iona?

To lat. 56° 20′ N. and $\alpha = 70°$ log O″, Table XXI.	6.458479
D″ = 17′ 52″ = 1072″ log × 2 = ...	6.060390
$d\,h$ = 330.2 feet, log	2.518869
h' = —3.5 = height of circle.	
H = 326.7 feet, the height of the ground.	

This method is sufficient for most cases. However, as circumstances will occur where the greatest possible accuracy may be required, then $\frac{1}{2}\,\varrho$, from Table XIX. $= \frac{\frac{1}{2}\,R''}{f}$, in which f is the factor to convert feet into seconds, will give, when combined with $(1 + n)^2 \tan^2 D$, the true height, with all the accuracy that can be expected from this method when n is computed by formula (12), page 389.

2. TO DETERMINE THE HEIGHT OF THE TIDES.

1. The heights have hitherto been generally referred to the mean level of the sea, supposed to be the same as if there were no tides.

Let p be a convenient station near the sea, from which direct levelling can be conducted to the shore. In a calm bay, well protected, but sufficiently open to admit the ingress and egress of the tide freely, let there be placed a deal or glass tube, having a wooden float with a divided stem fixed to it, projecting above the wooden tube, showing the height of the surface within it on divisions on the glass tube, easily read at the surface of the water—admitted by a small aperture near the bottom of each, to prevent the effect of the undulations within them. Then let h, h' be the measures of the difference of level of two successive high waters, and H that of the intermediate low water between h, h', H and the fixed point p near the shore, $\frac{1}{2}\,(h+h')$ will give the difference of level between the mean of the two high waters and p, while—

$$\tfrac{1}{2}\{\tfrac{1}{2}\,(h + h') + H\} = \tfrac{1}{4}\,(h + h' + 2\,H) \qquad . \quad . \quad . \qquad (1)$$

will give $d\,h$, the difference of level between the given point p and the mean level of the sea. From a series of observations of this kind during a lunar month, the height of the station p will be very accurately found, from which a series of levels may be carried over the country to be surveyed and levelled.

2. Observations by the barometer and thermometer should always accompany observations on the height of the tides, since the pressure of the atmosphere affects their rise. In fact, the tide rises sensibly less with a high barometer than with a low one, all other circumstances being alike. The formula for this purpose is—

$$-13.25\,(b - 30^{in}) \qquad . \qquad . \qquad (2)$$

Combining these two, we have the reduction—

$$\tfrac{1}{4}\,(h + h' + 2\,H) - 13.25\,(b - 30^{in}) \qquad . \qquad (3)$$

in which b is the observed height of the barometer, reduced for temperature to the standard point in the given country.

3. Let a be the *absolute* height of the tide at the time of syzygy, or at new and full moon, above the mean level of the sea, then for the coasts of Britain and France—

$$a = u\,k \qquad . \qquad . \qquad . \qquad (4)$$

an expression of which the double will represent the amplitude of that tide at the given place.

The quantity u is called the unit of the height of the tide at that place, and represents the height, above the mean level of the sea, of that tide which takes place about *thirty-six hours* after new and full moon, when the sun and moon are on the equator at the time of syzygy, at their mean distances from the earth. The co-efficient k, whose magnitude depends on the declinations of the sun and moon, and their distances from the earth, unfortunately, is not given in the *Nautical Almanac*, but is recorded in the *Connaissance des Tems*, computed by the formula of Laplace, given in the *Mécanique Celeste*, tome ii., page 289, and varies between the limits of 0.670 and 1.178.

The value of this unit may be deduced from formula (4.)

$$u = \frac{a}{k} \qquad . \qquad . \qquad . \qquad (5)$$

by substituting for a the quantity $\frac{h+h'}{4}$ thirty-six hours after any syzygy, and for k, the number in the *Connaissance des Tems*, from article *Tableau des plus grandes Marées*. Knowing the value of k for each day in the year, by its aid, and that of the mean level of the sea, we can calculate the height of high and low water to which soundings are referred. For let H′ be the height of the mean level above zero, then—

$$\text{High water} = H' + u\,k \quad . \quad . \quad . \quad (6)$$
$$\text{Low water} = H' - u\,k \quad . \quad . \quad . \quad (7)$$

4. When, by observations taken as directed in section (1,) we determine how much the syzygial tide falls below the mean level of the sea, and we wish to find the lower level of a great equinoxial tide, which serves for the *datum* of departure for the reduction of soundings, we have—

$$a = u\,k \quad . \quad . \quad . \quad . \quad . \quad (8)$$
$$a' = u\,k' \quad . \quad . \quad . \quad . \quad . \quad (9)$$

k and k' being the coefficients given in the *Connaissance des Tems*, corresponding, the *first* to the equinoxial tide, the *second* to the syzygial tide, which has furnished the value $a = \frac{1}{2}(h+h')$. From these relations, calling a' the distance of the mean level from the zero of the scale of reduction, we derive—

$$a'' = \frac{k'}{k} \times \frac{h+h'}{4} \quad . \quad . \quad . \quad (10)$$

In practice, k' is generally taken at 1.16, so that the distance of the mean level, from that to which soundings are referred, always exceeds the unit of height.

If from the expression equation (3)—which represents the number on the scale to which the mean level under a mean pressure of 30 inches of the barometer corresponds, the value of a'' be subtracted, the remainder expressed by—

$$\frac{1}{4}(h + h' + 2\,H) - 13.25\,(b - 30^{\text{in}}) - \frac{k'}{k} \times \frac{h+h'}{4} \quad . \quad . \quad (11)$$

will designate the number to which the low water is referred, whence we ought to set out in reducing soundings.

This will be the constant number to be subtracted from all those which the scale has indicated in order to refer our observations to the level originally adopted.

Heights may also be very readily determined by the mountain-barometer. The calculations for this purpose are very expeditiously performed by a set of tables published by the author.

They may also for moderate heights be easily obtained by the following formula, in which B is the observed height of the barometer at the bottom, τ the corresponding temperature of the mercury by the attached thermometer, and t that of the air by the detached thermometer; while b is the height at the top, τ' its corresponding temperature, and t' that of the air; then H being the height in English feet—

$$H = \{48400 + 60\ (t+t')\}\ \frac{B-b}{B+b} - 2.75\ (\tau - \tau') \qquad (1)$$

See Galbraith and Rutherfurd's edition of Bonnycastle's *Algebra*, page 192, formula (8,) from which, in heights exceeding 4000 or 5000 feet, one or two more terms may be introduced, though the formula given will be sufficient for almost any heights in Britain. In the application of that formula, care must be taken of the sign of $\tau - \tau'$, that is, if τ' in a few rare cases exceed τ, the sign of $\tau - \tau'$ will be *negative*, which, multiplied by —2.75, also *negative*, will, by the rules of algebra, give a *positive* quantity to be *added*, instead of subtracted, as is usually the case.

	In.	°	°
Ex. Let	$B = 30.295$,	$\tau = 75.5$,	$t = 75.5$
	$b = 27.064$,	$\tau' = 60.2$,	$t' = 60.2$
	$B - b = 3.231$,	$\tau - \tau' = 15.3$,	$t + t' = 135.7$
	$B + b = 57.359$,		

ANS.	Between barometers, ...	3143 feet.
	Reduction to level of the sea, ...	+ 35 feet.
	Total height above the mean level of the sea,	3178 feet.

Galbraith's *Barometric Tables* will be found very convenient for computing heights by the mountain barometer.

In Marine Surveying, it is seldom convenient, and often impossible, to determine the direction of the meridian by the pole-star, as has been shown in the preceding pages; and in the practice of ordinary surveying, such a degree of precision is unnecessary. In this case, recourse may be had to the methods recommended in pages 345, 346, &c., as illustrated by the following examples :—

Example 1.—On the 10th of July 1837, at 7^h A. M., in latitude 7° 31′ 20″ S., and longitude 153° 10′ E., the observed altitude of the sun's lower limb was 10° 30′ 0″, and at the same instant the observed distance of the sun's nearest limb from a well-defined point of land on the same level with the eye to the left of the sun was 95° 16′ 0″. The index-error of the former sextant was

—0′ 50″, that of the latter +1′ 10″, the height of the observer's eye taking the sun's altitude being 14 feet; required the true bearing of the point of land, and the variation of the compass, when the magnetic bearing of the same point was N. 5° 10′ W.?

	h. m. s.		° ′ ″
Ship time, July 9,	19 0 0	To G. M. T. sun's P.D.	112 20 16
Longitude in time,	10 12 40E.	Sun's semidiameter,	15 45
Greenwich Mean T.	8 47 20		

	° ′ ″		° ′ ″
Obs. Alt. l. l., .	10 30 0	Observed distance, .	95 16 0
Index error, .	— 0 50	Index error, . .	+ 1 10
Dip to 14 feet, .	— 3 43	Sun's semidiameter, .	+ 15 45
Semidiameter, .	+ 15 45	Apparent central distance,	95 32 55
Apparent altitude,	10 41 12		
Correction, .	— 4 56		
True altitude, .	10 36 16		

Now by the rule of the circular parts of Napier, applied to the right-angled spherical triangle H O ⊙, fig. page 246,

$$R \times \cos H\odot = \cos \odot O \times \cos HO, \text{ or } \cos HO = \cos H\odot \sec \odot O.$$

	° ′ ″		
H⊙, or apparent distance,	95 32 55	cosine	8.985383
⊙O, or apparent altitude,	10 41 12	sec	0.007599
HO, or HZ⊙, .	95 38 49	cos	8.992982

the difference of the azimuths of the sun and the object.

The sun's true azimuth may be computed by a formula similar to that for time, in page 339, thus—

TO FIND THE AZIMUTH.

Rule.—Set down the polar distance, the true altitude, and the latitude, then find half their sum, and the difference between this half sum and the polar distance. To the log secant of the altitude add the log secant of the latitude, the log cosine of the half sum and the log cosine of the difference; half the sum of these four logarithms will be the log sine of half the azimuth from the meridian, to be reckoned from the *south* in *north* latitude, and from the north in south latitude.

	° ′ ″		
Polar distance, . .	112 20 16		
True altitude, . .	10 36 16	secant,	0.007481
Latitude, . .	7 31 20	secant,	0.003754
Sum, . . .	130 27 52		
Half, . . .	65 13 56	cosine,	9.622153
Difference, . .	47 6 20	cosine,	9.832924
			19.466312
Half, . . .	32 44 54	sine,	9.733156
	2		
Sun's true bearing, .	N. 65 29 48 E.		
Object to left of sun .	95 38 49		
True bearing of object, .	N. 30 9 1 W.		
Magnetic bearing, .	N. 5 10 0 W.		
Variation of compass, .	24 59 1 W.		

Example 2.—On the 1st of May 1834, in latitude 33° 8′ 0″ N., longitude 16° 10′ W., the height of the eye 18 feet, the following observations were made to determine the true bearing.*

	h. m. s.		° ′ ″		° ′ ″
Mean time,	9 35 52 A.M.	Obs. Alt., ⊙̲	52 25 30	Obs. Dist., ⊙	111 34 0
Longitude,	1 4 40 W.	Dip to 18 ft.,	— 4 12	S. D.,	+ 15 53
		Sun's sem.-dr.,	+ 15 53		
Red. G.T.,	10 40 32			App. Dist.	111 49 53
		App. Alt.	52 37 11		

	° ′ ″		
App. Alt., .	52 37 11		
Correction, .	— 39	⊙'s Pol. Dist.,	75° 0′ 10″
True Alt. .	52 36 32		
Polar distance, .	75 0 10		
True altitude, .	52 36 32	secant, . .	0.216631
Latitude, .	33 8 0	secant, . .	0.077067
Sum, . .	160 44 42		
Half, . .	80 22 21	cosine, . .	9.223345
Difference, .	5 22 11	cosine, . .	9.998090
Sun's bearing, .	S. 69 48 40 E.	Reduced versine, .	9.515133

* The marks ⊙̲ mean the sun's lower limb, and ⊙ shows the position of the sun in the cross wires of the telescope, &c.

	° ′ ″			
Apparent altitude,	52 37 11	secant, . .	0.216738
Apparent distance,	111 49 53	cosine, . .	9.570399
Z to the right, .	127 46 25	cosine, . .	9.787137
Sun's bearing, .	S. 69 48 40 E.		
True bearing of object, .	S. 57 57 45 W.		

Should the object in example 1 be not on the level of the eye, the following method of computing the angle H Z ☉ must be employed.

	° ′ ″		
App. Cent. Dist. (D)	95 32 55		
Z. D. of point observed,	90 0 0	cosecant, .	0.000000
Sun's apparent Z. D.	79 18 48	cosecant, .	0.007600
Sum, . .	264 51 43		
Half (H) . .	132 25 51.5	sine, .	9.868110
Difference = H — D=	36 52 56.5	sine, .	9.778277
			19.653987
Half, . .	47 49 24.5	cosine, .	9.826993
	2		
Angle at zenith, .	95 38 49 as before.		

and this plan must be always followed when both zenith-distances differ considerably from 90°, or even when it is *doubtful* if the object be on the same level with the eye.

Example 3.—At Dunii Cairn, Iona, on the 21st of August 1839, in latitude 56° 20′ 34″ N., longitude in time 25ᵐ 34ˢ W., observations were taken with an astronomical circle, and reduced as stated below.

	h. m. s.
Mean time by watch, ...	7 9 46.2
Error of watch, fast, ...	— 11 54.0
Mean time, ...	6 57 52.2
Equation of time, ...	— 3 0.2
Apparent time, or angle at the pole, *t*, . .	6 54 52.0

	° ′ ″		
Polar distance, . .	77 49 2		
True altitude, .	2 42 16	secant,	0.000484
Latitude, . .	56 20 34	secant,	0.256315
Sum, . .	136 51 52		
Half, . .	68 25 56	cosine,	9.565377
Difference, . .	9 23 6	cosine,	9.994148
			19.816324
	54 2 13.5	sine,	9.908162
	2		
Azimuth, . .	S. 108 4 27 W.		
	180 0 0		
Azimuth, . .	N. 71 55 33 W.		

The azimuth may also be determined by the formulæ in page 345.

	° ′ ″
Let l=the latitude, ...	= 56 20 34 N.
and d=the declination, ...	= 12 10 58 N.
$d+l$=	68 31 32
$\frac{1}{2}(d+l)$ = ...	34 15 46
$l-d$ = ...	44 9 36
$\frac{1}{2}(l-d)$ = ...	2 4 48

	h. m. s.				
$\frac{1}{2}t$=	3 27 26	cot	9.895022	. . .	9.895022
$\frac{1}{2}(l-d)$=	22° 4′ 48″	cos	9.966920	sine . . .	9.575073
$\frac{1}{2}(l+d)$=	34 15 46	cosec	0.249500	sec . . .	0.082776
$\frac{1}{2}(m+e)$=	52 16 18.5	tan	10.111442	$\frac{1}{2}(m-e)$ = 19° 39′ 18″.6 tan	9.552871
$\frac{1}{2}(m-e)$=	19 39 18.5				

m=N. 71 55 37.0 W.
1. e= 108 4 27.0
2. e=S. 108 4 23.0 W.

Mean=S. 108 4 25 W.
Arc= 68 54 3 or horizontal angle at Dunii by the circle to Carn Cul ri Eirn.

Diff. S. 39 10 22 W. the bearing of Carn Cul ri Eirn from Carn Dunii in Iona.

Change of azimuth in	
10 minutes	2° 3′ 54″.0
1 minute	12 23 .4
10 seconds	2 3 .9
1 second	12 .39

Example 4.—On the 11th of August 1841, at my station, in latitude 55° 27′ 56″.74 N. longitude, 4° 37 35″ W., at 4h 40m 17s.5 mean time, by observations on the limbs of the sun, taken alternately above and below the central horizontal wire, and to the right and left of the vertical wire, in opposite quadrants of the diaphragm, so that the mean of all might be that of the sun's centre at the intersection of the wires in the centre of the telescope, I found the true altitude of the sun's centre, deduced from the *vertical arc* of my circle, to be 24° 25′ 45″, when the polar distance was 74° 47′ 14″; while, by the *horizontal arc*, the angle between the sun's centre and Ayr High Spire, was 2° 52′ 23″.3 W., distant 1152 feet; required the latitude and longitude of the spire?

		°	′	″	
ANS. The azimuth of the sun by numerous obs.	S.	81	16	42.8	W.
" spire "	S.	84	9	6.1	W.
Reduction of station to spire, in lat.,	—	0	0	1.16	S.
" " " in long.,	+	0	0	19.88	W.
Latitude of spire, . . .		55	27	55.58	N.
Longitude of spire, . . .		4	37	54.88	W.

In a similar way the common theodolite may be employed, and the result will be within a few minutes of the truth: whence also the variation may be obtained by taking bearings at the same time by the needle. These will give results sufficiently correct for all the usual purposes of the common land and marine surveyor, who should never trust, in remote localities, to the ordinary received amount of the variation of the compass.

EDINBURGH, *March* 21, 1840.

The following observations were made with a five-inch theodolite, to determine the bearing of a point on the west front of Salisbury Crags, from a position near the middle of the Queen's Park, east of Holyrood palace. The author was assisted by two of the Hon. East India Company's engineers, then receiving instruction in surveying and levelling.

The theodolite was properly levelled, and in adjustment in every respect, the vertical wire of the diaphragm was made a tangent to the point whose bearing was required, while the vernier was set to zero. The bearing by the needle was N. 232° 20′ E. from a mean of the theodolite and a Schmalcalder compass needle.

The times by chronometer were also taken, so that the azimuth might be computed both by the altitude and time. Chronometer slow of mean time 12 .5.

	Times A. M. by Chron.		Sun's Hor. Angle.			Altitude.
	h. m. s.		° ′			° ′
	11 26 20	1	N. 141 45 E.			34 8 u. l.
	11 31 40	2	142 40			33 45 l. l.
	11 35 25	1	144 30			34 21 u. l.
	11 40 45	2	145 24			33 56 l. l.
Mean,	11 33 32.5		143 34 45″		App. Alt. C.	34 2 30″
Cor.,	+ 12.5		180 0 0		Correction,	— 1 19
E. M. T.,	11 33 45	From sun to Crags	36 25 15		Sun's true alt.	34 1 11
Long. Ed.,	+ 12 43				Sun's Dec.,	° ′ ″ 0 22 52 N.
G. M. T.,	11 46 28				Pol. Dist.,	89 37 8

		° ′ ″		
Sun's polar distance,		89 37 8		
True altitude,	.	34 1 11	secant, .	0.081527
Latitude, .	. .	55 57 15	secant, .	0.251924
Sum, .	.	179 35 34		
Half, .	.	89 47 47	cosine, .	7.550678
Difference,	.	0 10 39	cosine, .	9.999998
				17.884127
		° ′ ″ 84 58 46	cosine, .	8.942063
		2		
From altitude,		N. 169 57 32 E.		

	h. m. s.
Mean time at Edinburgh, . . .	11 33 45 A.M.
Equation of time, ...	— 7 15.8
Apparent time, ...	11 26 29.2
	12 0 0.0
Apparent time from noon, or t = . .	0 33 30.8

		° ′ ″		° ′ ″
Sun's polar dist.,	.	δ, 89 37 8	. .	89 37 8
Co-latitude,	.	κ, 34 2 45	. .	34 2 45
$\delta+\kappa$.	.	= 123 39 53	$\delta-\kappa$, .	= 55 34 23
$\frac{1}{2}(\delta+\kappa)$.	.	= 61 49 56	$\frac{1}{2}(\delta-\kappa)$.	= 27 47 11

BY NAPIER'S ANALOGIES.

	° ′ ″				
$\frac{1}{2}(\delta + x) =$	61 49 56	cosec	0.054744	secant	0.326007
$\frac{1}{2}(\delta - x) =$	27 47 11	sine	9.668550	cosine	9.946792
	h. m. s.				
$\frac{1}{2} t =$	0 16 45.4	tan	8.864780	tangent	8.864780
$\frac{1}{2}(s + x) =$	87 46 55″	cot	8.588074	$\frac{1}{2}(s - x)$ cot	9.137579
$\frac{1}{2}(s - x) =$	82 11 2				

1st Bearing,	N. 169 57 57	E. of Sun.
2d Bearing,	169 57 32	page 406.
Mean,	N. 169 57 45	E.
Angle,	. 36 25 15	between Sun and Crags.
Bearing,	. N. 206 23 0	E. of the point observed.
	180 0 0	
Or,	. S. 26 23 0	W.

	° ′
True bearing of the point observed, . .	N. 206 23 E.
Magnetic bearing, ...	N. 232 20 E.
Variation of the compass, ...	25 57 W.

So far as the accuracy of the construction of the instrument employed can be trusted, this method is recommended to the attention of land-surveyors, in preference to trusting to the account given of the amount of variation in any locality, because it is not a constant but a variable quantity.

CONTOURS ON MAPS.

1. A distinct idea may be readily formed of the nature of contours by conceiving the irregular conical mass, represented by figure 1, Plate XXXI., to be a rock in the ocean, whose bottom, *o h*, is on a level with the surface of the sea at low water. After a short time, the tide rises gradually, say to one foot, and the surface of the water then corresponds with the first level line, 1, *g*; then with 2, *f*; 3, *d*, *e*; 4, *c*; and *a*, *b*, in succession. If these successive level lines be projected orthographically upon a plane by the dotted lines *h*, *h'*; *g*, *g'*; *f*, *f'*, &c., the irregular curves passing through the projected points, *h'*, *g'*, *f'*, *e'*, *d'*, *c'*, *b'*, *a'*, will give the contours of equal height surrounding the original mass, as shown by the figure in the corresponding elevation and projection.

2. If the ground to be represented be of moderate extent, the horizontal curves, which in a certain sense form the frame-work, are sensibly level. The line A B, in figure 2, on the ground, at once perpendicular to all their curves, is often one of double curvature, and is named *the line of greatest slope*, because, in fact, the *inclination* of a minuter portion, as A *m* of that line is *greater* than that of any other portion A *m'* of every other line A B', proceeding from the same common point A. Hence, when a plane is inclined to the horizon, its lines of greatest slope are lines perpendicular to all the horizontal lines drawn in this plane. The horizontal projection of this line of greatest slope is also perpendicular to the projections of the lines of level. These equidistant contours may also be filled in by numerous lines of slope, as in fig. 4, which, when neatly done, produces a good effect.

The ordnance maps, however, of this country, on a scale of six inches to a mile, have the contour curves only inserted, and want the line shading, called *hachures*. To those acquainted with the art of levelling, previously explained, the method of tracing these contours in the field is remarkably simple. Long flag staves or pickets are first erected, one at the top, and another at the bottom of such slopes as best define the ground, such as ridges or watershed lines, and water courses. Should no such sensible and easily-recognised lines exist, the poles must be placed at about equal distances, regulated in some degree by the minuteness required, and the various undulations in the surface of the ground. Almost close with the surface, a short stake or peg is driven in the intended line of contours, and also in a line between two of the long pickets. The spirit-level is then placed so as to command the best general view of this first line, and properly adjusted. Care must be taken that its axis is not so high as to be above the top of the levelling staff, if the *lower* contour is to be *first* traced, nor so low as to cut the ground under the picket, if the higher contour is first to be traced. The staff is then placed on or near this peg or short picket, and its vane raised or depressed till the proper mark on it is intersected by the cross wires of the telescope, when the vane ought to be *clamped to the staff, to retain it accurately in the same position.* The staff, with its vane kept to this height, is then shifted to another point, on or about the same level, and in a line between the next two long pickets; the staff itself, with its attached vane, being moved together up or down the slope till the proper mark on the vane coincides with the cross wires in the telescope of the level, turned in the proper direction, at which point another short stake or peg is driven. This operation is continued till the first contour is completed. To trace the next contour, the spirit-level must be placed on or near a picket in the first curve; and if the first contour is the lowest, the vane on the staff must be lowered such a number of feet as is equal to the vertical distance between each contour. If the staff is not long enough to effect this, the position of the next contour may be determined by the usual process of compound levelling, till the proper position is obtained. This being accomplished, the same process must be repeated to obtain the second contour; and so on, till the whole is completed.

The use of all these short pickets, indicating the contours in the same line down the slopes, is, when they are to be laid down on a plan, to facilitate the necessary measures between the original long pickets and entering the distance on these lines, with the offsets to the right or left of the short pickets or pegs which mark the horizontal lines.

When the difference of level between two points, A′ and B, fig. 3, has been determined, which belong to the *same slope*, it is evident that these two points are the extremities of the hypotenuse of a right-angled triangle, A′ H B, right-angled at H, whose base, H B, and height, H A′ are respectively the horizontal and vertical projections of that line. If, therefore, the equidistance of the horizontal and vertical slices or sections be fixed at 10 feet, on setting out from the highest point, A′, A, there may be drawn the horizontal lines, 1″, 1, 1′; 2″, 2, 2′; 3″, 3, 3′, &c., each at the distance of ten feet; and the points 1′, 2,′ 3′, &c. of the profile A′B, referred to the plane, will be those through which the projections of the horizontal slices will pass to the orthogonal representation indicated by the curves. Performing the same operation upon the profile A C, the corresponding points, 1″, 2″, 3″, and of the curves of level, 1′, 1″; 2′, 2″; 3′, 3″, &c., that are traced afterwards mentally, and in giving them the inflexions which the form of the ground requires in those parts which have not been instrumentally levelled.

In towns the vertical distance of these contours should not exceed 5 feet, in valuable rich ground about 10 feet, increasing in distance as they are carried higher, and the ground becomes less valuable.

The plans may be finished by lines called strokes or *hachures*, in such places as may be thought desirable, similar to figure 4, otherwise omitted, retaining the curve lines only.

The map of the Pyrenees will afford an interesting example of the method of finishing hill shading. The chain of these mountains was surveyed and levelled trigonometrically in the manner indicated in this work, by Colonel Coraboeuf, of the French engineers, in the years 1825, 1826, and 1827. The instruments used were ten-inch repeating circles of Borda's construction, and made by Gambey. They were constructed of small dimensions, for the sake of lightness and portability, that they might be readily transported to the top of these high regions, involving great difficulties of ascent. Of course the telescopes could not be very powerful, but they were found to be fully sufficient for the purpose in these elevated regions, extending to ten thousand feet in height, where the atmosphere was remarkably clear. The levelling showed the mean level of the ocean near Bayonne, and that of the Mediterranean in the neighbourhood of Perpignan, to be nearly the same. From this operation, detailed at length in *Nouvelle Déscription Géomètrique de la France*, by M. Puissant, première partie, page 334, &c., the best maps have been derived, but our limits will not permit us to give even an abstract here. The methods, however, given previously are quite sufficient to enable any intelligent engineer, sufficiently instructed, to execute similar operations in any part of the world.

In commencing or completing the survey of a distant country, the longitude ought to be fixed astronomically with all the accuracy possible. There are various methods of doing this, such as by lunar distances, by moon culminating stars, by eclipses and occultations. For the methods of performing such operations, see Galbraith's *Mathematical Tables*, Riddle's *Navigation*, Inman's *Navigation*, Mendoza Rios' *Nautical Tables*, or the French edition by Richard. The tables for computing time, &c., in the latter are not so convenient as in the English edition. To these may be added Coleman and Thomson's *Lunar and Horary Tables*, especially for finding the longitude by lunar distances in a simple manner, though less accurate in some cases.

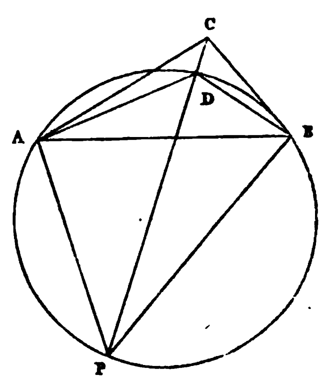
PRACTICAL EXERCISES

ON THE MEASUREMENT OF BASES, DISTANCES, ETC., UNDER PARTICULAR CIRCUMSTANCES.

EXAMPLE 1.—From a convenient station, P, there could be seen three objects, A, B, and C, whose distances from each other were A B = 8 miles, A C = 6 miles, B C = 4 miles; I took the horizontal angles A P C = 33° 45′, B P C = 22° 30′. It is hence required to determine the respective distances of my station from each object. Here it will be necessary, as illustrative and preparatory to the computation, to describe the manner of

Construction.

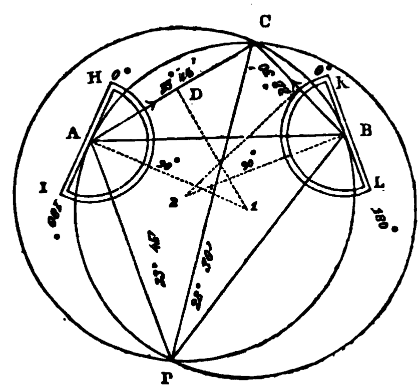
Draw the given triangle A B C from any convenient scale. From the point A draw a line A D to make with A B an angle D A B equal to 22° 30′, and from B a line B D to make an angle D B A equal to 33° 45′. Let a circle be described to pass through their intersection D, and through the points A and B. Through C and D draw a straight line to meet the circle again in P, which is the point required. For drawing P A, P B, the angle A P D is evidently equal to A B D, since it stands on the same arc A D; and, for a like reason, B P D = B A D. So that



P is the point where the angles have the assigned value.

GRAPHICAL SOLUTION FOR THE USE OF MARINE SURVEYORS, &c.

Let A B C be the given triangle, constructed as before: lay the centre of the protractor over the point A, with the division 33° 45′, on the side A C, and mark the point 90°; produce the line A 90° to 1; bisect A C in D, and draw D 1 at right angles to A C, the point of intersection 1, is the centre of the circle A C P which will contain an angle A P C, = 33° 45°.



Again, lay the protractor on the point B, with the division 22° 30′ on B C, and mark the point 90°: produce the line B 90° to 2; bisect the side B C in E, and at right angles to B C draw E 2, meeting B 2 in 2; the point 2 will be the centre of a circle C B P, which will contain an angle B P C = 22° 30′, and intersecting the former circle in P, the point required.

With a scale of chords this may also be easily effected, by making the angle C A 1 equal to the complement of the observed angle A P C, and C B 2 equal to the complement of B P C; the points of intersection 1 and 2 will be the centres of the required circles which determine the point P, as before.

The same thing may be done by placing 90° upon A C, and pointing off 33° 45′ from the other end of the protractor I, and 22° 30′ from L; that is, making the points 90° and 33° 45′ and 90° and 22° 30′ exchange places. The truth of this construction, which has been long used by the author, is obvious from Euclid III. and 32; that is, the angle between the chord A C and the tangent A H is equal to the angle A P C in the alternate segment of the circle A P C. In like manner, K B C = B P C.

Should the point of observation fall within the triangle A B C, or should P and D, in the first figure, interchange places, make A B P = A D P, equal to the supplement of A D C, the observed angle; and B A P = B D P, the supplement of B D C, the other observed angle: then about the triangle A P B, describe the circle A P B D; join P C, cutting the circle in D, the point required.

Computation.

In the triangle A B C, all the sides are given; to find the angles. In the triangle A B D, all the angles are known, and the side A B; to find both or one of the other sides A D. Take B A D from B A C, the remainder, D A C, is the angle included between two known sides, A D, A C; from which the angles A D C and A C D may be found. The angle C A P=180°—(A P C+A C D). Also, B C P = B C A — A C D; and P B C = A B C + P B A = A B C + sup A D C. Hence the three required distances are found by these proportions.

As sin A P C : A C : : sin P A C : P C, and : : sin P C A : P A; and, lastly, as sin B P C : B C : : sin B C P : B P. The operation at length is as under:—

By Rule II., Case III., Galbraith's *Mathematical Tables*, we have, using natural numbers when small—

Sin $\frac{1}{2}$ B A C = $\sqrt{\frac{1 \times 3}{8 \times 6}} = \sqrt{\frac{1}{16}} = \frac{1}{4} =$ ·25 = sin 14° 28′ 30″, and

B A C = 28° 57′ 18″.

Sin $\frac{1}{2}$ A B C = $\sqrt{\frac{1 \times 5}{8 \times 4}} = \frac{1}{8}\sqrt{10} =$ ·3952847 = sin 28° 17′ 1″ ½, and

A B C = 46° 34′ 3″.

Sin $\frac{1}{2}$ A C B = $\sqrt{\frac{3 \times 5}{6 \times 4}} = \sqrt{\frac{5}{8}} = \frac{1}{4}\sqrt{10} =$ ·7905694 = sin 52° 14′ 19″½, and

A C B = 104° 28′ 39″.

D A B =	22° 30′	C A B =	28° 57′ 18″		180° 0′ 0″
D B A	33 45	D A B =	22 30 0	D A C =	6 27 18
Sum	56 15	D A C =	6 27 18	A D C + A C D =	173 32 42
	180 0			½ (A D C + A C D) =	86 46 21
A D B	123 45				

As sin A D B 123° 45′ ar co log		0.0801536
Is to A B 8 miles		0.9030900
So is sin A B D 33° 45′		9.7447390
To A D log		0.7279826
A C 6 miles, log + 10		10.7781513
Arc . 48° 18′ 7″ tan		10.0501687
Subtract 45 0 0		
Remainder 3 18 7 tan		8.7611283
½ (A D C + A C D) = 86 46 21 tan		11.2487967
½ (A D B — A C D) 45 39 17 tan		10.0099250
A C D = 41 7 4		
A C D 41° 7′ 4″ sin	9.8179678	
A P C 33 45 0 ar co sin.	0.2552610	0.2552610
Sum 74 52 4		
180 0 0		
P A C 105 7 56 sin		9.9846740
A C 6 miles log	0.7781513	0.7781513
P A 7.10199 miles	0.8513801	
P C 10.42525 miles		1.0180863

A C B = 104° 28′ 39″		180° 0′ 0″
A C D = 41 7 4	B C P + B P C =	85 51 35
B C P = 63 21 35	P B C =	94 8 25

As sin B P C 22° 30′ 0″ ar co log	0.4171603
Is to B C 4 miles	0.6020600
So is sin B C P 63° 21′ 35″	9.9512594
To P B 9.34285 miles	0.9704797

The computation of problems of this kind, however, may be a little shortened by means of the following

*General Investigation.**

Put A C = a, B C = b, A P C = P, B P C = P′, A C B = C,

* See Cagnoli's Trig., § 802; Lacroix, Trigonometrie; Gregory's Trigonometry; Puissant, Geodesie, vol. i. page 234.

P A = D, P C = D′, and P B = D″, and let there be taken for unknown quantities P A C = x, P B C = y. The triangles P A C and P B C give

$$\text{Sin APC} : \sin \text{CAP} :: \text{AC} : \text{CP, and}$$
$$\text{Sin BPC} : \sin \text{CPB} :: \text{BC} : \text{CP; that is,}$$

$$\text{Sin P} : \sin x :: a : \frac{a \sin x}{\sin \text{P}} = \text{C P, and}$$

$$\text{Sin P}' : \sin y :: b : \frac{b \sin y}{\sin \text{P}'} = \text{C P}$$

Hence, $\frac{a \sin x}{\sin \text{P}} = \frac{b \sin y}{\sin \text{P}'}$; which may be reduced to

$$a \sin \text{P}' \sin x - b \sin \text{P} \sin y = 0.$$

In the quadrilateral A C B P, we have

$$\text{CBP} = 360° - \text{APC} - \text{BPC} - \text{ACB} - \text{CAP, or}$$
$$y = 360° - \text{P} - \text{P}' - \text{C} - x.$$

Make $360° - \text{P} - \text{P}' - \text{C} = \text{R}$, then we shall have $y = \text{R} - x$; and consequently,

$$a \sin \text{P}' \sin x - b \sin \text{P} (\sin \text{R} \cos x - \cos \text{R}' \sin x) = 0.$$

Dividing by $\sin x$, there results

$$a \sin \text{P}' - b \sin \text{P} \left(\sin \text{R} \frac{\cos x}{\sin x} - \cos \text{R}\right) = 0.$$

Whence we have $\frac{\cos x}{\sin x} = \cot x = \frac{a \sin \text{P}' + b \sin \text{P} \cos \text{R}}{b \sin \text{P} \sin \text{R}}$

This expression being separated into two parts, we have

$$\cot x = \frac{a \sin \text{P}'}{b \sin \text{P} \sin \text{R}} + \frac{\cos \text{R}}{\sin \text{R}}; \text{ or,}$$

$$\cot x = \frac{\cos \text{R}}{\sin \text{R}} \left(\frac{a \sin \text{P}'}{b \sin \text{P} \cos \text{R}} + 1\right); \text{ or,}$$

$$\cot x = \cot \text{R} \left(\frac{a \sin \text{P}'}{b \sin \text{P} \cos \text{R}} + 1\right) \quad . \quad . \quad . \quad (1)$$

$$\cot x = \cot \text{R}\left(\sin \text{P}' \operatorname{cosec} \text{P} \sec \text{R} \frac{a}{b} + 1.\right) \quad . \quad . \quad . \quad . \quad (2)$$

$$\cot x = \frac{a}{b} \sin \text{P}' \operatorname{cosec} \text{P} \sec \text{R} \cot \text{R} + \cot \text{R}. \quad . \quad . \quad . \quad (3)$$

or, lastly, Delambre's and Puissant's formulæ may be employed.

Hence, x being thus determined, we get y from the equation $y = R - x$; and C P from either of the expressions given above.

We shall now apply the foregoing formula to the solution of the question last proposed.

Here $a = 6$	P =	33° 45′ 0″		P A C = x
$b = 4$	P′ =	22 30 0		P B C = y
	A C B =	104 28 39		found by computation.
		160 43 39		
		360 0 0		
	R =	199 16 21		

By formulæ (2) and (3) using logarithms

we have	a' =		3 log	0.4771212	or,	0.477121
	b' =		2 *a. c. l.*	9.6989700		9.698970
	P′ =	22° 30′ 0″	sin	9.5828397		9.582840
	P =	33° 45′ 0″	ar co S.	0.2552610		0.255261
R whose cos is neg		199 16 21	ar co C	0.0250466		0.025047 —
					cot R	0.456359 +
	—	1.094557	log	0.0392385		0.495598 —
	+	1.000000		158		
					N =	— 3.13039
	—	0.094557	log	8.9756937	cot R =	+ 2.85995
cot R	+	199° 16′ 21″		10.4563587		— 0.27044
						— 105° 7′ 58″
cot x	—	105 7 57		9.4320524		
As sin 33° 45′ 0″		ar co		0.2552610		
Is to sin x 105 7 57		- - - -		9.9846734		
So is 6	- - -	- - -	- -	0.7781513		
To P C	10.42522			1.0180857		
				761		
				96		

whence, as before, the rest may be found.

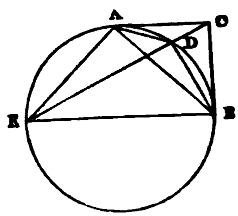
In using these formulæ, great attention must be paid to the signs of the quantities.

EXAMPLE 2.—From the station D, on Carn Dunii, in Iona, the angle Bein Heynish, Iona North Pile, Benmore, in Mull, or A D C, was observed to be 136° 48′ 38″; the angle Benmore, Iona North Pile, Jura, or C D B, was 84° 54′ 13″; while the distance of Bein

Heynish from Bein More, or A C, was 184,335 feet; the distance of Bein More from Jura, or C B, was 190,826 feet; and the distance of Jura from Bein Heynish was 275,405 feet: required the distance of Dunii Pile, or D, from these, that is D A, B D, and D C?

GEOMETRICAL CONSTRUCTION.

With the three given sides construct the triangle A B C. At the point A draw the straight line A E, making the angle B A E = B D E = to the supplement of B D C. Again, at the point B make the angle A B E = A D E = to the supplement of A D C, producing the straight line B E to meet A E in E, and join E C. About the triangle A B E, describe the circle A B E, cutting the E C in D; then will D be the position of Dunii. Finally, join A D and D B.



BY COMPUTATION.

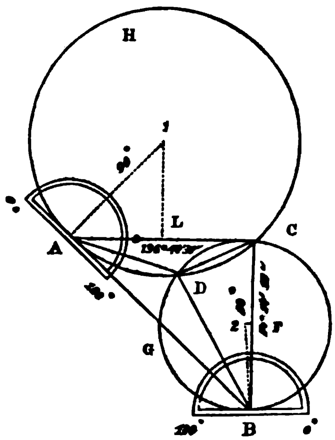
With the three sides of the triangle A B C compute the angles A = 43° 41′ 38″.50, B = 41° 51′ 34″.96, and C = 94° 26′ 46″.54.

1. As sin A E B : sin A B E : : A B, 275,405, : A E = 283,268 feet.
2. As sin A E B : sin B A E : : A B, 275,405, : B E = 412,248 feet.
3. With the two sides A E, A C, and the contained angle E A C, 138° 47′ 26″, find the angle A C E = 25° 9′ 10″, and A E C, or A B D = 16° 3′ 24″.
4. 180° − (A D C + A C D) = D A C = 18° 2′ 12″.
   As sin A D C : sin D A C : : A C 184,335, : D C = 83392.6 feet.
5. A B C − A B D = D B C = 25° 48′ 11″.
   As sin A D C : sin D C A : : A C 184,335, : A D = 114,476 feet.
6. A C B − A C D = D C B = 69° 17′ 35″.
   As sin B D C : sin D C B : : B C, 190,826 : B D = 179,207.4 feet.
7. B C bears N. 0° 35′ 34″ W., which, added to C B D, gives B D N. 26° 23′ 45″ W. Latitude of B in Jura is 55° 54′ 8″ N., longitude 6° 0′ 2″ W.

GRAPHICAL SOLUTION.

Let A B C be the given triangle: lay the centre of the protractor over the point A, with the division 136° 48′ 38″ over the side A C, and mark the point 90°. Produce the line A 90° to 1; bisect A C in E, and draw D 1 at right angles to A C: the point of intersection 1 will be the centre of the circle A H C D.

Again, lay the centre of the protractor over the point B, with the division 84° 54′ 13″ upon B C, and mark off 90°: produce a line B 90°; bisect B C in F, and draw F 2 at right angles to B C, meeting B 2 in the point 2, the centre of the second circle. These two circles will intersect one another in the point D, the station on Dunii Carn required. As a check, put 90° upon A C; point off 136° 48′ 38″ from right to left in the protractor, to the left of 90°; through A, 136° 48′ 38″, draw a line, E 1, to intersect A 1 in 1 as before. In like manner, lay 90° on B C; set off 84° 54′ 13″ to the left of 90°; draw B 84° 54′ 13″; draw F 2 to intersect B 90° in 2 as before: these centres should be the same points as before. With the preceding bearing and distance, Dunii is in latitude 56° 20′ 32″ N., longitude 6° 23′ 36″ W.



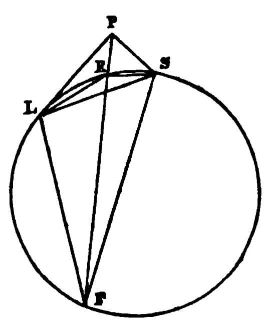
EXAMPLE 3.—From Plymouth the Lizard is distant 54.44 miles; from the Lizard the Start Point is distant 71.15 miles; and from the Start Point to Plymouth the distance is 23.31 miles.

			°	′	″	
From Eddystone light,	Plymouth bears	N.	25	4	5	E.
	Lizard, . .	S.	70	13	15	W.
	Start, . .	N.	83	52	20	E.

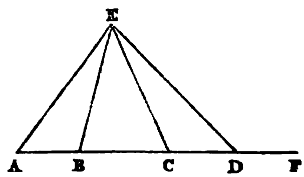
Let P be Plymouth, E Eddystone light, L the Lizard Point, and S the Start.

		°	′	″
Hence the angle	P E S =	58	48	15
	P E L =	134	50	50
	L P S =	127	47	29
	P L S =	15	0	17
	P S L =	37	12	14

Ans., . .	E L =	44.42 miles.
	E P =	13.10
	E S =	27.22



EXAMPLE 4.—To find the length of one of the three segments of a straight line when the two other segments, and the three angles which each segment subtends, at a given point without the line, are known.



Let A D be the straight line, A B, C D, the given segments, and B C that required, or a, b, and x; also E, the point at which the angles A E B $= \alpha$, A E C $= \beta$, and A E D $= \gamma$, were measured, it is required to find the interjacent segment B C or x?

Since the exterior angle of any triangle is equal to the sum ot the two interior and opposite angles, the angle E B C $=$ A $+ \alpha$, the angle E C D $=$ A $+ \beta$, and the angle E D F $=$ A $+ \gamma$. Now, by trigonometry, the triangles A B E, A C E, give

$$\frac{BE}{a} = \frac{\sin A}{\sin \alpha} \text{ and } \frac{CE}{a+x} = \frac{\sin A}{\sin \beta} \text{ whence, by division,}$$

$$\frac{BE}{CE} = \frac{a \sin \beta}{(a+x)\sin \alpha} \quad . \quad . \quad . \quad . \quad (1)$$

The triangles B E D, C E D, give, in like manner,

$$\frac{BE}{b+x} = \frac{\sin (A+\gamma)}{\sin (\gamma - \alpha)}, \text{ and } \frac{CE}{b} = \frac{\sin (A+\gamma)}{\sin (\gamma - \beta)}, \text{ whence}$$

$$\frac{BE}{CE} = \frac{(b+x)\sin (\gamma - \beta)}{b \sin (\gamma - \alpha)} \quad . \quad . \quad . \quad . \quad (2)$$

Equating these two values of $\frac{BE}{CE}$ in equations (1) and (2,) and there results—

$$\frac{a \sin \beta}{(a+x)\sin \alpha} = \frac{(b+x)\sin (\gamma - \beta)}{b \sin (\gamma - \alpha)}, \text{ from which is obtained}$$

$$x^2 + (a+b)\,x + a\,b = \frac{a\,b \sin \beta \sin (\gamma - \alpha)}{\sin \alpha \sin (\gamma - \beta)} \quad . \quad . \quad (3)$$

To resolve equation (3,) let $\tan^2 \varphi = \frac{4\,a\,b}{(a-b)^2} \cdot \frac{\sin \beta \sin (\gamma + \alpha)}{\sin \alpha \sin (\gamma - \beta)}$, and $x^2 + (a+b)\,x = \frac{1}{4}(a-b)^2 \tan^2 \varphi - a\,b$, whence

$$x = \pm \tfrac{1}{2}(a-b)\sqrt{(1 + \tan^2 \varphi)} - \tfrac{1}{2}(a+b), \text{ or finally,}$$

$$x = \pm \tfrac{1}{2}(a-b)\sec \varphi - \tfrac{1}{2}(a+b) \quad . \quad . \quad . \quad (4)$$

consequently x becomes known, and

$$a + x + b = + \tfrac{1}{2}(a-b)\sec \varphi + \tfrac{1}{2}(a+b) = AD \quad (5)$$

EXAMPLE 5.—Let $a = 2731$ feet, $b = 1987$, $\alpha = 19° 7'$, $\beta = 50° 12'$, and $\gamma = 65° 9'$, required x?

By formula (4), we have—

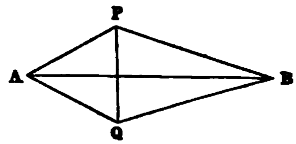
No.	4 . . .	log 0.602060			
$a =$	2731 . . .	log 3.436322			
$b =$	1987 . . .	log 3.298198			
$a-b =$	744 2 $a, c, l,$.	4.256854			
$b =$	50° 12′ 0″ sine .	9.885522			
$\gamma-\alpha =$	46 2 0 sine .	9.857178			
$\alpha =$	19 7 0 cosec .	0.484798			
$\gamma-\beta =$	14 57 0 cosec .	0.588421			
		2.409353			
$\varphi =$	86° 25′ 41″.6 tan .	11.204676 sec	. .	1.205519	
$\frac{1}{2}(a-b) =$	372 . log .	. . . . .	.	2.570543	
	5971.2 log .	. . . . .	.	3.776062	
$\frac{1}{2}(x+b) =$	∓ 2359.0				
$x =$	3612.2 = B C the segment required ;				
$a+x+b =$	8330.2 = A D the whole line.				

EXAMPLE 6.—Let $a=527.167$ feet, $b=315.063$ feet, $\alpha=2°\ 10'\ 30''.1$, $\beta=40°\ 13'\ 1''.9$, and $\gamma=42°\ 14'\ 2''.7$.

The separate measures of these angles were $\alpha=2°\ 10'\ 30''.1$, $\beta-\alpha=38°\ 2'\ 31''.8$, and $\gamma-\beta=2°\ 0'\ 57''.7$.

From the first of these series $\gamma=42°\ 14'\ 2''.7$, from the sum of the last $\gamma=42°\ 13'\ 59''.6$. The difference of these two values is 3″.1. Now, as the value of γ in the first and each of the three last angles was measured with equal care, one-fourth of 3″.1, or 0″.8 may, as the probable error, be added to the values of α, β, and γ respectively, they become $\alpha=2°\ 10'\ 30''.9$, $\beta-\alpha=38°\ 2'\ 32''.6$, $\gamma-\beta=2°\ 0'\ 58.5$, making the sum, or $\gamma=42°\ 14'\ 2$.

No.	4 log .	0.602060		
$a =$	527.167 log .	2.721948		
$b =$	315.063 log .	2.498397		
$a-b =$	212.104 2, $a, c, l,$	5.346902		
$\beta =$	40° 13′ 3″.5 sine	9.810026		
$\gamma-\alpha =$	40 3 31 .1 sine	9.808597		
$\alpha =$	2 10 30 .9 cosec	1.420718		
$\gamma-\beta =$	2 0 58 .5 cosec	1.453668		
		3.662316		
$\varphi =$	89 9 17.46 tan	11.831158 sec	. .	1.831205
$\frac{1}{2}(a-b) =$	106.052 log	. . . . .	.	2.025519
	7189.92 log	. . . . .	.	3.856724
$\frac{1}{2}(a+b) =$	∓ 421.12			
$x =$	6768.80 = B C, the segment required ;			
$a+x+b =$	7611.04 = A D, the whole line.			



Example 6.—In extending the measured base to the sides of adjacent triangles of greater magnitûde, the following method may be advantageously employed :—

Let P Q be the measured base, which, by means of triangulation, is to be extended to A B, the side of a larger triangle, constituting one of the series to be extended over a country; then

1. $\text{Sin } PAQ : \sin APQ :: PQ : AQ = \dfrac{\sin APQ.\ PQ}{\sin PAQ}$

2. $\text{Sin } PAQ : \sin AQP :: PQ : AQ = \dfrac{\sin AQP.\ PQ}{\sin PAQ}$

3. $\text{Sin } PBQ : \sin BPQ :: PQ : BQ = \dfrac{\sin BPQ.\ PQ}{\sin PBQ}$

4. $\text{Sin } PBQ : \sin BQP :: PQ : BP = \dfrac{\sin BQP.\ PQ}{\sin PBQ}$

5. $\text{Sin } PAB : \sin APB :: PB : AB = \dfrac{\sin APB.\ PB}{\sin PAB}$ (1)

6. $\text{Sin } QAB : \sin AQB :: QB : AB = \dfrac{\sin AQB.\ QB}{\sin QAB}$ (2)

Or, by substitution in (1.),

$$AB = \frac{\sin APB \times \dfrac{\sin BQP.\ PQ}{\sin PBQ}}{\sin PAB} = \frac{\sin APB \sin BQP.\ PQ}{\sin PAB \sin PBQ} \qquad (3)$$

$$AB = \ldots\ldots \frac{\sin APB \sin AQP.\ PQ}{\sin PAQ \sin PBA} \qquad (4)$$

$$AB = \ldots\ldots \frac{\sin AQB \sin PDQ.\ PQ}{\sin PBQ \sin QAB} \qquad (5)$$

$$AB = \ldots\ldots \frac{\sin AQB \sin APQ.\ PQ}{\sin PAQ \sin QBA} \qquad (6)$$

Any one of these last four equations will give A B, or the whole may be confined in one.

$$AB^4 = \frac{\sin^2 APB \sin^2 AQB \sin BQP \sin AQP \sin BPQ \sin APQ.\ PQ^4}{\sin^2 PBQ \sin^2 PAQ \sin PAB \sin PBA \sin QAB \sin QBA} \qquad (7)$$

By transformation there will be obtained—

$$PQ^4 = \frac{\sin^2 PBQ \sin^2 PAQ \sin PAB \sin PBA \sin QAB \sin QBA,\ AB^4}{\sin^2 APB \sin^2 AQB \sin BQP \sin AQP \sin BPQ \sin APQ} \qquad (8)$$

which may be employed to find the base of verification.

This formula may be applied to the trigonometrical survey—

though not to its full extent, for want of the necessary angles connecting the base measured on Hounslow Heath with the sides of the adjacent triangles.

Mean of the angles measured by Roy and Mudge,—

		°	′	″
St Ann's Hill reduced, or	PAQ =	44	18	51.62
Hampton Poorhouse, .	AQP =	61	26	33.53
King's Arbour, . .	APQ =	74	14	34.85
		180	0	0.00

Angles observed by Mudge and reduced,—

		°	′	″
Hanger Hill tower, or	ABQ =	24	39	16.33
Hampton Poorhouse, .	AQB =	130	3	3.08
St Ann's Hill, . .	QAB =	25	17	40.59
		180	0	0.00

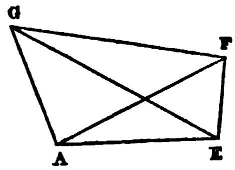
The mean length of PQ at the level of the sea is 27405.35 feet, whence, by formula (6),—

AQB	= 130° 3′ 3.08″ sine	9.8839303
APQ	= 74 14 34.85 sine	9.9833656
PAQ	= 44 18 51.62 *a, c, b*, sine . . .	0.1557749
ABQ	= 24 39 16.33 *a, c, b*, sine . . .	0.3797119
PQ	= 27405.35 feet, log	4.4378354
AB	= 69281.63 feet, log	4.8406181

the distance between Hanger Hill and St Ann's Hill.

EXAMPLE 7.—To determine the distance between two inaccessible points, when the angles between a given base and each of the points at both extremities of the base are given.

Let A E be the given base a, and G F the required distance b, G A E = α, F A E = β, A E F = γ, A E G = δ.



Whence A G E = θ, and A F E = φ: hence,

As sin A G E : sin G A E : : A E : G E = A E sin G A E cosec A G E,

As sin A F E : sin E A F : : A E : E F = A E sin E A F cosec A F E.

But Euclid II. and 12, $GF^2 = GE^2 + EF^2 - 2\,GE\,.\,EF \cos GEF$, which, by substitution, becomes

$GF^2 = AE^2 \sin^2 GAE \operatorname{cosec}^2 AGE + AE^2 \sin^2 EAF \operatorname{cosec}^2 AFE - 2\,AE^2 \sin GAE \operatorname{cosec} AGE \sin EAF \operatorname{cosec} AFE \cos (AEF - AEG)$

Hence,

$$b = a\,\{(\sin \alpha \operatorname{cosec} \theta)^2 + (\sin \beta \operatorname{cosec} \phi)^2 - 2 \sin \alpha \operatorname{cosec} \theta \sin \beta \operatorname{cosec} \phi \cos (\gamma - \delta)\}^{\frac{1}{2}} \quad . \quad . \quad (1)$$

and conversely,

$$a = b \div \{(\sin \alpha \operatorname{cosec} \theta)^2 + (\sin \beta \operatorname{cosec} \phi)^2 - 2 \sin \alpha \operatorname{cosec} \theta \sin \beta \operatorname{cosec} \phi \cos (\gamma - \delta)\}^{\frac{1}{2}} \quad . \quad . \quad (2)$$

Similarly,

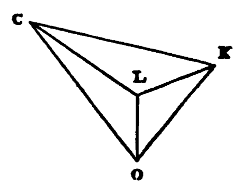
$$b = a\,\{(\sin \delta \operatorname{cosec} \theta)^2 + (\sin \gamma \operatorname{cosec} \phi)^2 - 2 \sin \delta \operatorname{cosec} \theta \sin \gamma \operatorname{cosec} \phi \cos (\alpha - \beta)\}^{\frac{1}{2}} \quad . \quad . \quad (3)$$

and conversely,

$$a = b \div \{(\sin \delta \operatorname{cosec} \theta)^2 + (\sin \gamma \operatorname{cosec} \phi)^2 - 2 \sin \delta \operatorname{cosec} \theta \sin \gamma \operatorname{cosec} \phi \cos (\alpha - \beta)\}^{\frac{1}{2}} \quad . \quad . \quad (4)$$

EXAMPLE 8.—To determine the same thing, if the given base have a perpendicular instead of a parallel direction to the required distance.

Let $OL = a$, $LOK = \alpha$, $LOC = \gamma$, $CKL = \theta$, $OKC = \psi$, $OLK = \kappa$: $CK = b$, $LKO = \beta$, $LCO = \delta$, $KCL = \phi$, $OCK = \omega$, $OLC = \rho$.



By a similar method of investigation,

$$b = a\,\{(\sin \alpha \operatorname{cosec} \beta)^2 + (\sin \gamma \operatorname{cosec} \delta)^2 - 2 \sin \alpha \operatorname{cosec} \beta \sin \gamma \operatorname{cosec} \delta \cos (\theta + \phi)\}^{\frac{1}{2}} \quad . \quad . \quad (5)$$

$$a = b \div \{(\sin \alpha \operatorname{cosec} \beta)^2 + (\sin \gamma \operatorname{cosec} \delta)^2 - 2 \sin \alpha \operatorname{cosec} \beta \sin \gamma \operatorname{cosec} \delta \cos (\theta + \phi)\}^{\frac{1}{2}} \quad . \quad . \quad (6)$$

Similarly,

$$b = a\,\{(\sin \kappa \operatorname{cosec} \beta)^2 + (\sin \rho \operatorname{cosec} \delta)^2 - 2 \sin \kappa \operatorname{cosec} \beta \sin \rho \operatorname{cosec} \delta \cos (\psi + \omega)\}^{\frac{1}{2}} \quad . \quad . \quad (7)$$

$$a = b \div \left\{ (\sin \kappa \operatorname{cosec} \beta)^2 + (\sin \rho \operatorname{cosec} \delta) - 2 \sin \kappa \operatorname{cosec} \beta \sin \rho \operatorname{cosec} \delta \cos (\psi + \omega) \right\}^{\frac{1}{2}} \quad . \quad . \quad (8)$$

Example illustrative of Formulæ (1) and (2).

Let α = 139° 15′ 45″	Whence $\alpha - \beta$ = 85° 45′ 22″	θ = 8° 55′ 15″
β = 53 30 23	$\gamma - \delta$ = 82 35 55	ϕ = 12 4 42
γ = 114 24 55		a = 6265,88 feet.
δ = 31 49 0	required b.	

α = 139° 15′ 45″ sin	9.814643	constant logarithm, or log 2,			0.301030
θ = 8 55 15 cosec	0.809473	$\gamma - \delta$ = 82° 35′ 55″ cosine,			9.109982
	0.624116	. . . .			0.624116
	2	β = 53° 30′ 23″ sine,	9.905215		
No. 1, + 17.71054 log	1.248232	ϕ = 12 4 42 cosec,	0.679337		
			0.584552		0.584552
			2		No.—0.619680
No. 2, + 14.76064		. . . . . log +	1.169104		—4.16562
No. 3, — 4.16562					

Sum, 28.30556	log	1.451872
	half	0.725936
a = 6265.88 feet,	log	3.796982
b = 33336.4 feet,	log	4.522918

Formula 3 would give the same result.

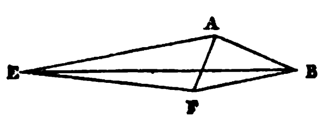
It is obvious that if b had been given, it is only necessary to subtract half the log of the sum from the log b to obtain the log a.

Thus from log b, ...	4.522918
Subtract log half, ...	0.725936
a = 6265.88 feet, log, ...	3.796982

By these means a series of triangles may be made to rest on a measured base of verification.

Example 9.—When the measured base intersects the required distance.

Let F A B = α =	108° 2′ 18″
A F B = β =	47 10 28
A B F = θ =	24 47 14
	180 0 0



FAE = γ = 47° 9′ 46″
AFE = δ = 121 13 20
AEF = ϕ = 11 36 54

180 0 0

AF = a = 386.55 feet.*

It may be supposed that θ and ϕ have been inferred if they cannot be directly observed.

By another investigation as formerly—

$$b = a\left\{(\sin\alpha \operatorname{cosec}\theta)^2 + (\sin\gamma \operatorname{cosec}\phi)^2 - 2\sin\alpha \operatorname{cosec}\theta \sin\gamma \operatorname{cosec}\phi \cos(\beta+\delta)\right\}^{\frac{1}{2}} \quad . \quad . \quad (9)$$

or,

$$b = a\left\{(\sin\beta \operatorname{cosec}\theta)^2 + (\sin\delta \operatorname{cosec}\phi)^2 - 2\sin\beta \operatorname{cosec}\theta \sin\delta \operatorname{cosec}\phi \cos(\alpha+\gamma)\right\}^{\frac{1}{2}} \quad . \quad . \quad (10)$$

By formula 9—

α = 108° 2′ 18″ sin 9.9781118 log 2 . . . . . . 0.3010300 —
θ = 24 47 14 sin 9.6224727 $\beta + \delta$ = 168° 23′ 48″ cosine 9.9910326 —

Diff. 0.3556391 . . . . 0.3556391
2 γ = 47 9 46 sin 9.8652748

No. 1 = + 5.14373 log 0.7112782 ϕ = 11 36 54 sin 9.3039179

Diff. 0.5613569 0.5613569

2 1.2090586 +

No. 2 = + 18.26520 . . log . . . 1.1227138 log No. 3
No. 3 = + 16.18299

Sum = 34.59192 log 1.5389747

Half 0.7694873
a = 386.55 feet log 2.5872057

b = 2273.49 feet log 3.3566930

By formula (10)

No. 1 = + 3.06033
2 = + 18.04107

3 = + 13.49051

Sum as before 34.59191
and would give the same final result when carried out.

* There is a typographical error in page 320, when A F = 387.55 instead of 386.55.

EXAMPLE ILLUSTRATIVE OF FORMULA (5) AND (7).

O L = a = 7703 feet.

	° ′ ″		° ′ ″
L O K = α =	17 39 0.5	L O C = γ =	41 56 9.0
L K Q = β =	5 48 52.9	L C O = δ =	9 38 29.1
$\alpha + \beta$ =	23 27 53.4	$\gamma + \delta$ =	51 34 38.1
	180 0 0.0		180 0 0.0
O L K = x =	156 32 6.6	O L C = ρ =	128 25 21.9
O L C = ρ =	128 25 21.9		
	284 57 28.5		
	360 0 0.0		
K L C = $\theta + \phi$ =	75 2 31.5		

substituting these in the proper formula—

and b = ...	33323.3 feet.
former value of b, page 425, . . .	33336.4
mean of both	33329.85 feet.

ON THE DETERMINATION OF THE LONGITUDE OF AN IMPORTANT POINT ASTRONOMICALLY.

In conducting a series of trigonometrical observations, as previously observed, in a distant country, it will be necessary to determine the longitude of some convenient and well-known point astronomically with regard to a given meridian, as that of Greenwich. I had not an opportunity of fixing with sufficient precision astronomically the longitude of any point in Arran in this manner, but to show the method of procedure in regard to eclipses and occultations, the following example for Edinburgh may be useful, and the calculations for this purpose, as well as for others, may be readily performed by the rules and formulæ given in my *Mathematical and Astronomical Tables*. To insure the utmost possible accuracy, numerous observations should be made on solar eclipses, occultations of fixed stars by the moon, moon culminating stars, &c. &c. by a combination of which the most minute errors may be eliminated, and thence the longitude of the point of observation determined with the utmost precision, especially if corresponding observations at the two places can be obtained.

I.—Determination of the Longitude of Edinburgh Observatory by the Solar Eclipse of 15th May 1836, from the Observations of WILLIAM GALBRAITH.

The chronometer used was set to mean time shortly before the commencement.

	h.	m.	s.
Suspected contact, by chronometer,	1	33	0
Certain—somewhat on,	1	33	10
Annulus complete,	2	57	20
Annulus broke,	3	1	10
End of eclipse very distinct,	4	19	20

At commencement the chronometer was nearly correct, but was, on the termination, 1.6s fast of the clock, while by observations taken that day, the clock was 4.0s slow; whence the chronometer lost 2.4s during the continuance of the eclipse, or in about $2^h\ 46^m\ 20^s$. Hence the chronometer lost 0.86s hourly, and the corrected times will be—

	h.	m.	s.
Commencement,	1	23	0.00
Annulus complete,	2	57	20.35
Annulus broke,	3	1	11.26
End of eclipse,	4	19	22.40

As the commencement was lost by Mr Henderson, late astronomer, from an oversight, no comparison could be made for a check at that phase, but the others agreed as nearly as could be expected. The first observation, therefore, may be a few seconds out, on account of the difficulty of getting the time of first contact correctly. The longitude will therefore be determined by the last.

	h. m. s.
Mean time of last contact,	4 19 22.40
Longitude in time by estimation West,	+ 12 43.50
Mean time at Greenwich,	4 32 5.90

For computation 4^h 32^m will be assumed.

	h. m. s.
☉'s R. A. at 4 32 (h. m.) by Nautical Almanac,	3 29 46 .76
Declination,	18° 59′ 13″.75 N.
Semi-diameter,	15′ 49″.8
Augmentation for about 26° of alt.,	— 7 .2
Reduced semi-diameter,	15 42 .16

	h. m. s.
☽'s R. A. at the assumed time, 4 32 (h. m.),	3 33 48.14
„ at 3^h afterwards, or, 7 32,	3 39 51.98
Declination at 4 32,	19 46 39.8 N.
Semi-diameter, at 4 32,	14 59.2
Equatorial horizontal parallax,	54 22.6
Reduction to latitude 55° 57′ 23″ N.,	— 7.2
Reduced horizontal parallax for latitude,	54 15.4
☉'s horizontal parallax by Nautical Almanac,	— 8.4
Difference of sun and moon's horizontal parallax,	54 7.0

	h. m. s.
Mean time of observation,	4 19 22.40
Equation of time at 4^h 20^m,	+ 3 55.90
Horary angle from the meridian,	4 23 18.30

Diff. of par. . 0° 54′ 7″.0 P. L. . 0.52194 P. L. . 0.52194 P. L. 0.52194 C. L. 1.17609
Red. latitude . 55 46 23.0 sec . 0.24996 sec . 0.24996 cosec 0.06256
Hor. angle (a) 4h 23m 18s.3 cosec . 0.03986, 18° 59′ 13″.8 ∂ cosec } 0.45764 sec . 0.02439
Sum 0.81176 . . . . 0.81176
C. L. 1.17609 + 42 18 .6 . . P. L. 0.62880
Sun's dec. . 18° 59′ 13″.8 cos . 9.97570 19 41 32 .4

	h.	m.	s.		
Arc (1) (2 b)	0	1	57.46	P. L.	. 1.96355
Half (b)	0	0	58.73		

(a—b) . − 4 22 19.57 sec 0.38363
− 4 5 .6 1.64317 P. L.
Moon's true app. dec. . . . . . . 19 37 26 .8 cosine 9.97401
Moon's red. declination . . . . . . 19 46 39 .8 . . . 0h 1m 57s.92 P. L. 1.96186
Sun's semi-diameter 15′ 42″.6 Difference − . 9 13 .0 ⊙'s R. A. 3 29 46.76
Moon's semi-diameter 14 49 .2 3 31 44.68
Sum . . . 30 31 .8 . . . 30 31 .8 + 2 3.66
Moon's app. dec. 19° 37′ 26″.8 Sum . . 39 44 .8 P. L. 0.65596 3 33 48.34
Moon's red. dec. 19 46 39 .8 Diff. . . 21 18 .8 P. L. 0.92665 3 33 48.14
Sum . . 39 24 6 .6 Sum 1.58260 3 29 51.98
Half 0.79130 Δ_1 0.20 P. L. 4.73239
Half . 19 42 3 .3 cosine . . . . . . 9.97380 Δ_2 6 3.84 P. L. 1.47251
Const log . . . . 1.17609 5.94 } P. L. 3.26968
2m 3s.66 P. L. 1.94119 4 32 0.00 }
End of eclipse at Greenwich . . − 4 32 5.94

	h.	m.	s.
End of eclipse at Greenwich, ...	4	32	5.94
End of eclipse at Edinburgh, ...	4	19	22.40
Longitude of Edinburgh in time, . . .	0	12	43 .54
„ in arc, . . .	3°	10′	53″.10 W.

II. To find the Longitude by a Lunar Observation, or by taking the Angular Distance between the Moon and the Sun or a Fixed Star, or one of the four Planets selected for this purpose.

April 20th, 1842, about 5h 52m 30s P.M., in latitude 17° 54′ N., longitude by account 4h 20m 30s E., when the barometer stood at 30 inches, and the thermometer at 50° Fahrenheit, the observed altitude of the sun's lower limb was 4° 57′ 46″, that of the moon's lower limb was 53° 15′ 10″, and the observed distance between the sun and moon's nearest limbs was 120° 11′ 6″, the height of the observer's eye was 20 feet. The instrument with which the sun's altitude was observed, had an index error of + 1′ 10″; that with which the moon's was observed was — 30″, while that with which the distance was taken was correct. Required the true longitude of the position?

	h. m. s.
Longitude by account, ...	4 20 30 E.
Mean time of observation, ...	5 52 30 P.M.
Estimated Greenwich time, ...	1 32 0

TO THIS TIME BY NAUTICAL ALMANAC.

	° ′ ″			′ ″	
Sun's declination, . .	11 29 48	N.	Moon's horizontal parallax,	59 50.0	
Polar distance, . .	78 30 12		Reduction for latitude,	1.0	
	m. s.				
Equation of time . .	— 1 6.6		Reduced horizontal par.,	59 49.0	
Sun's semi-diameter, .	15 55.7		Moon's semi-diameter, .	16 18.3	
			Augmentation, . .	+ 14.1	
			Moon's augmented semi-diameter, . .	16 32.4	
	° ′ ″			° ′ ″	
Sun's observed altitude,	4 57 46	*l. l.*	Moon's observed altitude	53 55 10	*l. l.*
Index error, . .	+ 1 10		Index error, . .	— 30	
	4 58 56			53 54 40	
Dip to 20 feet, . .	— 4 24		Dip to 20 feet, . .	— 4 24	
Sun's apparent altitude,	4 54 32	*l. l.*	Moon's apparent altitude,	53 50 16	*l. l.*
Sun's correction, . .	— 9 54		Moon's correction, .	+ 34 38	
Sun's true altitude, .	4 44 38	*l. l.*	Moon's true altitude, .	54 24 54	*l. l.*
Sun's semi-diameter, .	+ 15 56		Moon's semi-diameter, .	+ 16 32	
Sun's true altitude, .	5 0 34	centre	Moon's true altitude, .	54 41 26	centre
	° ′ ″			° ′ ″	
Sun's apparent altitude,	4 54 32	*l. l.*	Moon's apparent altitude,	53 50 16	*l. l.*
Sun's semi-diameter, .	+ 15 56		Moon's semi-diameter, .	+ 16 32	
Sun's apparent altitude,	5 10 28	centre	Moon's apparent altitude,	54 6 48	centre

	° ′ ″
Observed distance, nearest limbs, ...	120 11 6
Sun's semi-diameter, ...	+ 15 55.7
Moon's augmented semi-diameter, ...	+ 16 32.4
Apparent central distance, ...	120 43 34.1

TO FIND THE TRUE DISTANCE.

	° ′ ″			
Apparent distance,	120 43 34			
Sun's apparent altitude,	5 10 28	secant,		0.0017735
Moon's apparent altitude,	54 6 48	secant,		0.2319662
Sum,	180 0 50			
Half,	90 0 25	cosine,		6.0835149
Difference,	30 43 9	cosine,		9.9343375
Sun's true altitude,	5 0 34	cosine,		9.9983380
Moon's true altitude,	54 41 26	cosine,		9.7619219
Sum,	59 42 0		(Sum),	16.0118520
Half,	29 51 0			
Arc 1,	89 25 9	cosine,	(Half),	8.0059260
Sum,	119 16 9	sine,		9.9406821
Difference,	59 34 9	sine,		9.9356288
				19.8763109
	° ′ ″			
	60 8 35.5	sine,		9.9381554
	2			
True distance,	120 17 11			
Distance at 0^h,	119 25 15	P. L. 2533		
Difference,	0 51 56	P. L. 5398		
	h. m. s.			
	1 33 4	P. L. 2865		
Preceding time,	0			
Greenwich mean time,	1 33 4			
	° ′ ″			
Sun's true altitude,	5 0 34			
Sun's polar distance,	78 30 12 N.	cosecant,		0.008802
Latitude,	17 54 0 N.	cosecant,		0.021548
Sum,	101 24 46			
Half,	50 42 23	cosine,		9.801606
Difference,	45 41 49	sine,		9.854704
	h. m. s.			
Apparent time,	5 53 35.6	R. V. S.,		9.686660
Equation of time,	— 1 6.6			
Position, mean time,	5 52 29.0			
Greenwich mean time,	1 33 4.0			
Longitude in time,	4 19 25 = 64° 51′ 15″ East.			

EXERCISES.

1.—In the process of conducting satisfactorily a trigonometrical survey, on a somewhat extensive scale, it is convenient to select some standard point from which series of triangles may be carried over the country. Of this point the latitude and longitude must be accurately determined astronomically, together with the azimuth of a well-defined signal, from which, by reference, the azimuth of the different sides of the consecutive triangles may be obtained by simple angular measures.

For this purpose, in the subsequent examples, the place of observation here selected was a position in the village of Broddick, in the island of Arran, and the point of which the azimuth is to be found is that of the pile on the summit of Goatfell, the standard point from which the other conclusions are to be deduced. During several years, numerous observations for these ends were made by an altitude and azimuth astronomical circle, of which one or two instances will be given for the purpose of elucidating the whole process.

It was on these series of observations that the corrected latitudes, longitudes, and heights of many points in Messrs W. & A. K. Johnston's map of Scotland were founded.

2.—Previous to determining the latitude by astronomical observations, it is requisite to have the error of the watch by which the observations made are reduced to the meridian. For this purpose an approximate value of the latitude must be determined by previous meridional observations, in order to compute the time from observations taken in the morning or evening of the given day on objects near the prime vertical, for which the sun is the most convenient. To accomplish this, the latitude of the station at Broddick was found to be 55° 35′ 20″ N., nearly, and the longitude $20^m\ 37^s$ west. The circle had three verniers, each reading 10″, and a fixed level reading from a central zero, the value of each division being 2″. The circle was so graduated as to read zenith-distances.

Broddick, August 23, 1843.

Barometer, $29^{in}.6$; Thermometer, 56°.3 Fahrenheit.

The following observations were made on the sun in the morning, to determine the error of the watch :—

No.	Times.		Level.	Ver.	Z. Distance.	
	h. m. s.		+ −		° ′ ″	
1.	8 28 55	A.M.,	17 30	A	59 57 50	upper limb.
				B	57 10	
				C	57 10	
2.	8 31 51		19 27	A	60 2 10	lower limb.
				B	2 50	
				C	2 50	
	2) 60 46		36 57	6)	180 0	
			36		30 0	
	8 30 23		2) 21		59 30 0	
Longitude . .	+ 20 37		− 10.5		60 0 0	
Error of watch .	+ 12		level −		10.5	

Greenwich mean time 8 51 12 ; 23d, obs. Z. D. 59 59 49.5 = θ.*
Or, ... 20 51 12 ; 22d.

Refraction computed, Table V., VI., VII., and VIII.

θ=60°.0′, log $\delta\theta$.	= 2.0037	Sun's declension, Aug. 22,	11° 56′ 17″.9 N.
b=29in.6, log b . .	= 9.9942	Reduction for 20h.85, = −	17 33.5
τ=56° Fahr., log τ .	= 9.9997		
t=56° Fahr., log t .	= 9.9946	Sun's reduced declension,	11 38 44.4 N.
			90 0 0.0
r''= 98″.22 log . .	1.9922		
π''= 7 .22		Sun's polar distance, .	78 21 15.6
Cor. = 91 .00 . .	= +0° 1′ 31″.0	Equation of time, June 22, =	+ 2m 47s.97
Observed Z. D. .	= 59 59 49.5	Reduction for 20h.85,	− 12.99
	60 1 20.5	Reduced equation, . +	2 34.98
	90		

Sun's true altitude, .	29 58 39.5		
Sun's polar distance, .	78 21 15.6	cosecant	0.009023
Latitude, . . .	55 35 20.0	secant	0.247854
Sum, ...	163 55 15.1		
Half, ...	81 57 37.5	cosine	9.145685
Difference, ...	51 58 58.0	sine	9.896430
App. time, Aug. 22, .	20 28 1.0	R. V. S., ...	9.298992
Equation of time, . .	+ 2 35.0		
Mean time, ...	20 30 36.0		
Time by watch, . . .	20 30 23.0		

Error of watch, . . 13.0 slow, and losing 3s.3 daily, as determined by previous similar observations.

* θ is the zenith-distance, log $\delta\theta$ is the logarithm of the mean refraction, b the height of the barometer, τ the temperature by the attached thermometer, t that by the detached, r'' the refraction in seconds, and π'' the parallax in zenith-distance here.

TIME OF TRANSIT OF THE SUN BY THE WATCH.

		h.	m.	s.
Apparent noon, ...		12	0	0
Equation of time at Broddick, apparent noon,	+		2	32.8
Error of chronometer, slow at noon, . . .	−			13.4
Time of transit by watch, ...		12	2	19.4

EXAMPLE 1.—Determination of the latitude by circum-meridian observations on the sun by the same instrument.

In this example the readings were, and in all other cases have been, recorded in the same manner as the last, but, to save room, the means of the three verniers A, B, C, only are recorded here.

No.	Times by Watch. h. m. s.	Level. +	Level. −	Z. D. ° ′ ″		Refraction and Parallax, or Correction.
1.	11 45 47	15	29	43 52 5.0	u. l.	θ=44° 1′ log ∂θ 1.7512
2.	11 48 30	16	28	44 17 33.3	l. l.	b=29in.5 log b 9.9927
3.	11 57 35	14	30	43 45 33.3	u. l.	τ=62° F. log τ 9.9995
4.	12 0 10	17	25	44 12 48.3	l. l.	t=62° F. log t 9.9893
5.	12 5 3	18	25	43 45 13.3	u. l.	r″=+54″.04 log 1.7327
6.	12 7 50	25	18	44 13 23.3	l. l.	π″=− 5 .94
	6)71 44 55	105	155	6) 24 6 36.5		cor.=+48 .10
	11 57 29.2		105	44 1 6.1		
			6) 50			

Cor. W.	+ 13.4	− 8.3	l=−	8.3	Dec. at G.N. 11° 36′ 5″.0 N.
Longitude,	+ 20 37.0			44 0 57.8	Red. to 18m.3 =− 15.8
G. M. T.,	12 18 19.6	cor.	= +	48.1	Reduced dec. 11 35 49.2 N.
		true Z. D.		44 1 45.9	

		h. m. s.		
Sun's transit by watch,		12 2 20		
Time of 1st observation,		11 45 47	Table XVIII.	
			V.	v.
Δ_1 or 1st difference,	.	16 33	26062	6792
Δ_2	.	13 50	18211	3316
Δ_3	.	4 45	2148	46
Δ_4	.	2 10	447	2
Δ_5	.	2 43	703	5
Δ_6	.	5 30	2879	83
			50450	10244

Computation of the Reduction to the Meridian.

			—		+
	° ′ ″				
Latitude,	55 35 20 N.	cosine	9.752146		
Declination,	11 35 50 N.	cosine	9.991042		
Time Z. D.	43 59 30	cosec.	0.158294	cot.	0.015289
			9.901482	× 2 =	9.802964
Σ V =	50450	log . . .	4.702861	r v log	4.010470
N =	6	log . . .	7.536274	log 6	5.235244
Chron. rate	$3^s.3$ losing, log	.	0.000033	log	9.063967
1st term = —	138″.24	log . . .	2.140650	+0″. 12	
2d term = +	0.12				
					° ′ ″
Reduction = —	138.12	=	. . . .	—	0 2 18.12
Corrected observed zenith-distance,			. . .	+	44 1 45.90
True meridian zenith-distance,			. . . .		43 59 27.78N.
Declination for mean of times of obs.			. . . . . .		11 35 49.20N.
True latitude, by obs., of the sun,			. . . . .	=	55 35 16.98N.

Example 2.—Determination of the latitude by observations on Polaris, at a distance from the meridian, in the evening.

Broddick, August 23, 1843.

$b = 29^{in}.40$; $\tau = 56°$ Fahrenheit; $t = 56°$ Fahrenheit.

No.	Times by chron. h. m. s.		Level. e. +	o. —	Z. D. ° ′ ″	Refraction, or correction.	
1.	8 52 23	P.M.	23	25	34 32 53.3	θ = 34° 26′.4	log ϑ θ = 1.6025
2.	8 59 25		21	27	25 26.7	b = 29.4°	log b = 9.9912
3.	9 5 55		23	27	27 23.3	τ = 56° F.	log τ = 9.9997
4.	9 13 27		19	31	20 11.3	t = 56 F.	log t = 9.9946
	4) 11 10		86	110	4) 105 54.6	r'' = 38″.73	1.5880
	9 2 47.5			86	34 26 28.65		
Chron. fast,	— 6 31.7			4) 24	l = — 6.00		
Brod. M. T.,	8 56 15.8	l =	—	6	34 26 22.65		
Longitude,	+ 20 37.2	r''	=		+ 38.73		
G. M. T.,	9 16 53.0				34 27 1.38		

To Determine the Stars' time of Transit.—Tables XXVI. and XXVII.

Polaris.	h. m. s.
1. Sidereal time at Greenwich, M.N.	10 4 31.00
Broddick mean time, . .	8 56 15 80
Reduction for Green. mean time,	1 31.48
Sidereal time of observation, .	19 2 18.28
Star's right ascension, or Polaris R. A.	1 3 49.98
Diff. — sidereal time after transit of Polaris, —	17 58 28.30
	° ′ ″
Declination by N. A. — . .	88 28 26.1

δ Aquilæ.	h. m. s.
2. Star's R. A. in S. T. . .	19 43 11.36
Red. to 19^h 43^m $11^s.36$ . .	— 3 13.83
Star's R. A. mean time, .	19 39 57.53
Transit of Aries, . . .	+ 13 57 8.04
Sum — 24^h . . .	— 9 37 5.57
Red. or accln. for long. . .	— 3.39
M. T. of transit, . . .	9 37 2.18
Chron. fast on 9^h 37^m . .	+ 6 31.76
Transit by chron. . . .	9 43 33.94

CALCULATION OF THE LATITUDE AND AZIMUTH BY THE FORMULÆ, PAGE 344.

$d =$	88° 28′ 26″.1	cot	8.4255583	cosec	0.0001541	cot	8.4255583
$t =$	17h 58m 28s.3	cos	7.8240322			sin	9.9999903
$u = +$	0° 0′ 36″.65		6.2495905	cos	10.0000000		
	$\delta =$ 34° 27′ 1″.38			cos	9.9162520		
$\lambda = +$	55 34 45.40			sin	99164061	seq	0.2477476
$l =$	55 35 22.05 N, $a =$			N	2° 41′ 54″.05 E	tan	8.6732962
		Referring lamp		E	1 42 21.95		
		Lamp		N	4 24 16.00		
Angle.	Goatfell Station, lamp			W	31 7 57.20		
	Azimuth of Goatfell Pile			N	26 43 41.20 W		from station.

EXAMPLE 3.—DETERMINATION OF THE LATITUDE BY α AQUILÆ.

Broddick, August 23, 1843, $b = 29^h.4$ τ and $t = 56°$ F.

Transit by chronometer, 9h 43m 34s. Daily rate, 3s.3 losing.

No.	Times by Chron. h. m. s.	Level a. +	Level o. −	Z. D. ° ′ ″	Refraction or correction. ° ′		
1	9 25 57 P.M.	25	25	47 17 13.3	$\theta = 47$ 11.1	log $\delta\theta$	=1.7992
2	9 30 57	20	30	7 48.3	$b = 29.4$	log b	9.9912
3	9 38 35	43	7	9 50.0	$\tau = 56°$	log τ	9.9997
4	9 43 38	35	15	3 10.0	$t = 56°$	log t	9.9946
5	9 50 5	32	18	10 26.7	$r'' = 1' 0''.91$	log	1.7847
6	9 54 40	29	22	6 36.7			
7	10 0 27	32	18	16 35.0			
8	10 5 41	28	23	15 45.0			
8)	78 10 0	244	158	8) 87 25.0			
	9 46 15	158		47 10 55.62			
		8) 86	$l = +$	10.75			
		$l = +$ 10.75		47 11 6.37			
		Cor.		+ 1 0.91			
		Cor. Z. D.		47 12 7.28			

REDUCTION TO THE MERIDIAN BY A SPECIAL TABLE, FOR. (9), (10), p. 344.

	h. m. s.		
Transit by watch, page 436, ...	9 43 34		
First observation, ...	9 25 57		″
Δ 1, or first difference, ...	17 37	R =	464.07
Δ 2, ...	12 37		238.22
Δ 3, ...	4 59		37.19
Δ 4.	0 4		0.01
Δ 5, ...	6 31		63.59
Δ 6, ...	11 6		184.43
Δ 7, ...	16 53		426.30
Δ 8, ...	22 7		732.20
		Sum,	2146.01

				° ′ ″	
Sum, ...	″ —2146.01				
One-eighth, or R . . .	— 268.25				
Red. of M. Sol. to Sid T. .	+ 1.47	= $\frac{1}{183}$ R + $\frac{1}{15}$ of $\frac{1}{183}$ R			
Correction for 3ˢ.8 losing .	+ 0.02	daily or mean time.			
Reduction, ...	— 269.74	=	—	0 4 29.74	N
Corrected zenith-distance,			+	47 12 7.28	N
True meridian zenith-distance,		=		47 7 37.54	N
Stars' declination from N. A.		=		8 27 43.38	N
Latitude, ...				55 35 20.92	N

RECAPITULATION AND FINAL MEAN RESULTS, BY GIVING WEIGHT TO THE NUMBER OF OBSERVATIONS IN EACH SERIES.

	° ′ ″		″
1 Series,	55 35 16.98 N	Seconds, × 6 =	161.88
2 Series,	22.05	Seconds, × 4 =	88.20
3 Series,	. 20.92	Seconds, × =	167.36
		18	) 357.44

Hence the true latitude is . . . . . . 55° 35′ 19″.86 N

By combining a greater number of observations in different years, on various days, the latitude was found to be 55° 35′ 19″.43, differing by 0″.43 from the preceding.

LONGITUDE OF BRODDICK STATION BY CHRONOMETER.

EXAMPLE 4.—*Broddick, August* 4, 1843.

Barometer, $29^{in}.4$; Thermometer, 65° Fahrenheit.

	No.	Times by Chron. h. m. s.	Level. e. o. + —	Z. D. ° ′ ″	Refraction. ° ′	
	1	9 6 15 A.M.	13 31	50 37 20.0	θ = 50 39.3 log βθ	= 1.8520
	2	9 9 23	0 44	50 42 13.3	b = 29.4 log b	9.9912
					τ = 65° F. log τ	9.9994
	2)	15 38	13 75	79 33.3	t = 65° F. log t	9.9866
T. by Watch,		9 7 49	13	50 39 46.67	r″ = 67″.5 log	1.8292
Cor. Watch,		+ 1 30	2)62			
Longitude,		+ 20 37				
			l — 31	l = — 31.00	π = 6″.5	
G. M. T.		9 29 56				
				50 39 15.67	Cor. = 61 .0	= 1′ 1″.00
			Cor.	+ 1 1.00		
			Sun's true Z.D.	50 40 16.67		
				90 0 0.00		
Sun's true altitude, . .				39 19 43.33		

To Greenwich mean time, August 3, 21^h 29^m 56^s ☉'s dec. = 17° 24′ 8″.0 N
90

Sun's polar distance, 72 35 52.0 N

	° ′ ″		
Sun's true altitude, . .	39 19 43		
Sun's polar distance, . .	72 35 52	cosecant .	0.020347
Latitude, ...	55 35 19	secant .	0.247851
Sum, ...	167 30 54		
Half, ...	83 45 27	cosine .	9.036376
Difference, ...	44 25 44	sine .	9.845113
	h. m. s.		
Time past, March 3, . .	21 3 27.5	R. V. S., .	9.149687
Equation of time, . +	5 51.4		
Mean time, ...	21 9 18.9		
Time by watch, ...	21 7 49.0		
Watch slow, ...	0 1 29.9		
Watch slow of chronometer, .	0 8 32.5		
Chronometer fast, . .	0 7 2.6		

		h. m. s.		° ′ ″
Brought over,		0 7 2.6		
Correction for rate,	+	51.7		
Longitude from Edinburgh,		0 7 54.3 W.		
Longitude of Edinburgh,		0 12 43.0 W.		
Longitude of Broddick,		0 20 37.3 W.	=	5 9 19.5 W.
By very numerous observations,				5 9 16.95
				h. m. s.
Or in time,				0 20 37.13

Example 5.—*Broddick, August* 3, 1844.

$b = 29^{in}.96$, τ and $t = 65°$ Fahrenheit.

	Times by Watch. h. m. s.	Level. e o + —	Z. D. ° ′ ″		Bearing of Sun and Goatfell. ° ′ ″	
	5 13 12	15 29	68 22 53.3	T. R.,	36 17 36.7 sun,	1
					100 48 3.3 Goatfell,	2
	5 22 15	19 24	69 3 56.7	T. L.,	218 39 26.7 sun,	3
	2) 35 27	34 53	137 26 50.0		280 44 46.7 Goatfell,	4
	5 17 43.5	34	68 43 25.0	2—1=	64 30 26.6	
Watch fast,	— 2 13.5	2) 19 l —	9.5	4—3=	62 5 20.0	
Brod. M. T.,	5 15 30.0	l = — 9.5	68 43 15.5	Mean,	63 17 53.3	
Longitude,	+ 20 37.3	sun's cor. +	2 16.1	= refraction — parallax.		
Green. M. T.,	5 36 7.3	T. Z. D.,	68 45 31.6			
		True alt.,	21 14 28.4			

	° ′ ″		
Sun's polar distance,	72 37 22		
Sun's true altitude,	21 14 29	secant	0.030555
Latitude,	55 35 19	secant	0.247851
Sum,	149 27 10		
Half,	74 43 35	cosine	9.420663
Difference,	2 6 13	cosine	9.999707
			19.698776
Half azimuth,	= 45 0 46	cosine	9.849383
	2		
Azimuth of sun,	N. 90 1 32.0 W.		
Angle to Goatfell pile,	N. 63 17 53.3 W.		
Azimuth of Goatfell,	N. 26 43 38.7 W.		
Azimuths, page 437,	N. 26 43 41.2 W.		
Mean,	N. 26 43 39.95 W.		

By more numerous observations, the azimuth was found to be N. 26° 43′ 38″.56 W. from the station of the instrument at Broddick.

The height of the soil at the same station is 18.5 feet above mean tide nearly. The height of the axis of the circle above the ground generally varied from about 4.8 to 5.0 feet.

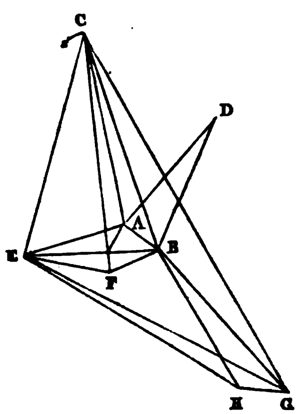
The latitude, longitude, and height of what may be termed the primary station being now determined, together with the bearing of the second point of the triangulation, in a conspicuous situation, its distance and position must next be obtained by triangulation, whence a complete series may be extended in all directions as far as necessary or convenient.

For this purpose must be measured the *base* or primary side of the first triangle with all possible accuracy, from the best means at command, as has been described in a preceding part of this work, because on this, in an important degree, depends the accuracy of all succeeding results, and the final conclusion of the whole operation.

In this diagram, A is Broddick station, and C is Goatfell pile.

Fig. 1.

A B = 676.22 feet. }
A F = 386.55 ... } Parts of the base E B connected by angles.
A E = 1407.88 ... }

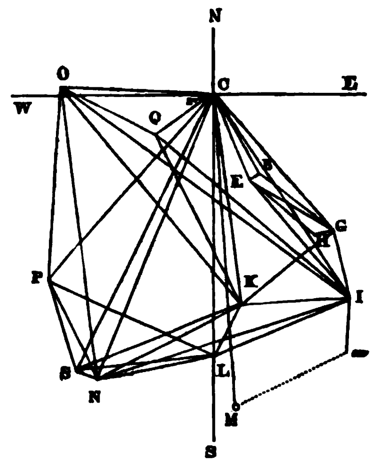


Note.—The line should have been drawn from A to F, and not to the intersection of the lines C F and E B, as has been done by an oversight of the engraver. See pages 320 and 425, *s* is the circle's position, near Goatfell pile, at C.

TRIANGULATION OF A PART OF ARRAN.

See also Plates XXIX. and XXX.

Fig. 2.



A Station at the south end of Broddick village.
B That at the boat-hut near the shore. } Fig. 1.
C Goatfell pile.
D The staff on Broddick castle.
E Station at the bridge near the church. Fig. 2.
F Station at Strabane gate.
G Dunfion pile.
H Dundubh pile.
I Holy Isle pile.
K Dunurie pile.
L Halfway-hill pile.
M Ailsa Craig.
N Brownhill.
O Beinbharin pile.
P Kingshill pile.
Q Beinnorah.
s Circle's position on Goatfell.
s C = 14.56 feet.

Though the triangles in Nos. 1 and 2 are of a shape unfavourable to accuracy, it has been thought proper to give them; but their results, on that account, have not been used in the subsequent calculations.

Several of these triangles will show how difficult or impossible it is to render practice conformable to theory. See Article IV., page 321.

o.		Angles.	Opposite Sides in Feet.	No.		Angles.	Opposite Sides in Feet.
1.	A B C	73° 50′ 43″.30	15377.2	12.	B H E	9° 11′ 17″.90	2273.497
	B A C	103 44 1.67	15552.3		B E H	69 27 47.83	13333.00
	A C B	2 25 15.03	676.22		E B H	101 20 54.27	13959.49
		180 0 0.00				180 0 0.00	
2.	A F C	31° 2′ 6″.70	15498.14	13.	B H G	148° 48′ 35″.16	15823.58
	F A C	148 13 40.00	15623.07		B G H	25 52 20.68	13333.00
	A C F	0 44 13.30	386.55		H B G	5 19 4.16	2831.80
		180 0 0.00				180 0 0.00	
3.	C E F	84° 43′ 8″.85	15788.03	14.	C B E	91° 28′ 39″.08	15835.19
	C F E	90 11 13.30	15856.25		C E B	80 16 14.23	15612.62
	E C F	5 5 38.35	1407.88		E C B	8 15 6.69	2273.497
		180 0 0.00				180 0 0.00	
4.	C A F	148° 13′ 40″.00	15788.03	15.	B C G	9° 7′ 59″.77	15823.58
	A F C	31 2 6.70	15450.80		B G C	9 0 37.73	15612.62
	C F A	0 44 13.30	386.55		C B G	161 51 22.50	31043.00
		180 0 0.00				180 0 0.00	Rejected from bad shape.
5.	A B D	125° 51′ 45″.00	2634.56	16.	E G C	16° 33′ 9″.46	15856.25
	B A D	42 7 53.35	2180.77		E C G	17 22 24.68	16618.95
	A D B	12 0 21.65	676.22		C E G	146 4 25.86	31063.07
		180 0 0.00				180 0 0.00	
6.	A F B	47° 10′ 27″.52	676.22	17.	C I G	9° 19′ 25″.27	31063.07
	A B F	24 47 14.13	386.55		C G I	166 20 53.28	45253.33
	F A B	108 2 18.35	876.68		G C I	4 19 41.45	11469.96
		180 0 0.00				180 0 0.00	
7.	F A E	47° 9′ 46″.26	1407.88	18.	C I M	140° 19′ 14″.18	137061.60
	F E A	11 36 53.74	386.55		I C M	27 30 41.15	99173.66
	A F E	121 13 20.00	1641.87		I M C	12 10 4.67	45253.33
		180 0 0.00				180 0 0.00	
8.	E A B	155° 12′ 46″.60	2273.497	19.	K G I	67° 29′ 26″.14	21385.46
	A B E	17 37 55.78	1641.87		K I G	73 49 17.22	22232.28
	A E B	7 9 59.62	676.22		G K I	38 41 16.64	11469.96
		180 0 0.00				180 0 0.00	
9.	E F B	168° 23′ 47″.52	2273.497	20.	I K L	97° 54′ 13″.40	22837.68
	F B E	7 9 18.36	1407.88		K L I	68 6 47.44	21385.46
	F E B	4 26 54.12	876.69		K I L	13 58 59.16	5568.89
		180 0 0.00				180 0 0.00	
10.	B C E	8° 15′ 6″.68	2273.497	21.	C K I	87° 20′ 2″.73	45253.33
	B E C	80 16 14.23	15612.62		K C I	28 10 5.33	21385.46
	C B E	91 28 39.09	15856.25		C I K	64 29 51.94	40688.55
		180 0 0.00				180 0 0.00	
11.	B G E	7° 31′ 49″.97	2273.497	22.	C L I	72° 43′ 25″.35	46436.93
	B E G	65 48 11.62	15823.58		I C L	28 47 43.55	22837.68
	E B G	106 39 58.41	16618.95		C I L	78 28 51.10	45253.33
		180 0 0.00				180 0 0.00	

No.		Angles.	Opposite Sides in Feet.
23.	K C L C K L K L C	0° 37′ 38″.35 174 45 43.87 4 36 37.78	5568.89 46437.17 40888.55
		180 0 0.00	
24.	K L N L K N K N L	112° 42′ 31″.38 57 4 42.10 10 12 46.52	28973.35 26364.40 5568.89
		180 0 0.00	
25.	K C O O K C K O C	97° 15′ 39″.86 34 20 28.08 48 23 52.06	54241.97 30846.31 40888.55
		180 0 0.00	
26.	C I O C O I I C O	21° 42′ 19″.45 32 51 31.53 125 26 9.02	30846.31 45253.33 67955.92
		180 0 0.00	
27.	C O N C N O N C O	77° 52′ 48″.26 30 7 18.18 71 59 53.56	60096.50 30846.30 58457.60
		180 0 0.00	
28.	C I N C N I N C I	78° 55′ 2″.57 47 38 36.87 53 26 20.56	60096.50 45253.33 49188.24
		180 0 0.00	
29.	O N I O I N N O I	77° 45′ 52″.40 57 12 46.40 45 1 21.20	67955.92 58457.60 49188.24
		180 0 0.00	
30.	Benlomond Holy Isle Goatfell	8° 54′ 43″.93 53 52 8.57 117 13 7.50	45253.33 235926.30 259760.20
		180 0 0.00	
31.	Benlomond Beinbharin Goatfell	6° 15′ 8″.00 56 24 12.00 117 20 40.00	30846.30 235926.30 251591.19
		180 0 0.00	
32.	Benlomond Holy Isle Beinbharin	15° 9′ 51″.68 75 34 30.32 89 15 38.00	67955.92 251591.10 259760.20
		180 0 0.00	
33.	Benlomond Bencampsie Goatfell	86° 55′ 45″.60 65 35 15.10 27 28 59.30	258772.6 235929.7 119569.0
		180 0 0.00	

No.		Angles.	Opposite Sides in Feet.
34.	Benlomond Bencleuch Tinto	47° 5′ 22″.14 96 39 59.61 36 14 38.25	217990.9 295620.1 175967.7
		180 0 0.00	
35.	Bencampsie Calton Tinto	51° 29′ 17″.12 59 48 16.40 68 42 26.48	165378.2 182672.8 196923.4
		180 0 0.00	
36.	Bencleuch Bencampsie Tinto	56° 14′ 21″.85 97 12 8.65 26 33 29.50	182672.8 217990.9 98240.5
		180 0 0.00	
37.	Benlomond Bencleuch Bencampsie	32° 11′ 43″.23 40 25 37.63 107 22 39.14	98240.5 119569.0 175967.7
		180 0 0.00	
38.	Bencleuch Calton Bencampsie	105° 33′ 41″.56 28 43 26.10 45 42 52.34	196923.4 98240.3 146335.5
		180 0 0.00	
39.	Bencleuch Calton Tinto	49° 19′ 22″.32 88 31 40.24 42 8 57.44	165378.8 217990.9 146334.6
		180 0 0.00	
40.	Calton Kellielaw Bencleuch	91° 58′ 40″.28 41 47 57.10 46 13 22.62	202548.4 146335.0 135083.5
		180 0 0.00	
41.	Bencleuch Calton Inchkeith Lt.	11° 53′ 54″.95 73 16 29.13 94 49 35.92	30278.8 140643.7 146335.0
		180 0 0.00	
42.	East Lomond Bencleuch Calton St.	83° 55′ 45″.39 45 38 58.74 50 25 15.87	146334.6 105230.8 113423.0
		180 0 0.00	
43.	East Lomond Kellie Law Inchkeith Lt.	78° 42′ 10″.32 45 52 54.59 55 24 55.09	106847.7 78222.2 89704.5
		180 0 0.00	
44.	East Lomond Calton Hill Kellie Law	87° 21′ 0″.11 41 33 23.04 51 5 36.85	135083.6 89704.5 105230.8
		180 0 0.00	

No.		Angles.	Opposite Sides in Feet.	No.		Angles.	Opposite Sides in Feet.
45.	Isle of May Light	42° 5′ 39.17″	105230.8	56.	Cairnsmuir .	37° 46′ 19.00″	277762.0
	East Lomond	77 21 38.20	153174.5		Goatfell .	109 40 37.00	426974.0
	Calton Hill .	60 32 42.63	136688.2		Knocklayd .	32 33 4.00	243998.5
		180 0 0.00				180 0 0.00	
46.	Goatfell .	35° 16′ 47.18″	182672.5	57.	Benlomond .	78° 15′ 2.40″	336845.4
	Bencampsie .	89 49 58.41	316276.3		Goatfell .	58 27 21.80	293217.6
	Tinto . .	54 53 14.41	258722.7		Benmore, Mull	43 17 35.80	235929.7
		180 0 0.00				180 0 0.00	
47.	Benlomond .	72° 2′ 6.42″	316276.3	58.	Benlomond .	56° 43′ 17.04″	426974.1
	Tinto . .	45 12 7.10	235929.7		Cairnsmuir D.	79 42 17.54	502504.5
	Goatfell .	62 45 46.48	295690.1		Knocklayd .	43 34 25.42	352037.6
		180 0 0.00				180 0 0.00	
48.	Benlomond .	40° 41′ 53.74″	195427.8	59.	Benlomond .	40° 41′ 53.74″	195427.8
	Goatfell .	87 22 40.17	299388.8		Goatfell .	87 22 40.17	299388.8
	Jura . .	51 55 26.09	235922.9		Jura . .	51 55 26.09	235922.9
		180 0 0.00				180 0 0.00	
49.	Goatfell .	22° 54′ 33.85″	111136.5	60.	Jura . .	41° 51′ 31.00″	184334.4
	Jura . .	113 53 33.60	261030.6		Benmore .	94 26 54.00	275409.2
	BenTartevil, Islay	43 11 52.55	195427.8		Ben Heynish	43 41 35.00	190826.3
		180 0 0.00				180 0 0.00	
50.	Ben Oe, Islay	86° 57′ 46.60″	277762.0	61.	Benmore .	74° 40′ 22.00″	283870.9
	Goatfell	38 6 0.80	171630.2		Ben Tartevil	38 46 32.00	184336.7
	Knocklayd, Ireland	54 56 12.60	227672.7		Ben Heynish	66 33 6.00	270033.2
		180 0 0.00				180 0 0.00	
51.	Jura . .	90° 22′ 26.80″	227672.7	62.	Ben Tartevil	74° 17′ 19.00″	275400.3
	Goatfell .	30 29 37.20	115533.5		Jura . .	82 51 8.00	283865.3
	Ben Oe .	59 7 56.00	195428.0		Ben Heynish	22 51 33.00	111136.1
		180 0 0.00				180 0 0.00	
52.	Ben Oe .	72° 55′ 0.40″	111136.5	63.	Benmore .	12° 37′ 41.00″	101995.0
	Jura . .	23 31 9.20	46396.7		Ben Heynish	10 38 42.00	86180.5
	Ben Tartevil	83 33 50.40	115533.5		Lunga, .	156 43 37.00	184335.5
		180 0 0.00				180 0 0.00	
53.	Benlomond .	13° 0′ 25.00″	277762.0	64.	Benmore .	25° 9′ 13.00″	114479.1
	Goatfell .	155 58 18.20	502514.0		Ben Heynish	18 2 9.00	83388.9
	Knocklayd .	11 1 16.80	235930.0		Iona N. Pile	136 48 38.00	184335.5
		180 0 0.00				180 0 0.00	
54.	Benlomond .	30° 26′ 58.80″	227672.9	65.	Benmore .	69° 17′ 40.00″	179209.0
	Goatfell .	117 52 17.50	397138.5		Jura . .	25 48 7.00	83388.9
	Ben Oe .	31 40 43.70	235928.0		Iona N. Pile	84 54 13.00	190826.3
		180 0 0.00				180 0 0.00	
55.	Tinto . .	49° 41′ 4.00″	243999.0	66.	Benmore .	93° 2′ 39.0″	205412.8
	Goatfell .	31 35 16.00	167609.0		Jura . .	18 52 49.0	66563.8
	Cairnsmuir on Deugh	98 43 40.00	316276.3		StaffaPile or Cave	68 4 32.0	190826.3
		180 0 0.00				180 0 0.0	

No.		Angles.	Opposite Sides in Feet.
67.	Benmore .	49° 31′ 11.0″	225025.7
	Ben Tartevil	16 22 19.0	83389.3
	Iona N. Pile	114 6 30.0	270036.7
		180 0 0.0	
68.	Jura . .	22° 58′ 42.0″	117800.0
	Ben Heynish	42 53 59.0	205410.0
	Staffa Pile .	114 7 19.0	275404.8
		180 0 0.0	
69.	Ben Heynish	42° 12′ 15.0″	214865.2
	Ben Tartevil	20 21 29.0	111270.3
	Iona S. Pile	117 26 16.0	283861.7
		180 0 0.0	
70.	Jura . .	12° 12′ 16.0″	275404.8
	Ben Heynish	19 20 43.0	111270.0
	Iona S. Pile	148 27 1.0	174357.7
		180 0 0.0	
71.	Jura . .	95° 3′ 27.0″	214876.1
	Ben Tartevil	53 55 54.0	174366.5
	Iona S. Pile	31 0 39.0	111136.9
		180 0 0.0	
72.	Benmore .	15° 53′ 38.0″	119742.0
	Jura . .	23 25 56.0	89462.1
	Colonsay . .	140 40 26.0	190826.3
		180 0 0.0	
73.	Benmore .	78° 33′ 14.0″	198892.2
	Ben Heynish	36 9 50.0	119747.3
	Colonsay .	65 16 56.0	184335.5
		180 0 0.0	
74.	Benmore .	75° 26′ 41.0″	208803.0
	Ben Heynish	45 51 8.0	154793.4
	Oronsay Pile	58 42 11.0	184335.5
		180 0 0.0	
75.	Benmore .	20° 44′ 9.0″	138085.1
	Ben Heynish	7 28 14.0	50707.1
	Ulva Pile .	151 47 37.0	184335.5
		180 0 0.0	
76.	Jura . .	101° 16′ 47.0″	150785.2
	Ben Tartevil	32 25 59.0	89461.2
	Colonsay Pile	46 17 14.0	111136.5
		180 0 0.0	
77.	Benmore .	19° 0′ 10.0″	67215.0
	Jura . .	48 34 48.0	154794.2
	Oronsay Pile	112 25 2.0	190826.1
		180 0 0.0	
78.	Jura . .	76° 7′ 55.0″	115274.3
	Ben Tartevil	34 28 39.0	67214.0
	Oronsay .	69 23 26.0	111136.5
		180 0 0.0	
79.	Jura . .	124° 42′ 42.0″	270036.6
	Benmore .	19 46 28.0	111137.7
	Ben Tartevil	35 30 50.0	190836.1
		180 0 0.0	
80.	Ben More .	25° 9′ 13.0″	114479.1
	Ben Heynish	18 2 9.0	83389.0
	Iona N. Pile	136 48 38.0	184335.5
		180 0 0.0	
81.	Colonsay .	43° 42′ 39.0″	83389.0
	Benmore .	53 24 2.0	96862.0
	Iona N. Pile	82 53 19.0	119745.0
		180 0 0.0	
82.	Colonsay .	107° 45′ 2.0″	283867.0
	Ben Tartevil	41 51 31.0	198892.0
	Ben Heynish	30 23 27.0	150785.0
		180 0 0.0	
83.	Oronsay .	101° 58′ 22.0″	89462.0
	Colonsay .	52 52 46.0	67215.0
	Jura . .	25 8 52.0	35822.0
		180 0 0.0	
84.	Oronsay .	47° 58′ 24.0″	92820.0
	Colonsay .	115 22 2.0	112906.0
	Iona S. Pile	16 39 34.0	35822.0
		180 0 0.0	
85.	Ben Tartevil	21° 29′ 55.0″	92820.0
	Colonsay .	121 57 50.0	214871.0
	Iona S. Pile	36 32 15.0	150675.0
		180 0 0.0	
86.	Ben Tartevil	19° 27′ 15.0″	112906.0
	Oronsay .	140 40 0.0	214871.0
	Iona S. Pile	19 52 45.0	115274.0
		180 0 0.0	
87.	Oronsay .	145° 54′ 1.0″	225026.0
	Ben Tartevil	17 24 34.0	120091.0
	Iona N. Pile	16 41 25.0	115274.0
		180 0 0.0	
88.	Ben Tartevil	2° 2′ 41.0″	12832.0
	Iona S. Pile	141 16 29.0	225026.0
	Iona N. Pile	36 40 50.0	214871.0
		180 0 0.0	

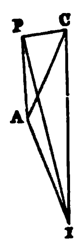
No.		Angles.	Opposite Sides in Feet.	No.		Angles.	Opposite Sides in Feet.
89.	Ben Heynish	6° 18′ 36″.0	12830.0	93.	Ben More .	37° 46′ 52″.0	54964.0
	Iona N. pile	72 23 23.0	111271.0		Lunga .	68 21 17.0	83389.0
	Iona S. Pile	101 18 1.0	114479.0		Iona N. Pile	73 51 51.0	86181.0
		180 0 0.0				180 0 0.0	
90.	Benmore .	23° 44′ 59″.0	34975.0	94.	Benmore .	41° 34′ 3″.0	64721.0
	Staffa . .	106 12 34.0	83389.0		Iona S. Pile	62 4 6.0	86181.0
	Iona N. Pile	50 2 27.0	66564.0		Lurga .	76 21 51.0	94795.0
		180 0 0.0				180 0 0.0	
91.	Benmore .	45° 53′ 22″.0	60632.0	95.	Colonsay .	122° 43′ 35″.0	120091.0
	Iona N. Pile	37 7 33.0	50707.0		Oronsay .	42 44 26.0	96882.0
	Ulva . .	96 59 5.0	83389.0		Iona N. Pile	14 31 59.0	35822.0
		180 0 0.0				180 0 0.0	
92.	Iona N. Pile	36° 35′ 25″.0	36684.0	96.	Iona N. Pile	65° 53′ 45″.6	174362.0
	Ulva .	63 16 9.0	54964.0		Iona S. Pile	110 15 6.9	179209.0
	Lunga . .	80 8 26.0	60632.0		Jura Pile .	3 51 7.5	12833.0
		180 0 0.0				180 0 0.0	

From the station of the circle, near Goatfell pile, the following observations were taken.

Let A be Broddick station, I Holy Isle pile, C Goatfell pile, and P the point on which the circle stood when the observations were made.

Then, by measurement and a little provisory calculation, it was found that

$$
\begin{aligned}
PC &= d = 14.56 \text{ feet.} \\
PA &= r = 15460 \text{ feet approximately.} \\
PI &= l = 45250 \text{ feet approximately.} \\
API &= P = 7^\circ\ 8'\ 13''.77 \\
IPC &= p = 74\ 40\ 58.33 \\
APC &= P + p = 81\ 49\ 12.10 \\
PAI &= A = 169\ 11\ 18 \\
IPC &= p = 74\ 40\ 58 \\
A - p &= 94\ 30\ 20
\end{aligned}
$$



These are the data employed in formulæ (3) and (4), page 329, of Trigonometrical Surveying, to reduce the observed angle A P I to A C I—that required at the axis of the signal.

By (3) log R″ =	5.314425			
d = 14.56 feet	log. 1.163161			
	6.477586 +	. . . . .		6.477586 —
P + *p* = 81° 49 12.1 sin	9.995559	*p* = 74° 40′ 58″.3 sin		9.984292
r = 15460 ft. a. c. l.	5.810791	*l* = 45250 a. c. l.		5.344381
1st term + 192″.28 log	2.283936 +	2d term = — 64″.01		1.806259 —
2d term — 64 .01				
Reduction + 128 .27				

By (4) log R″	. . . . . .	5.314425 +
A = 169° 11′ 18″ cosecant	. . . . .	0.726810 +
A — *p* = 94 30 20 sine	. . . . .	9.908656 +
P = 7 8 14 sine	. . . . .	9.094283 +
d = 1456 feet log	. . . . .	1.163161 +
r = 15460 feet a. c. l.	. . . . .	5.810710 +
Reduction + 128″.21	. . log	2.108045 +

Mean cor. + 128″.24 =	+ 0° 2′ 8″.24	
P	7 8 13.77	
A C I . . . =	7 10 22.01	while by observation
A C bears	N. 26 43 38.56	W. from A, and dis-

tant 15459.8 feet, as will be found by subsequent calculation.

Bearings and Distances.

1. In the preceding figure, as shown in pages 440, 441,

A C bears N. 26° 43′ 38″.56 W. distant 15459.8 ft.
Latitude of A 55° 35′ 19″.43 N. longitude 5° 9′ 16″.95 W.

(1) Latitude of C or *l* = 55° 37′ 36″ approximately.
(2) „ of A or *l* = 55 35 20

½ (*l* + *l′*) = 55 36 28

½ (*l* + *l′*) =	55° 36′ 28″	log M =	7.993750	log P =	7.992822
α =	N 26 43 38.56 W.	cos	9.950927	sine	9.652967
A =	15459.8 feet	log	4.189204	log	4.189204
m″ =	+ 0° 2′ 16″.11	log	2.133881	log *p″*	1.834993
l′ =	55 35 19.43				
λ =	55 37 35.54				
r″ =	— 0.02 see page 370, note.				
l =	55 37 35.52 secant	. . . . .			0.248271
l′ =	55 35 19.43 Δ *p*	. . + 0° 2′ 1″.13 log			2.083264
½ (*l′* + *l*) =	55 36 27.48 *p′*	. . = 5 9 16.95			

Brought over,		. . +	0° 2′ 1.13″	log 2.083264
½ (l + l′) = 55° 36′ 27.48″ p′		. =	5 9 16.95	
	p		5 11 18.08	
	½ (l + l′)	=	55 36 27.48	sine 9.916554
	Δz . .	=	0 1 39.96	log 1.999818
	z . .	=	N. 26 43 38.56 W.	
	C A bears		S. 26 45 18.52 E.	
	A C I .	=	7 10 22.01 E.	

From Goatfell, Holy Isle bears S. 33 55 40,53 E.
Distant, page 443, Δ 17, . . 45253.33 feet.

Hence the definitive latitude of Goatfell is

55° 37′ 35″.52 N., longitude 5° 11′ 18″.08 W.

that is here assumed as the starting point from which others are to be derived by means of these bearings and distances from it.

Broddick Castle.

A C bears	N. 26° 43′ 38.56″ W.	Fig., p. 441.
C A D Δ's (1) (5)	61 36 8.32 E.	
A D bears	N. 34 52 29.76 E.	distant 2634.56 feet.

l Broddick castle, lat.	55° 35′ 40″ N.	by approximation.
l′ Broddick station, lat.	55 35 20 N.	
½ (l + l′) . .	55 36 30 N.	

C I	or Holy Isle pile bears . .	S. 33° 55′ 40.53″ E.	
G C I	Δ 17, ...	4 19 41.45 E.	
C G	or Dunflon bears . .	S. 38 15 21.98 E.	distant 31063.07 ft.

From Goatfell Pile.

C I	Holy Isle pile bears . .	S. 33° 55′ 40.53″ E.	distant 45253.33 ft.
K C I	Holy Isle, Goatfell, Ailsa, .	27 30 41.15	
	1, ...	S. 6 24 59.38 E.	
	2, ...	S. 6 25 6.00 E.	
	3, ...	S. 6 25 8.93 E.	
	Mean, ...	S. 6 25 4.76 E.	distant 137081.60 ft.

	Holy Isle pile bears . .	S. 33° 55′ 40″.53	E.	
△ 21	Holy Isle, Goatfell, Dunurie,	28 10 5.33	W.	Feet.
C K	Dunurie bears . . .	S. 5 45 35.20	E. distant	40888.55
K C L	Half-way Hill, or S.W. pile,	0 37 28.35	W.	
C L	Half-way Hill, or S.W. pile,	S. 5 7 56.85	E. distant	46437.17
L C N	Half-way Hill, Goatfell, Brownhill,	24 38 27.60	W. △ 22, 28, &c.	
C N	Brownhill bears . . .	S. 19 30 30.75	W. distant	60096.50
N C O	Brownhill, Goatfell, . .	71 59 53.56	W. △ (27)	
C O	Beinbharin bears . .	S. 91 30 24.31	W.	
	or, ...	N. 88 29 35.69	W. distant	30846.30

POSITION OF BRODDICK CASTLE.

$\frac{1}{2}(l+l')=$	55° 35′ 30″	log M =	7.993750	log P to l	7.992823
s =	N. 84 52 29.76	cos	9.914027	sine . .	9.757234
A =	2634.56	log	3.420708	log . .	3.420708
$m''=$	+ 0° 0′ 21″.31	log	1.328485	$\log p''$ . .	1.170765
l' =	55 35 19.43				
l =	55 35 40.74	secant		. . . . .	0.247917

Δp	= −	0° 0′ 26″.22	log	1.418682
p	=	5 9 16.95		
p'		5 8 50.73		
$\frac{1}{2}(l+l')$	=	55 35 30.08	sin	9.916471
Δs	=	0° 0′ 21″.63	log	1.335153
s	=	N. 84 52 29.76	E.	

Broddick station bears . S. 84 52 51.39 W. from castle.

Broddick castle lat. . . 55° 35′ 40″.74 N. long. 5° 8′ 50″.73 W.
In time $0^h\ 20^m\ 35^s.38$ W.

In this manner the results in the following table were obtained.

No.	Names of Stations.	Azimuth from the north.	Latitude W.	Longitude W.
1	Broddick Station .	34° 52′ 29″.76	55° 35′ 19″.43	5° 9′ 16″.95
2	Broddick Castle, . .	214 52 51.39	55 35 40.74	5 8 50.73
3	Broddick Station, .	333° 16′ 21″.44	55° 35′ 19″.43	5° 8′ 50″.73
4	Goatfell, . . .	153 14 41.48	55 37 35.52	5 11 18.08
5	Goatfell, . . .	141° 44′ 38″.02	55° 37′ 35″.52	5° 11′ 18″.08
6	Dunfion, . . .	321 49 14.05	55 33 34.94	5 5 43.57
7	Goatfell, . . .	146° 4′ 19″.47	55° 37′ 35″.52	5° 11′ 18″.08
8	Holy Isle, . . .	326 10 21.50	55 31 25.20	5 3 59.19
9	Goatfell, . . .	173° 34′ 55″.24	55° 37′ 35″.52	5° 11′ 18″.08
10	Ailsa Craig, . .	353 38 38.40	55 15 12.67	5 6 52.69
11	Goatfell, . . .	174° 14′ 24″.80	55° 37′ 35″.52	5° 11′ 18″.08
12	Dunurie, . . .	354 15 23.60	55 30 54.51	5 10 6.80
13	Goatfell, . . .	174° 52′ 3″.15	55° 37′ 35″.52	5° 11′ 18″.08
14	Halfway-hill, . .	354 53 2.65	55 29 59.62	5 10 5.84
15	Goatfell, . . .	199° 30′ 30″.75	55° 37′ 35″.25	5° 11′ 18″.08
16	Brownhill, . . .	19 25 43.57	55 28 16.98	5 17 6.34
17	Goatfell, . . .	271° 30′ 24″.31	55° 37′ 35″.52	5° 11′ 18″.08
18	Beinbharin, . . .	91 23 0.88	55 37 43.17	5 20 15.34
19	Goatfell, . . .	28° 51′ 15″	55° 37′ 35″.5	5° 11′ 18″.1
20	Benlomond, . . .	209 19 2	56 11 27.7	4 37 45.6
21	Goatfell, . . .	56° 20′ 15″	55° 37′ 35″.5	5° 11′ 18″.1
22	Bencampsie, . .	237 12 30	56 0 2 9	4 8 9.0
23	Goatfell, . . .	91° 37′ 3″	55° 37′ 35″.5	5° 11′ 18″.1
24	Tinto, ...	272 52 45	55 35 33.2	3 39 34.6
25	Goatfell, . . .	123° 12′ 19″	55° 37′ 35″.5	5° 11′ 18″.1
26	Cairnsmuir on Deugh,	304 1 8	55 15 24.2	4 12 34.5
27	Goatfell, . . .	232° 52′ 56″	55° 37′ 35″.5	5° 11′ 18″.1
28	Knocklayd, Ireland, .	52 0 35	55 9 46.7	6 14 52.0
29	Goatfell, . . .	270° 59′ 0″	55° 37′ 35″.5	5° 11′ 18″.1
30	Ben Oe, . . .	90 4 26	55 37 56.2	6 17 24.6
31	Goatfell, . . .	278° 34′ 5″	55° 37′ 35″.5	5° 11′ 18″.1
32	Bein Tartevil, . .	97 32 2	55 43 35.9	6 26 26.6
33	Goatfell, . . .	301° 28′ 38″	55° 37′ 35″.5	5° 11′ 18″.1
34	Jura, ...	120 48 20	55 54 11.7	6 0 2.5
35	Goatfell, . . .	330° 23′ 48″	55° 37′ 35″.5	5° 11′ 18″.1
36	Benmore, Mull, . .	149 42 54	56 25 32.1	6 0 37.6

PRIMARY STATION, JURA PILE.

No.	Names of Stations.	Azimuth from the north.	Latitude N	Longitude W.
37	Jura, ...	359° 24′ 26″	55° 54′ 11″.7	6° 0′ 2″.5
38	Benmore, Mull, .	179 23 57	56 25 32.4	6 0 37.6
39	Jura, ...	333° 36′ 19″	55° 54′ 11″.7	6° 0′ 2″.5
40	Iona North Pile, . .	153 16 45	56 20 31.7	6 23 36.4
41	Jura, ...	329° 45′ 16″	55° 54′ 11″.7	6° 0′ 2″.5
42	Iona South Pile, . .	149 23 43	56 18 53.7	6 26 0.0
43	Jura, ...	317° 32′ 55″	55° 54′ 11″.7	6° 0′ 2″.5
44	Bein Heynish, Tiree, .	136 47 6	56 27 22.0	6 55 11.5
45	Jura, ...	234° 41′ 45″	55° 54′ 11″.7	6° 0′ 2″.5
46	Ben Tartevil, Islay, .	54 19 55	55 43 35.7	6 26 26.6
47	Jura, ...	310° 49′ 39″	55° 54′ 11″.7	6° 0′ 2″.5
48	Oronsay, ...	130 37 17	56 1 23.9	6 14 57.7
49	Jura, ...	335° 58′ 30″	55° 54′ 11″.7	6° 0′ 2″.5
50	Colonsay, ...	155 50 19	56 6 33.7	6 9 54.7
51	Jura, ...	340° 31′ 37″	55° 54′ 11″.7	6° 0′ 2″.5
52	Staffa Cave, ...	160 14 45	56 25 58.9	6 20 20.6

PRIMARY STATION, DUNII CAIRN, IONA.

No.	Names of Stations.	Azimuth from the north.	Latitude N.	Longitude W.
53	Iona North Pile, . .	18° 20′ 5″	56° 20′ 31″.7	6° 23′ 36″.4
54	Staffa Cairn, over cave,	198 22 48	56 25 58.9	6 20 20.7
55	Iona North Pile, . .	31° 14′ 59″	56° 20′ 31″.7	6° 23′ 36″.4
56	Ulva Pile, . . .	211 22 43	56 28 59.6	6 14 19.0
57	Iona North Pile, . .	354° 30′ 43″	56° 20′ 31″.7	6° 23′ 36″.4
58	Lunga Pile, . . .	174 29 25	56 29 30.9	6 25 10.0
59	Iona North Pile, . .	68° 22′ 32″	56° 20′ 31″.7	6° 23′ 36″.4
60	Benmore, Mull, .	248 41 43	56 25 32.5	6 0 34.8

HEIGHTS.

Broddick, August 15, 1844.

At Station B.—Fig. p. 441.

Barometer, $30^{in}.10$ Fahrenheit; Thermometer, $63^{\circ}.5 = \tau$ and t.

NOTE.—T. R. means telescope right, and T. L. telescope left, in reference to the divided circle after reversing it.

Level. *e* +	Level. *o* —	Telescope on Vertical Arc.	Zenith-distance. ° ′ ″	Level. *e* +	Level. *o* —	Depression of Sea. ° ′ ″
22	20	T. R.	79 37 28.33	28	18	0 0 50.00
11	29	T. L.	79 43 3.33	31	17	0 5 3.33
33	49		80 31.66	59	35	0 5 53.33
	33					
	2)16		79 40 15.88	35		0 2 56.66
Level = —	8	Level, —	8.00	24		
			79 40 7.88	+ 12	l = +	12.00
						3 8.66

Computation of n by Table XI. — Height of Station.

$b = 30^{in}.10$ and $t = 63^{\circ}5$	log	7.45008	D = 8′ 8″.66 = 188″.66 log × 2 = 4.5514
$\tau = 63^{\circ}.5$	log × 2 =	9.99884	Log *o*, Table XXI.. = 6.4579
$t = 63^{\circ}.5$	log	= 9.98795	
$b = 30^{in}.10$	log	1.47857	$d h$ = 10.22 feet, log 1.0093
			the height of the axis of the circle
$n = 0.08231$	log	8.91544	above mean tide.
-0.5			

$n - 0.5 = -0.41769$	log	9.620854 —	
$\frac{1}{2}(l + l')$ and $\alpha = 29^{\circ}.1$	log *o*	7.993474	
$\kappa = 15612.62$ feet	log	4.193476	4.193476
$v' = -0^{\circ}\ 1'\ 4''.24$	log	1.807804 —	
$\delta = 79\ 40\ 7\ .88$			
$\delta = 79\ 39\ 3\ .64$	cotangent	. . . .	9.261534
$\Delta h' = 2851.12$ feet	. . . .	log	3.455010
$d h = 10.22$ feet.			

H = 2861.34 feet, the height of Goatfell above mean tide.

2. From the top of Goatfell, Beinbharin had a zenith-distance of $90^{\circ}\ 59'\ 31''.35$ when the barometer stood at $27^{in}.50$, the attached and detached thermometer at 54° Fahrenheit, and the distance was 30846.31 feet.

$b = 27^{in}.50$ and $t = 54^{\circ}$, give log	. . . .	7.45085
$\tau = 54^{\circ}$	log $\tau \times 2$ = ...	9.99966
$t = 54^{\circ}$	log t	9.99640
$b = 27.50$	log b	1.43933
$n = 0.07696$	. . . .	8.88624

Brought over, +0.07696
−0.5

$n - 0.5 = -0.42304$ log	. . . . . .		9.6263814 —
Log o to latitude 55°.6 and $\alpha = 88°.4$	. . .		7.9928302
K = 30846.31 log . . .		4.4892030	— 4.4892030
$v = -$ 0° 2′ 8″.35 log	. . .		2.1084146
$\delta =$ 90 59 31 .35	. . .	593	= $m h$ Table xxiv.
		3	= S
$\delta_1 =$ 90 57 23 .00 cot	.	8.2225522	—
— 515.01 log	.	2.7118148	—
Goatfell + 2861.34			

H = 2346.33 feet, the height of Beinbharin.

3. From Goatfell, Benlomond bears N. 28° 51′ 12″ E., distant 235926.3 feet, $\frac{1}{2}(l + l') = 55° 54'.5$.

Goatfell pile, August 30, 1843, $b = 27^{in}.25$, τ and $t = 50°$ Fahrenheit; mean zenith of distance from two series, 90° 12′ 6″.16.

$b = 27^{in}.25$, and $t = 50°$ Fahrenheit, log b	.	7.45122
$\tau = 50°$ Fahrenheit, . .	log $\tau \times 2 =$	0.00000
$t = 50°$ Fahrenheit, . .	log .	0.00000
$b = 27^{in}25$	.	1.43537
$n = 0.07702$ log		8.88659
— 0.5		
$n - 0.5 = -0.42298$ log		9.6263198 —
K = 235926.3 log		5.3727764
$\frac{1}{2} l') = 55°.91$, and $\alpha = 28°.84$ log o . . .		7.9935088
$v' = -$ 0° 16′ 23″.12 log		2.9926050 —
$\delta =$ 90 12 6 .16 cot		
$\delta_1 =$ 89 55 43 .04 cotangent		7.0954404
K = 235926.3 log		5.3727764
$m h$ Table xxiv.		594
S Table xxiv. . . .		48
$d h =$ 293.96 feet, log		2.4682810
$h =$ 2861.34 feet, the height of Goatfell.		

H = 3155.30 feet, the height of Benlomond.

4. From the top of Dunii Cairn in Iona, 329.6 feet above the mean level of the sea, the zenith-distance of Benmore in Mull was observed to be 88° 9′ 36″.3, the barometer $b = 30^{in}.02$, the thermometer 50° 0′, at the distance of 83389 feet: required the height of Benmore? *Ans.*—3150 feet.

In like manner the following heights were determined :—

5. Dunfion,	538.83	feet.
6. Dundubh,	741.72	„
7. Holy Isle,	1031.72	„
8. Dunurie, west of Lamlash,	1338.27	„
9. Pile south-west of Dunurie near Half-way Hill,	1510.30	„
10. Brown Hill, south of Blackwater foot,	752.61	„
11. Ailsa Craig,	1113.33	„

12. DETERMINATION OF THE HEIGHT OF MONT BLANC, FROM THE OBSERVATIONS OF M. PLANA.

Names of Stations.	Spherical Angles.	Spherical excess.	Mean Angles.	Logs of opposite sides in feet.
Mont Colombier,	70° 56′ 14″.52	— 3″.33	70° 56′ 11″.19	5.4395635
Mont Granier, .	76 9 14.00	— 3.33	76 9 10.67	5.4512512
Mont Blanc, .	Concluded.	— 3.33	32 54 38.14	5.1991231
			180 0 0.00	

From	Zenith-distance of Mont Blanc.	English Barometer.	Faht.'s Therm.	Latitude.	Bearing.
		Inches.			
Mont Colombier,	88° 5′ 23″.00	25.445	55°.04	45° 52′ 56″.4 N	S 86° 44′ 10″.5 E
Mont Granier, .	88 21 25.00	23.729	52.70	45 27 53.3 N	N 60 28 30.3 E
Mont Blanc, .				45 49 58.3 N	

Dist. of Mont Blanc from Mont Colombier, . . .	log K =	5.4512512
Height of Mont Granier, 6356.64 feet, ...	m h	+ 984
Dist. of Mont Blanc from Mont Granier, 282656.5 feet,	S.	+ 69
Log K reduced, ...		5.4513565

Dist. of Mont Blanc from Mont Granier, . . .	log K	5.4395635
Height of Mont Granier, 6356.64 feet, ...	m h	+ 1320
Dist. of Mont Blanc from Mont Granier, 275146.2 feet .	S.	+ 65
		5.4397020

Mont Col. and M. Blanc } ($l + l'$ = 45° 51′ 27″.35 and a = S. 86° 44′ 10″.5 E. log O = 7.9930770
Mont Gran. and M. Blanc } ($l + l'$ = 45 38 55.80 and a = N. 60 28 30.3 E. log O = 7.9934144

Computation of n at Colombier.

b = 25.445 in. and t = 55.04° F.	log	7.45051
τ = 55°.04	log × 2 =	9.99956
t = 55°.04	log × 7 =	9.99554
b = 25.445 in.	log	1.40560
n = 0.070992	log	8.85121
− 0.5		
− 0.429008	log	9.6324654
Log O		7.9930770
Red. log K, . . .		5.4513565
v' = − 0° 19′ 53″.81	log	3.0768989
δ = 88 5 28 .00		
δ' = 87 45 34 .19	cot	8.5924442
Log K reduced . . .		5.4513565
$d\,h$, 11061.15	log	4.0438007
h' 4743.64	soil	
h'' 4.43	instrument.	
1, h = 15809.23 feet.		

Computation of n at Granier.

b = 23.729 in. and t = 52.7° F.	log	7.45058
τ = 52°.7 F.	log × 2 =	9.99976
t = 52° 9.	log × 6 =	9.99757
b = 23 .729 in.		1.37528
n = 0 .066556	log	8.82319
− 0 .5		
− 0 .433444	log	9.6369330
Log O		7.9934144
Red. log K, . . .		5.4397920
$v_{,}$ = − 0° 19′ 35″	log	3.0700494
δ' = 88 21 25		
$\delta_{,}$ = 88 1 50	cot	8.5363922
Log K reduced, . .		5.4397020
$d\,h$, 9464.43	log	3.9760542
h' 6356.64	soil	
h'' 4.43	instrument.	
2, h = 15825.50 feet.		
1, h = 15809.23		

Mean height of Mont Blanc, ... 15817.36 feet above the mean level of the sea.

Two determinations by Colonel Coraboeuf give a mean height of 15798.90 feet through the Mole at Geneva and Mont Chervin.

The aggregate of these give 15808.13 feet, which cannot differ much from the truth. This result was communicated to my friend, Mr A. K. Johnston, taken at 15810 feet in round numbers, for the use of his *Physical Atlas*.

DESCRIPTION AND USE

OF

THE INSTRUMENTS

EMPLOYED IN

TRIGONOMETRICAL SURVEYING AND LEVELLING.

DEFINITIONS

NECESSARY TO BE KNOWN IN ORDER TO UNDERSTAND THE USE OF INSTRUMENTS.

1. WHEN angles are measured on a level plane, similar to the surface of the sea or a lake, they are called *horizontal* angles.

2. When angles are measured on a plane perpendicular to the level plane, they are called *vertical* angles.

3. If angles are measured in neither of these planes, they are said to be taken in *oblique* or on *inclined* planes.

4. If the angles be measured in the vertical plane, above the straight line passing through the eye of the observer perpendicular to the plumb-line, they are called angles of *elevation;* their complements to 90° are called *zenith-distances;* and the angular instruments, such as theodolites, altitude and azimuth circles, &c., are commonly constructed so as to read either way, according to the orders of the observer.

5. When angles are taken below the level or horizontal line defined above, they are called angles of *depression;* though, when the instrument reads zenith-distances, this distinction is unnecessary, because the excess above 90° is the depression.

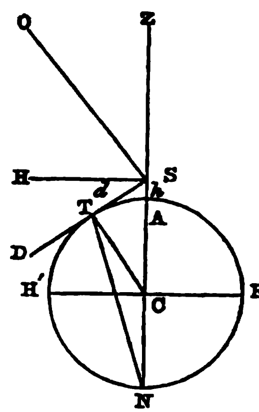
6. These respective positions are known by means of the *plumb-line* or *spirit-level*, one or other of which is generally applied to all

instruments requiring, in their application, a knowledge of these planes.

7. Those points are said to be in the *same level* which are equidistant from the earth's centre considered as a *sphere*. The earth, however, is really a *spheroid*, having its polar axis less than its equatorial diameter by $\frac{1}{300}$, and in the more refined operations, it is the surface of this spheroid that is accounted the level.

8. In the measurement of altitudes, the height of the instrument must generally be added to the result from calculation when situated at the *bottom*, but subtracted when at the *top*. The mean level of the sea, or that at half-tide, is generally adopted as the standard from which heights are estimated. If high or low water at spring-tides be assumed, this should be stated, and the rise of the tide recorded.

9. To illustrate the preceding definitions and terms used in the mensuration of heights and distances on the earth's surface trigonometrically, let A H′ N R be a section of the earth at the surface of the sea, considered as a sphere, which for this purpose is sufficiently near the truth, then, if S be the station of the observer at the height A S, or h, above the mean level of the sea, Z N will be pointed out by the plumb-line hanging freely, H S perpendicular to Z N will be indicated by the spirit-level; the point T will be the utmost limits of vision, or the surface of the sea at the distance T S or d, and to these lines distinctive names have been appropriated. The point Z is called the zenith, the opposite point N is called the nadir; H S, perpendicular to Z N, is called the horizontal line; H′ C R, parallel to it, and passing through the earth's centre C, is called the true horizon; and S T the distance of the visible horizon where the sky and the extreme limits of the surface of the sea appear to meet. When observations are made with angular instruments, as the theodolite, the altitude and azimuth circles, the reflecting circle, &c., on any object O, in the direction S O; the angle O S Z is called the zenith-distance, O S H the altitude, and H S T the depression of the horizon T S, below the horizontal line H S marked by the spirit-level, called also by seamen the *dip* of the horizon. Independent of refraction, it is equal to the angle



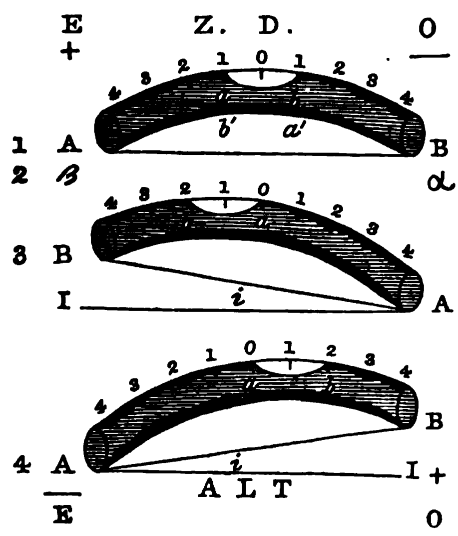
T C S measured by the arc A T. This arc has by Horsburgh, &c., been improperly given as the definition of the dip, though, as has been shown, it is equal to it, and may be taken as a measure of it only, without allowing for the effects of terrestrial refraction.

THE SPIRIT-LEVEL.

The spirit-level is a cylindrical glass tube AOB, of a uniform diameter throughout, which is carefully ground into the form of a circular arc of large radius, occasionally of several hundred feet, which makes it appear almost straight.

It is then nearly filled with some fluid, as alcohol or ether, and the ends are artificially closed or hermetically sealed. To the upper surface of fine instruments there is adapted a scale having divisions cut on a slip of ivory, or even on the surface of the glass itself, showing single seconds, or some multiple of the second, though in all the smaller portable instruments *two* seconds is the best, and by far the most convenient in application, and the reading from a central zero is commonly preferred.

If the cylindrical arc be placed in a vertical plane with the convex



side uppermost, and the extremities A B resting on a horizontal surface as in the figure (1), the bubble of air a b left in the tube will rise to the highest part of it, and will remain, from the principles of gravity, steadily between the same divisions; while the plane on which it is placed revolves round a truly vertical axis, by that means retaining the plane in a perfectly horizontal position. If it be necessary to bring the plane of an instrument, such as that of a theodolite, readily into a horizontal position, it is generally provided with two levels nearly equal to each other in every respect, which are placed at right angles to one another, and permanently attached to the plane, though still capable of adjustment by screws for that purpose. In the more ordinary instruments, the maker marks the position of the bubbles when the plane is horizontal, and, therefore, when the bubbles occupy these positions, the plane on which they are fixed must be horizontal. For common instruments these marks are reckoned sufficient, and the divided scales are thought to be unnecessary.

In fine instruments, if the plane of the level be inclined, by the unequal action of heat upon its supports or other unavoidable causes, to the vertical, and the position of the extremities of the bubble be noted, then if, upon reversing the instrument by turning it half round the vertical axis, at a second observation, they occupy the same positions as in (1) (2), where A and B merely exchange places and occupy those of α and β in a reversed position, the plane will be truly level, and have the same inclination to the vertical ZN in the preceding figure as it had before. This, however, from different causes, almost never happens, and then it becomes absolutely necessary to record the reading of both ends of the level reckoned most conveniently, as in the figure, from a central zero, indicated from the positions marked (3) and (4). If the verniers of the instrument read zenith-distances, the reading of the extremities of the bubble on the scale of the level next the observer, called the *eye-end*, is marked +, and that farthest from him, or the *object-end*, —. If the instrument reads altitudes, the signs must be reversed, that is, the eye-end must be reckoned —, and the object-end +. If the divisions on the scale of the level do not show single seconds, the difference between the positive and negative sums must be multiplied by the value of *one* division, and the result divided by *twice* the number of the observations, and applied to the degrees, &c., read from the circle, according to its sign, to give the true reading corrected for the inclination of the vertical axis.

Example 1. Suppose the circle reads zenith-distances, then the reading of the level in the figure is marked thus:—

	e	o
	+	—
No. 1 of the figure A, B gives,	1	1
— 2 of the figure,	1	1
— 3 by a slight inclination B, A,	2	0
— 4 by an opposite inclination A, B,	0	2
Sums,	4	4

These sums being equal, and having opposite signs, prove that no error arises from the inclination of the vertical axis of the circle in the use of a fixed level.

EXAMPLE 2. In a course of operations made at Broddick, in Arran, by the writer, with a circle having three verniers, each showing 10″, and a fixed level, each division of the scale of which indicated 3″, the following observations on Polaris were taken in latitude by estimation 55° 35′ 30″ N., longitude 20^m 40^s W., on the 6th of August 1836, by a watch 9^m 5^s fast.

Broddick Bridge, August 6, 1836.

English Barometer $b = 29^{in}.98$ Fahrenheit's thermometer $t = 49°.5$.

Obs.	Times.	Ver.	Z.D.	Level. e	Level. o	Circle.
	h. m. s.		° ′ ″	+	—	
1.	10 23 45	A	34 17 20	20	14	Direct.
		B	17 30			
		C	17 20			
2.	10 44 45	A	34 12 20	23	11	Reversed.
Mean	10 34 15	B	12 30			
		C	12 10	Sums + 43,	−25	
				−25		
Mean			34 14 51.7			
Effect of level			+ 13.5	Diff. + 18		
				3″ = value of a division.		
Z.D. correc. for lev.			= 34 15 5.2			
			2 no. obs. =	4)54		
				+ 13.5 = l = effect of level.		

	h.	m.	s.
To the mean of the times of observation,	10	34	15
Apply the error of watch fast,	—	9	5
Mean time at place of observation,	10	25	10
Longitude in time west,	+	20	40
Mean time at Greenwich,	10	45	50

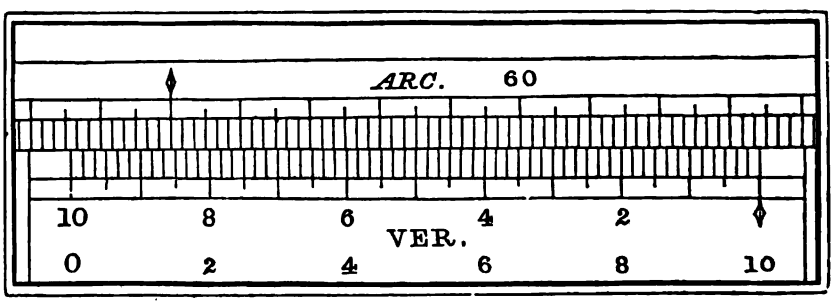
from which the latitude may be found by the method explained in the *Nautical Almanac* for 1836, p. 524, or by the formula given for the same purpose in this work. These observations, with the assistance of *Mathematical and Astronomical Tables*, and the *Nautical Almanac*, give the latitude 55° 35′ 28″.6 N. from this series, which, where great accuracy is required, ought to be continued for a considerable time on stars both to the north and south of the zenith, in pairs nearly equidistant from it, to destroy any error from a bias in the instrument, or a faulty habit of observing.

The mean of the whole, combined according to the number of observations in each series, will, even with a moderate-sized instrument, give the final latitude with considerable accuracy. The most convenient division of the scale of the level is 2″ for each, because the effect of the level would be got by dividing the difference of the sums of the columns *e* and *o* by the number of observations simply, whereby both the multiplication by the value of one division of the scale and the operation of doubling the number of observations for a divisor is avoided.

THE VERNIER.

The vernier is a small scale sliding against a divided scale or arc, in such a manner as to subdivide those parts of the arc into smaller divisions than can be conveniently and distinctly executed on the arc itself.

Thus, if an arc be divided into single degrees, then a small scale, having an extent equal to 59 of these degrees divided into 60 equal parts, each part on the vernier will be $\frac{1}{60}$ less



than one on the arc. But $\frac{1}{60}$ part of one degree is equal to 1′, consequently such a vernier would, by the coincidence of any two lines—one on the vernier with one on the arc—show single minutes. This vernier, however, would be rather inconveniently long. If, therefore, the arc, as in the common theodolite, be divided into half degrees of 30′ each, then a vernier scale of 29 of these half degrees, divided into 30 equal parts, will also show minutes, and the vernier scale, being shorter, is more convenient. In this case, care must be taken not to forget half a degree in recording the reading indicated by the arrow which marks the degrees and parts on the arc. Generally, if $n-1$ divisions on the arc be divided into n divisions on the vernier, then this vernier shows $\frac{1}{n}$ part of the divisions on the limb, for $1-\frac{n-1}{n}=\frac{1}{n}$. Thus, if one degree, as in the figure, be divided into six equal parts of 10′ each, and if 59 of these be divided into 60 on the vernier, then $\frac{10'}{60} = 10''$; consequently, such a vernier shows 10″ directly, and 5″ may be easily estimated.* Finer subdivisions than these are generally obtained by the reading microscope. If observations be repeated, however, on different parts of the limb, a degree of precision sufficient for almost the nicest purposes may be easily obtained even by this vernier. Indeed, repetitions should be taken on different days, to avoid the irregularities to which the most powerful instruments are liable from the effects of refraction. In using the different kinds of verniers, it will be found more easy, and less liable to error in reading off the arcs, when the degree on the limb and the minute on the vernier are similarly divided. Thus, if the limb be divided to 20′, the vernier should show 20″; if the limb read 10′, the vernier should read to 10″, as in the figure, &c. By this arrangement, the mind is less liable to be distracted during the operation of reading, than when the limb is read according to one arrangement and the vernier to another.

THE READING MICROSCOPE.

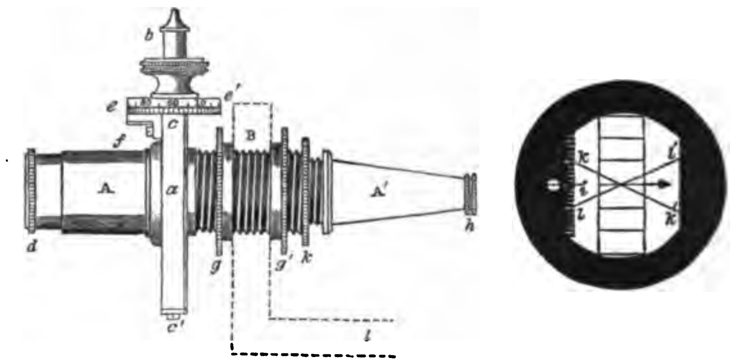
When the reading microscpe is applied to read the divisions of an astronomical circle, the graduations in the arc generally indicate spaces of five minutes each, which are read along with the degrees by means of an index pointer. The remaining minutes and seconds are determined by the reading microscope.

METHOD OF ADJUSTMENT AND APPLICATION TO PRACTICE.

In the figure, A, A′ represents the microscope, attached to the instrument by the arm l, and passing through its support B, formed by a collar embracing it, where it is firmly held by the milled nuts

* In the figure above, the coincidence takes place at 56° 35′ 5″. The 5″ is put down by estimation, since 5′ 0″, and 5′ 10″ are about equally distant from a coincidence.

g, g', acting on screws cut upon the tube of the microscope. These nuts also serve the purpose of placing the instrument at the proper distance from the divisons which it is employed to read, in order to obtain distinct vision, and destroy parallax. In the body



of the instrument, at *a*, the common focus of the object and eye-glasses, are placed two wires, *k k*, *l l*, crossing each other diagonally at acute angles, which are made to traverse the field of view, backwards or forwards, by turning the micrometer head *b*, whose axis works in the box *c c'*, in the first figure. In the second figure is shown the field of view, with the magnified divisions on the circle, as seen through the microscope. The shaded part represents the diaphragm, with its cross wires, the angle between which may, by turning the micrometer-screw *b*, be bisected by any line on the circle within the field of view, as is shown in the figure. On the left hand of the diaphragm appears the scale of minutes, from its shape called a *comb*, in which each tooth represents a minute. Movable with the wires along the comb, is a small index or pointer *i*, which in the figure is represented at zero, the centre of the scale, known to be correct when it bisects the small hole at the back of the comb, while at the same time the cross wires bisect a division. Now one revolution of the screw *b* moves the point connected with the wires over one tooth of the comb—that is, over a space on the divided arc of the circle equal to one minute—and therefore part of a revolution moves them only over a part of a minute. To determine the value of this fractional part of a minute in seconds, a large cylindrical head, *e e'*, is attached to the screw, having its exterior circumference divided into 60 equal parts, representing seconds, and read by an index opposite the eye of the observer at *f*. In reading off an angle by this instrument, observe, first, the degrees and nearest five minutes shown by the pointer on the graduated circle, then this will be the true angle, if, as in the figure, a division on the graduated circle bisect the angles of the cross wires. But if the cross wires be not thus bisected, read the degrees and nearest five minutes as before, then apply to the microscope, and, by turning the screw *b* in the order of the numbers upon the head *e e'*, make the nearest division in the reverse order of the numbers upon the graduated circle, nicely bisect the acute angles formed by the intersection of the cross wires; the number of teeth which the pointer *i* has passed over from its zero, to produce such a bisection, will be the number of minutes to be added to the degrees and minutes read off the circle by the pointer; and, lastly, the odd seconds *and estimated tenths* to be added are taken from the divided head *e e'*, as shown by the index *f*. In cases of great nicety, the run of the microscope may be taken to the next division in the *direct* order of the numbers upon the circle, which, subtracted from *five minutes*, ought to give the same number of minutes and seconds as formerly, to be added to the arc shown by the pointer on the circle. If there is a slight discrepancy, the mean of both may be taken and so applied.

ADJUSTMENTS OF THE MICROSCOPE.

1. To make the cross wires in the focus of the microscope and the divisions on the circle appear both at the same time distinct and free from parallax, draw out the eye-piece *d*, until distinct vision of the wires is obtained, and the divisions on the instrument are equally well defined and free from parallax; that is, whether any motion of the eye causes the least apparent displacement of the wires with respect to the graduations. If such a dancing motion be observed, the microscope must be moved to or from the circle, by turning the nuts *g g'*, easing the one and tightening the other, till the wires and graduations appear both distinct, and no parallax can be detected.

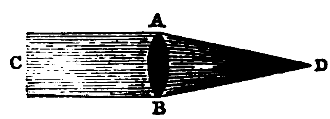
2. To make five revolutions of the micrometer-screw measure a five-minute space upon the graduated circle exactly. If the run of the screw has been carefully adjusted by the maker, and no alteration made in the body of the microscope, the image of the space between two divisions will be exactly equivalent to five revolutions of the screw, when the wires and divisions are both seen distinctly. Suppose, for example, however, that the microscope has been deranged, and the run is too great, and that the 5′ space

on the arc is equal to 5′ 5″, when measured by the micrometer, thus making the image too large. But the magnitude of the image formed by the object-glass of the microscope depends entirely on the distance of the object-glass from the limb, and, in the ordinary construction of the microscope, is diminished by increasing the distance between the object-glass and the limb, and conversely. In the case supposed, the image is too large, consequently the object-glass must be removed farther from the limb, by turning the screw at *h* inwards in the direction of B.* The image will not now be formed at *a*, as it ought to be, but nearer to B, and distinct vision must again be obtained by bringing the whole body of the microscope, by the screws *g g′*, nearer to the limb. By a repetition of two or three more cautious attempts in this way, five revolutions of the screw carrying the cross wires will correspond exactly with the image of the space between two divisions, which, for greater security, may be read to the right and left on each side of zero. The screw *c′* gives motion to the comb or scale of minutes; and the micrometer-head, being adjustable by friction, can be made to read either zero or any required second, when the cross wires bisect any particular division, by holding fast the milled head *b*, and at the same time turning the divided arc *e e′* round till any required division, as zero, coincides with *f*, the index.

THE TELESCOPE.

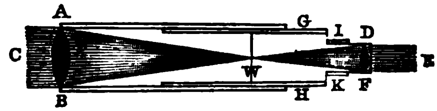
All instruments now capable of giving results possessing the requisite accuracy are furnished with one or more telescopes. The rays of light proceeding from distant objects move in straight lines, unless they are reflected or refracted by some *medium*, such as metal, glass, &c., and also in *parallel lines* nearly, especially if the object from which they come be remote.

Let AB represent the section of a lens, such as the object-glass of a telescope. Let the parallel rays coming from some distant object on the left beyond C strike the glass lens AB, they will pass through it, suffering refraction, and, on leaving the lens at the opposite side, they will converge and meet in the straight line C D at a certain point D, called the focus of the lens, where the eye, by



* This is sometimes done by moving the part A′, and fixing it by *k*.

a little practice in selecting the proper distance, will see an inverted image of the distant object in the air. Now, suppose two of these lenses are applied to the construction of a telescope, the image of a distant object will be formed at W, the focus of



the object-glass A B, where, by moving the eye-glass or lens D F till its focus comes to the same point W, by means of two slides GH, IK, the eye of the observer at E will view a magnified inverted image of the object formed by the object-glass AB, with the eye-glass DF as a microscope. Since both these lenses are capable of motion, they may always be moved in such a manner that their foci will meet exactly at W, making the central line CWE a straight line, technically called the optical axis, or line of collimation of the telescope, from which the readings in all mathematical and astronomical instruments are taken. This point, W, is marked by fine wires, hairs, silk fibres, or spider lines, fixed to a circular perforated piece of brass, called a diaphragm; and this is the reason why both glasses must be moved till the telescope produce distinct vision, and the wires are well seen, in which case the telescope is said to have no parallax.

If this adjustment is imperfect, the object will, on moving the eye up and down a little, start from the intersection of the wires, thus causing an uncertainty in all observations, which must be instantly corrected. The point W, or focus of the object-glass, varies with every change in the distance of the object, and therefore this adjustment must be frequently examined, and, if necessary, corrected for terrestrial objects, though it remains constant for celestial. This instrument is commonly called the astronomical or inverting telescope, because it wants other two lenses between the object and eye glasses to view objects erect as they appear to the naked eye. They are, however, almost universally employed for astronomical purposes, where it is less necessary to see objects erect, and because they appear more distinct, from a greater quantity of light being attainable, since each lens absorbs a portion of it. It is scarcely necessary to add, that no attempt at adjustment should be made *during*, but always *before*, an observation.

The diaphragm is of the shape of some one of the figures 1, 2, or 3. Fig. 1 is generally the form best suited for the theodolite; fig. 2 for the spirit-level; fig. 3 for the altitude and azimuth circle.

A small notch may be made to the right in fig. 2, for estimating distances by the number of divisions it embraces on the levelling-staff.

Fig. 1.

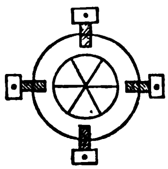


Fig. 2.

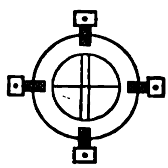
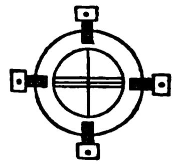


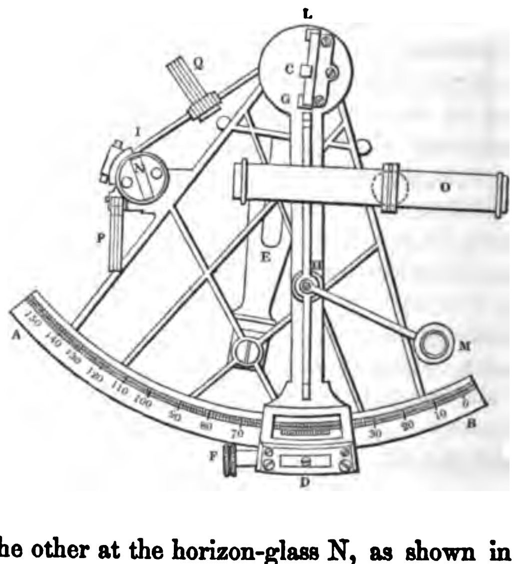
Fig. 3.



The screws in the circumference are for adjusting the line of collimation according to the given directions.

THE SEXTANT.

1. The sextant, as its name implies, is the sixth part of a circle, and therefore contains really an arc A B in the figure of sixty degrees; but on account of two reflections—one at the index-glass



L G, and the other at the horizon-glass N, as shown in books on optics—the deviation of a ray of light, after two reflections at C and N, is equal to *twice* the angle between the reflectors; consequently, an arc of sixty degrees on the limb A B must be divided into one hundred and twenty equal parts, each reckoned a degree; so that the real angle between two given objects, when brought in contact, may be read by the index of this instrument.

2. The best instruments are commonly made of metal. The frame consists of a metal of a hard composition, generally called gun-metal; the index C D and limb A B are of brass; the handle E is alone made of wood, and attached to the back, by which, when an observation is made, it is held with one hand, while the other is employed to regulate the motion of the index.

The arc on the limb, as well as the vernier, in fine instruments, is divided on a ring of silver, gold, or platina, to prevent the tarnish arising from the action of the atmosphere.

3. In good instruments, a degree on the limb A B is commonly divided, as shown in the figure, *vernier*, into six equal parts, each containing 10′, and each minute on the vernier above D also into six equal parts, whence it reads to 10″.

In many, the degree is divided into four equal parts, each of 15′, and the vernier shows 15″.

The smaller class have the degree divided into three equal parts, each of 20′, and the vernier shows 20″.

4. That end of the index next the limb at D is furnished with an adjusting, or, from its position in reference to the limb, a tangent screw, F, by which the index, when clamped to the limb by a screw behind D, is moved slowly and regularly, whence the contact of two objects may be made as perfect as the eye is able to accomplish, when assisted with one of the telescopes, O, accompanying the instrument.

The arc on the instrument is then read from the vernier by the assistance of the movable magnifying lens at M turning on its axis, round which is placed a reflector, to throw the requisite light from a lamp, in night observations.

The sextant, and many octants, or, as they are commonly called, quadrants, are not now fitted up for the back observation, and in some the horizon-glass is incapable of adjustment. In this case it is indispensable to determine the index error.

5. One set of coloured glasses, generally four in number, are placed at Q, three being tinged red of different tints of colour, and one green, of which one or any combination are turned on an axis when placed between the index and horizon glasses.

The other set, consisting commonly of three, two red and one green, placed at P behind the horizon-glass N, any one or more of which may, as before, be turned behind it, when observing the sun's altitude, by reflection from a basin of water or mercury at land, or determining the index error by measuring the sun's diameter at right and left of zero on the limb.

There is also one or more coloured glasses that may be screwed on the eye-end of the telescope for the same purpose, and less liable to vitiate the result by errors arising from a want of parallelism in the planes of the former, that by the position is not affected by the latter.

6. There are generally two telescopes belonging to the sextant, one of which is of the common astronomical construction, showing objects inverted, and another showing them in their natural position. To accommodate less experienced observers, a plain tube, without any glasses, also accompanies this instrument.

The tube or either telescope, O, is screwed into a ring, as shown in the figure, which is connected with another ring by means of screws, in such a manner as to raise or lower the telescope in order that it may be directed to a proper part of the horizon-glass, while at the same time the line of sight or *collimation* may be made parallel to the plane of the sextant, by which means the contact of any two objects may be more accurately observed.

7. The arc has been extended to the whole circumference, and divided into 720 equal parts, from the principle of double reflection already explained, each being estimated a degree. These have been constructed in different forms, as those of Mayer, Borda, Troughton, Hassler, Dollond, &c.; and from being complete circles, called *Reflecting Circles*, each having its peculiar advantage, give on the whole more accurate results.

ADJUSTMENTS OF THE SEXTANT.

The adjustments of the sextant are to set the index and horizon glasses perpendicular to its plane, and parallel to one another, when the index is at zero on the limb, and to rectify the line of collimation, by rendering the optical axis of the telescope parallel to the plane of the instrument.

The deviation of each of these from its true position may be found, and the resulting error of observation computed, but it will be found convenient to have the instrument as correctly adjusted as possible.

1. *To set the index-glass perpendicular to the plane of the sextant.*

Move the index towards the middle of the limb, and hold the plane of the instrument nearly parallel to the horizon, placing the index-glass near the eye-end, and the limb from the observer. Direct the sight obliquely towards the silvered part of the horizon-glass; then, if the direct and reflected limb appear to be exactly in the same plane, that glass is perpendicular to the plane of the instrument. If not, turn the screws in the projecting plate behind the speculum till they are so, and then the index-glass will be adjusted.

2. *To set the horizon-glass perpendicular to the plane of the sextant.*

The index-glass being now adjusted, set the first division on the index to the first on the limb, or 0° to 0', and hold the plane of the instrument in a horizontal position; then direct the sight through the eye-vane to the horizon-glass, and if the direct and reflected horizons are apparently in the same straight line, that glass is also perpendicular to the plane of the sextant. If not, turn the adjusting screw of the horizon-glass, at the back of the instrument, till the coincidence is perfect.

3. *To set the horizon-glass parallel to the index-glass when the index is at zero.*

Set the first division on the vernier to zero on the limb, or 0' to 0°; fasten the index in this position by the clamping screw behind the index, and make the coincidence perfect by the tangent screw at its extremity, the eye being assisted by the magnifying lens; screw the telescope into its support, and turn the adjusting screw at the back of the instrument till the field of the telescope is bisected by the line which separates the silvered and transparent portions of the horizon-glass. Now, hold the plane of the sextant vertically, direct the sight to the horizon or other well-defined distant object, and make the direct and reflected objects coincide, by the screws in the frame of the horizon-glass for that purpose, and it will then be parallel to the index-glass at zero.

If, after this adjustment is accomplished, at least approximately, it does not appear perfect, it will be necessary to determine the *index error*.

This error is found by measuring the sun's or moon's diameter twice with a motion of the index, in contrary directions, on both sides of zero.

As all materials are liable to *bend* by pressure, it will be advisable to finish the motion of the index on both sides of zero, by the tangent screw, in the same direction, to avoid errors from this cause both in determining the index error and making observations. If both measures are taken either to the *right* or to the left of *o* on the limb, half their sum will be the index error, and is *additive* or subtractive accordingly. But if one of the measures is taken to the right and the other to the left of *o*, which is the more common case, half their difference will be the index error, and is additive when the measure to the right exceeds that to the left—otherwise, subtractive when the measure to the left exceeds that to the right.

Also, in the latter case, one-fourth of the sum will be the semidiameter.

When altitudes are taken at land by the method of reflection from a basin of water or mercury, the index error must be applied to the arc read from the limb before taking the half for the observed angle or arc.

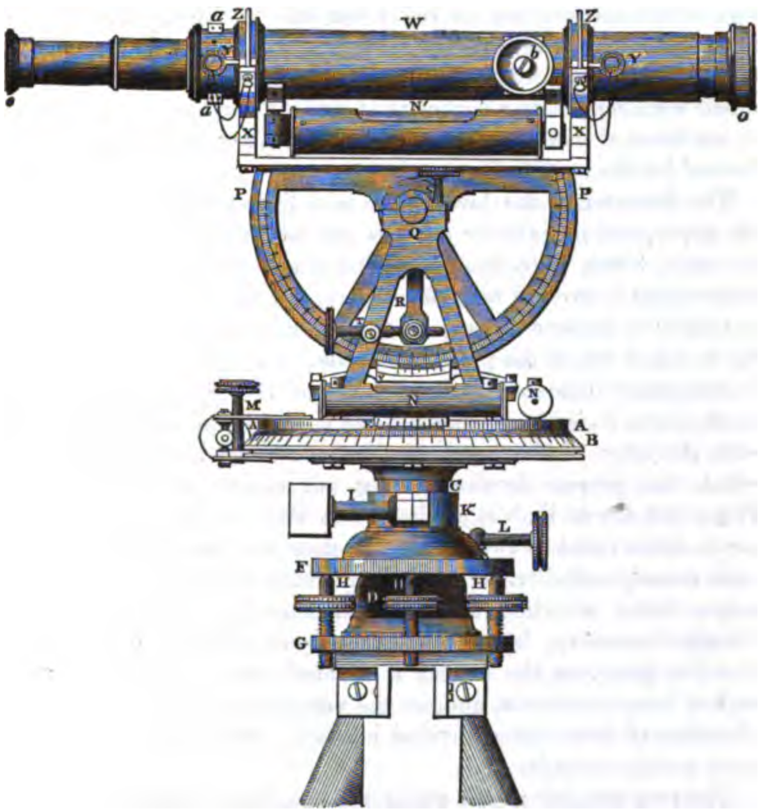
4. *To make the line of collimation, or optical axis of the telescope, parallel to the plane of the sextant.*

Turn the eye-end of the telescope round till one pair of parallel wires is placed parallel to the plane of the instrument, and let two distant objects be selected, such as two stars of the first magnitude, whose distance is not less than 90° or 100°; move the index till the coincidence of these objects is as perfect as possible at the wire nearest the plane of the instrument; move the instrument till the objects are at the other wire; and if the objects are still in contact, the axis is parallel to the plane of the instrument. If the objects are either apparently separated, or pass each other, correct half the error by the screws in the circular part of the support, one of which is above and the other below the telescope. Now turn the tangent screw till the objects are in contact. Examine the coincidence at both wires, and if it is perfect, the adjustment is complete; if not, proceed as before till the contact is perfect at both wires, when the instrument is in a state fit for observation. It is used, as described in bringing any two objects together by reflection, as in the above adjustment, by making the plane of the sextant pass through the eye of the observer and the two objects observed at the same time, then moving the index, assisted by the clamping and tangent screws, till as perfect a coincidence is made as can be distinguished by the power of the elescope.

The prices of metal sextants vary from £10 to £20, according to size and workmanship.

THE THEODOLITE.*

Of all angular instruments, the theodolite, properly constructed, is that best suited to the purposes of the surveyor. It has been



formed on a great variety of plans, but that most approved for general purposes is the five-inch theodolite, divided by one or two verniers to minutes in both the horizontal and vertical arcs. It would be an improvement, for nice purposes, if the vertical arc were an entire circle, having the telescope passing through its centre, and capable of reversion like the common portable astronomical circle, though this is perhaps unnecessary for the ordinary practice of a common land-surveyor.

* Prices:—Best, five inches in diameter, £25. Best, six inch, divided to 20", £30.

DESCRIPTION.

The instrument consists of an under circular plate divided commonly into degrees and half-degrees, usually called the horizontal limb at A A, on which an upper circular plate, called the vernier-plate, turns freely, that by means of one or two verniers, E, subdivides the half-degrees on the lower into minutes. Both plates have an easy though steady motion round the axis, which, for that purpose, is slightly conical. The internal centre also fits into a ball working within a socket at D, and the parts are held together by an internal screw at the lower end of the axis, within the tripod formed by the legs.

The diameter of the lower plate is a little greater than that of the upper, and its exterior edge is cut off in a plane inclined to the axis, which is technically called chamfered, and in the best instruments is covered with *silver*, to receive the graduations, being less liable to become obscure by the action of the atmosphere than the metal of which the plates are made. On the opposite ends of an imaginary diameter, at the distance of 180° from each other, a small space, E, is also chamfered and covered with silver, forming, with the edge of the lower plate, a continued inclined plane, on which the proper divisions being cut constitute the verniers. When the lower limb is graduated to thirty minutes, the vernier has a space equal to twenty-nine of them divided into thirty, and each, consequently, reads to minutes, which, by means of a microscope, either attached or detached, may by estimation, when thought necessary, be carried to thirty or even twenty seconds. For fine purposes, the degree is divided into three equal parts, each of twenty minutes, and, on the vernier-plate, a space equal to fifty-nine of these being divided into sixty, then this vernier indicates twenty seconds.

The two parallel plates under the graduated limb at F and G are held together by a ball and socket at D, and are set firm by four milled-headed screws H, H, H, H, which turn in sockets fixed to the lower plate, while their heads press against the under side of the upper, thus, acting on the vertical axis by means of the ball and socket, render the horizontal and vernier plates truly level when the instrument is prepared for observation.

Beneath these parallel plates is a female screw, within which the male screw lays hold of the axis, and keeps it firmly to the stand. The lower parallel plate is connected by brass joints to three mahogany legs, having their lower ends pointed with metal, for entering the ground, and frequently so constructed, that, when shut up, they form one round staff, secured in that form for carriage by rings placed upon them. When the legs are opened out they make a firm stand, however uneven the ground may be. Sometimes the legs are round or cylindrical, and formed of two parts, which unscrew for packing in a box, to facilitate their carriage when travelling. In this case, shods should be prepared to screw on the upper half, to be used when any nice observations are to be made requiring great steadiness in the stand.

The lower horizontal limb can be fixed in any given position by the clamping-screw I, which causes the collar K to embrace the axis C, and prevent it moving. It is generally necessary that the telescope should be fixed in some precise position, more exactly than it can be by the hand alone. For this purpose, it is first made nearly correct by the hand, the parallel plates being previously clamped with the verniers, and by the tangent-screw accurately set to the given positions, as zero and 180°; then the instrument is moved a small quantity, by turning the slow-motion screw L, attached to the upper parallel plate, till the direction of the cross-wires of the telescope is perfected. In a similar manner, the upper or vernier plate being now released, the telescope may again be placed upon any other object whose angular distance from the first is required, which, by the clamping and tangent screws, may be rendered perfect as before, and the angle shown by the verniers must now be read and recorded. Before proceeding to measure the horizontal and vertical angles, the parallel plates carrying the divisions and verniers must be made perfectly horizontal by two spirit-levels N N, placed at right angles to each other, and rectified by their adjusting screws for this purpose. Upon the vernier-plate, too, is commonly placed a compass between the levels, for the purpose of taking magnetic bearings.*

The vertical frames Q Q support the pivots of the horizontal axis of the vertical arc P V P, on which the telescope is placed. There is sometimes an arm carrying a microscope for reading the altitudes and depressions measured by this arc, and determined by the vernier V, which has a motion of several degrees, so as to be placed opposite the divisions of coincidence. There are, on this

* In the older theodolites the movements of the arcs were accomplished by racks and pinions instead of clamping and tangent screws, as alluded to occasionally in previous parts of this work. These, however, are now almost entirely superseded by the clamping and tangent screws as being more accurate and convenient.

vernier, two sets of divisions reading in opposite directions, of which the upper reads elevations, and the lower depressions.

Another screw S clamps the end of the horizontal axis seen at Q, while a slow-motion or tangent-screw, T, moves the vertical arc and telescope, till a perfect observation be made. One side of the vertical arc is inlaid with silver, and is divided into single minutes, or lower, with the assistance of its vernier. On the other side there are sometimes placed divisions, to show the difference between the hypotenuse and base of a right-angled triangle, the hypotenuse being 100, or, which comes to the same thing, the number of links to be deducted from each chain's length in measuring up or down an inclined plane, to reduce it to the horizontal measure. If the angle of elevation and depression be taken, these afford data to take this reduction more accurately, from a table calculated expressly for this purpose, or the deduction may be readily made by a table of natural versed sines. The level which is shown at N′, under and parallel to the telescope, is attached to it at one end by a joint, and at the other by a capstan-headed screw, and will permit the level to be placed parallel to the optical axis of the telescope, commonly called the line of *collimation*. The screw at the opposite end is employed to adjust it laterally, so that it may be placed parallel to the axis also in a vertical plane. In this way the level is placed parallel to the axis of vision, both horizontally and vertically. The telescope has two collars or rings of gun-metal, ground truly cylindrical, on which it rests on its supports X X, called Ys, from their resemblance to that letter, and it is confined in its place by the clips Z Z, which may be opened by removing the pins Y Y, for the purpose of reversing the telescope in double observations, when great accuracy is required. These pins should, to prevent loss, be secured by silk strings connecting them with the frame.

In the focus of the eye-glass are frequently placed three fine wires or lines of spider-web, one horizontal and two crossing it nearly vertically, making with each other a small or acute angle. This method of fixing the wires is preferable to having one horizontal and another vertical, crossing one another at right angles, as is commonly the case, especially for horizontal angles, because a distant object can be made to bisect the small angle between the vertical wires with more certainty than the object can be bisected by the vertical wire. For many astronomical purposes, however, the second method is preferable; and for making observations on the sun, one or two coloured glasses should be provided, to be fitted on the eye-end of the telescope. The screws for adjusting the cross-wires are shown near the eye-end of the telescope at *a*, *a*, *a*, *a*, of which there are four, equidistant from each other. Hence the imaginary line joining any two opposite screws is at right angles to the line joining the other two; so that, by *first* easing the one, and *then* tightening the other opposite to it, the intersection of the cross-wires may be readily adjusted. The object-glass, *o*, is moved by turning the milled-head *b*, on the side of the telescope, till the object is seen well defined; while a corresponding motion is given to the eye-glass, *e*, by moving it with the hand in its slide till the wires are seen equally distinct, which will easily be effected in one or two trials. The reason and effects of this process will be readily comprehended by consulting the description of the figure, p. 467, though the arrangement there is somewhat different.

A brass plummet and line are also packed in the box with the theodolite, to be suspended from a hook truly under the centre of motion of the horizontal arc, by which it can be placed exactly over the station whence the observations are taken, an operation to be carefully performed in all fine work, otherwise considerable errors may arise, and surveys cannot close accurately. If required, two extra eye-pieces are furnished for the telescope, to be used in astronomical observations. The one inverts the object, has a greater magnifying power, but, with fewer lenses, possesses more light. The other is a diagonal eye-piece, which, without inconvenience, will enable an observer to see objects having a considerable altitude. A small cap, containing a dark-coloured glass, is made to apply to the eye-end of the telescope, or to either of the preceding lenses, to screen the eye of the observer from the effects of the sun's rays, when that object is observed. A magnifying-glass, a screw-driver, and a steel-pin to turn the capstan-screws for adjustments, are also furnished with the instrument. In some theodolites, the telescope passes through the horizontal axis, the supports are made sufficiently high to allow the telescope to pass under them when the instrument is reversed in azimuth, and it then becomes an astronomical altitude and azimuth circle. With these additions, a well-made theodolite may perform most of the problems in practical astronomy with considerable accuracy, though such an instrument would be rather too good for the usual purposes of surveying, which may be very well effected by an inferior instrument.

ADJUSTMENTS.

1. The first adjustment is to make the intersection of the cross-

wires coincide with the axis of the cylindrical rings on which the telescope turns, called rectifying the line of *collimation*. This is known to be correct when the eye, looking through the telescope, observes the intersection of the wires continue on the same point of a well-defined distant object during an entire revolution of the tube of the telescope in the Ys. First make the intersection of the wires, when the level is under the telescope, coincide with some well-defined distant object; then turn the telescope half round in its Ys till the level lies above it; and if the same point is still cut by the intersection of the wires, the adjustment is correct in that position. If not, move the wire one-half the deviation, by turning two of the opposite screws at *a a*, taking care to *release one* before tightening the other, and correct the other half by elevating or depressing the telescope. Proceed in like manner with the other position, by placing the level alternately on the right and left.

Now if the coincidence of the cross-wires with the mark remains exact during a complete revolution of the telescope in the Ys, the line of collimation is correct; if not, the same operations must be repeated till it is so.

2. The second adjustment is that which places the level attached to the telescope parallel to the *rectified* line of collimation. The clips Z Z, being open, and the vertical arc PVP clamped, bring the air-bubble, N′, of the level to the centre of its glass-tube by turning the tangent-screw T; when this is done, reverse the telescope in the Ys, that is, turn it end for end very carefully, so as not to disturb the vertical arc; then, if the bubble resume its former position in the middle of the tube, all is right; but if it rises to one end, bring it back one-half by the screw towards the eye-end of the telescope in the figure, which elevates or depresses that end of the level, and the other half by the tangent-screw T; and this process must be repeated till the adjustment is perfect. To make it completely so, the level should be adjusted laterally, that the bubble may remain in the middle of the tube when slightly inclined to either side of its usual position under the telescope. This is effected by giving the level such an inclination; and if the bubble does not continue still in the middle, it is necessary to make it do so by turning the two lateral screws in the end of the level next the eye. If, in making the lateral adjustment, the former should be deranged, the whole operation must be carefully repeated.

3. The third adjustment is that which makes the axis of the horizontal limb, or the azimuthal axis, truly vertical. Set the instrument, by the eye, as nearly level as possible; fasten the centre

of the lower horizontal limb by tightening the staff-head by the clamp I, while the upper limb is at liberty to be moved till the telescope is over two of the parallel plate-screws; when in this position, bring the bubble of the level under the telescope to the middle of its tube by the screw T; now turn the upper limb, or vernier-plate, half round, that is, through 180° from its former position, then, if the bubble returns to the middle of its tube, the limb is horizontal in that direction; but if not, half the difference must be corrected by the parallel plate-screws over which the telescope lies, and the other half by elevating or depressing the telescope from turning the tangent-screw T, of the vertical arc. When this is effected, turn the upper limb 90° from its present position, either forward or back, that the telescope may lie over the other two parallel plate-screws, and from their motion set it horizontal by means of its level. Having now levelled the limb-plates by means of the telescope's level, which is commonly the most sensible upon the instrument, the air-bubbles of the levels fixed upon the vernier-plate may be brought to the middle of their tubes by the screws which fasten them to their places.

4. The fourth adjustment is that which brings the zero of the vernier of the vertical arc to zero on the limb. When all the preceding adjustments are perfect, if zero on the vernier does not coincide with zero on the arc, the deviation must be rectified by releasing the screws by which the vernier is held, and then tightening them after having made the proper adjustment. As this is an operation difficult to be performed accurately, it will be perhaps better to call the quantity of deviation an *index error*, to be applied according to its sign, which must be carefully noted. This index-error is best determined by repeating the observation of an altitude, or depression in reversed positions of the telescope and vernier-plate, then half the difference will be the error; or half the sum of the observed altitudes or depressions before and after reversing the telescope, will be the true angle independent of index-error.

THE METHOD OF OBSERVING WITH THE THEODOLITE.

The instrument being placed, by means of its plumb-line, exactly over the station whence the angles are to be taken, and set level by the parallel plate-screws, then, by the clamping and tangent screws, set the vernier A exactly to zero, and B to 180°, or as near it as the construction of the instrument will permit; read off the verniers, and note them in a book for that purpose. Turn the telescope by hand till it is nearly on the left hand object by a motion of the head of the instrument fixed in one piece, round the lower axis C, tighten the clamping-screw I, and with the tangent-screw L, make the intersection of the wires nicely bisect the object. Now release the upper plate, and move it round by hand till the telescope is directed to the second object, whose angular distance from the first is required; then clamping it with the screw M, make, with its tangent-screw, adjacent to it, the cross wires bisect this object correctly, and read off the two verniers as before, the difference between the first and second means will be the true horizontal angle required; thus,

First Reading.			Second Reading.		
Vernier A	.	0° 0′ 0″	Vernier A	.	61° 45′ 30″
B	.	0 0 30	B	.	46 20
1st sum	.	0 0 30	2d sum	.	11 50
1st half	.	0 0 15	2d half	.	61 45 55
			1st half	.	—0 0 15
			Difference or true angle	. .	61 45 40

By means of the motion of the lower horizontal plate about the vertical axis, any angle required with great accuracy may have its value repeated as often as we please, and the amount of the whole, divided by the number of repetitions, will give the simple value, almost independent of errors of construction and dividing. To repeat an angle, therefore, after making the second bisection as directed above, let the upper plate remain clamped to the lower, while the clamp of the axis is released. Now move the whole head of the instrument by hand round upon the lower axis toward the first object, placing the cross wires in contact with it, and make, as at first, the bisection perfect with the lower tangent-screw. Leaving the instrument in this position, release the upper or vernier plate, turn the telescope towards the second object as formerly, and bisect it nicely with the aid of the upper clamping and tangent screws. This operation completes one repetition; and when the observation is read off and compared with the preparatory reading of the verniers, the difference will be twice the real angle.

Vernier A	123° 31′ 30″
B	32 0
Half the sum or mean	123 31 45
First reading	— 0 0 15
Difference equal to twice the arc . . .	123 31 30
Half or simple value	61 45 45

The correct angle from one repetition, and this process may be carried as far as five or ten times, if thought necessary, taking always care to read the first observation, and record it, so that when the last division is performed, as many circumferences of 360° may be first added as will render the quotient nearly the same, within a few seconds, as the first observation already recorded. When the art of constructing and dividing instruments was less perfect than at present, considerable advantage resulted from this repetition, though now little, in ordinary cases, will be obtained. Sometimes a telescope, capable of motion horizontally and vertically, is placed under the parallel plates as A′ B′, in the figure, page 87, of which the intersection of the cross wires is clamped upon a fixed object, to detect any movement of the instrument during the time of observation, should it occur, either from an unsteady support or undue pressure on the part of the observer.

The magnetic bearing of an object is taken by reading the angle pointed out by the compass-needle when the object is bisected by the telescope, recollecting that the north end of that needle is indicated by a notch or small brass pin passing through it horizontally; and in the usual construction of the instrument, the *south* end is generally that read, though, for greater accuracy, the mean of both may be taken.

The bearing may be obtained a little more accurately by clamping the lower plate, then by moving the upper plate till the needle reads zero, at the same time reading the horizontal limb; now, by turning the upper plate about, bisect the object and read again: the difference of these two readings will be the bearing required.

In determining the variation by the theodolite-compass, it would contribute to accuracy by destroying the errors of centring of the needle, to observe two objects whose azimuths had been accurately found astronomically both *forward* and *backward*.

In taking angles of elevation or depression, it may be added that the object must be bisected by the horizontal wire, or more accurately by the intersection of the wires. In cases requiring great accuracy, after an observation is made with the telescope in its usual position, it may be reversed in the Ys—that is, turned end for end, and the same observations repeated, and a mean of the whole taken for the true value.

The proof of the accuracy of a number of horizontal angles taken completely round one point or station, is that their sum should be exactly 360°.

If all the angles of a plane triangle be measured, their sum ought to be 180°; of those of a four-sided figure 360°; and, in general, when all the angles of any polygon of n sides are measured, the sum $s = 180° (n - 2)$, provided all the angles be *salient*, that is, projecting outwards from the body of the figure, or if the interior angle, when even greater than two right angles, be thus measured.

REPEATING THEODOLITE.

Suppose the theodolite, whose figure is given in page 87, to be placed on its stand, consisting of a tripod or other firm support, and adjusted so as to have its leading vernier A, for example, at zero, when the upper telescope A B is accurately directed by its tangent screw upon the object on the right, and the under telescope A′ B′ upon the object on the left, and then both clamped firmly to the instrument. Now, let the lower telescope A′ B′, on the object to the left, by a motion of the head of the instrument, be placed upon the object on the right, the upper telescope will be moved to the right by an angle equal to that required to be measured. When the instrument is in this position, let the upper clamping screw be slackened, and the corresponding telescope brought upon the object on the left, and made to bisect it nicely by its tangent screw, it is evident that the first, or upper telescope A B, has passed over an arc double of the angle required.

If this process be repeated, the arc read by the vernier will be quadruple of the required arc, thus doubling the measure each time. Hence this instrument is called a *doubly-repeating* theodolite, in contradistinction to the preceding instrument represented in page 473, which gives simple repetitions only. The doubly-repeating theodolite, of about twelve inches diameter, reading to ten decimal seconds, or 3″.24 sexagesimal seconds, is much used by the French engineer officers, and seems to give satisfactory results.

It is supposed to eliminate all errors of centring and dividing, when the repetition is carried to ten or twenty times, though I think this doubtful. In theory, the instrument is apparently perfect; but from the defects of materials and workmanship, and the insufficiency of the tangent and clamping screws to produce absolute stability, it is found from experience to repeat the error along with the angle measured, so that it is not equal to our larger and more perfect, though weightier instruments.

THE SPIRIT-LEVEL.

The spirit-level, as usually constructed, is an instrument in some respects similar to the theodolite, and by the latter the operations of the former may be readily performed. The spirit-level has a stand with clamping and tangent-screw, a telescope with its level, and a compass exactly similar to the theodolite, but without horizontal or vertical arcs, the compass alone being thought sufficient for every angular purpose required in the use of this instrument.

The method of setting up and adjusting for observation the Y level at least, being so similar to that followed for the theodolite, that it is not necessary to say much in regard to it here.* There are, however, several other kinds of levels, such as Troughton's, Gravatt's, &c., with more powerful telescopes than those generally applied to theodolites, in which some of the adjustments are effected by the maker, and do not so easily get out of order as those of the common Y level. These adjustments are generally made in the field by interchanging the position of the instrument and divided levelling-rod, half the difference of the reading is the correction of the level, which must be rectified by altering the adjusting screws of the telescope, till the intersection of the cross wires cuts the middle point between the two readings in both positions. Or a station, on as level a piece of ground as possible, may be chosen exactly half-way by measurement between two levelling-rods, perfectly vertical, at the distance of about two hundred yards from one another. The instrument being firmly placed upon its stand at one hundred yards from each staff, the air-bubble is made, by the screws of the parallel plates, to remain perfectly in the same position when turned to each staff in succession, and the divisions cut by the horizontal wire of the telescope in both instances recorded. These divisions are truly on the same level, though the line of collimation may not be parallel to the spirit-level. Next remove the instrument to one of the staves, raise the centre of the eye-aperture to the height recorded at the staff where it stands, then direct the telescope to the other staff, and if the reading agree with that first recorded on it, the instrument is truly adjusted. If not correctly the same, alter the adjusting screws till the difference of the readings is reduced to one-half the original, and the instrument will then be truly adjusted. It would be prudent to repeat

* See fig., page 239.

this operation till the observer is satisfied. In all instruments it is recommended to have the heads of the adjusting screws protected or covered, so that it is impossible or difficult to alter them by accident or mistake.

From the preceding observations it is obviously advantageous to place, when convenient or practicable, the instrument half-way between the fore and back staffs, to avoid error from bad adjustment, as well as to render any allowance for curvature and refraction unnecessary.

There are various levelling-rods constructed, to be used along with this instrument, having marks or vanes that slide up and down, and are moved by the bearer or assistant. It is, however, more convenient, and less liable to error, to have a rod divided into feet, tenths, and hundredths, and so distinctly marked that the principal observer may easily read them through his telescope at a moderate distance, and instantly rocord them. *In all observations the reading and writing should be re-examined, to see that both are correct.*

THE ALTITUDE AND AZIMUTH CIRCLE.

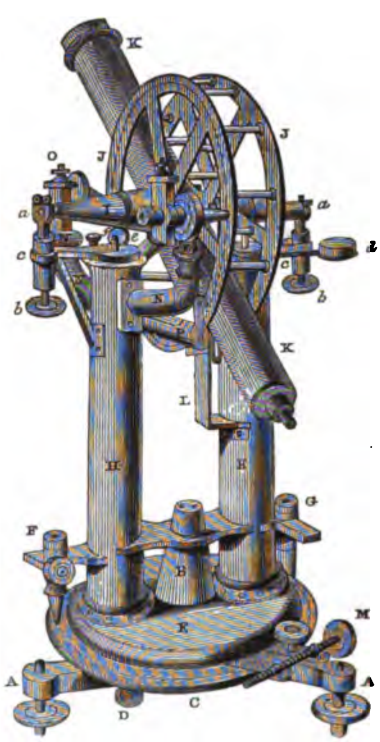
BY TROUGHTON AND SIMMS.*

The Altitude and Azimuth Circle, as now constructed, is an instrument of great utility and importance in practical astronomy and geodesy. It is made of all dimensions, varying from the small portable instrument, whose divided circles are five or six inches in diameter, to those of two or three feet. The smaller class have their arcs read by verniers, the larger by reading microscopes. The diameters of the divided circles of that whose figure is here given are generally about twelve or eighteen inches, and the divisions are subdivided by reading microscopes; while the smaller class of the same construction, varying from ten to twelve inches, have only verniers reading to about 10″: but both these are occasionally varied to suit the views of purchasers, and the work they are required to perform.

* This instrument still continues to be made in a superior manner by Mr Simms, 138 Fleet Street, London. Prices,—Both circles, twelve inches diameter, £100, with reading micrometers; both circles, eighteen inches diameter, £200, ditto. Ertel's universal instrument, having its horizontal circle twelve inches, and its vertical nine inches, with verniers reading to 4″, is a valuable instrument, price £150.

DESCRIPTION.

To the centre of the tripod A A, is fixed the vertical axis of the instrument of a length equal to about the radius of the circle passing up the interior of the frustum of the cone B.* On the lower part of the axis, in close contact with the tripod, is centred the azimuthal circle C, which, by means of a slow-motion screw, whose milled head appears at D, admits of a horizontal circular motion of some extent for the purpose of bringing its zero exactly into the meridian; though in some instruments, for the sake of permanent stability, this is omitted, as it is occasionally purchasing a convenience at the risk of some error.



Above the azimuth circle, and concentric with it, is placed a strong circular plate E, which carries the whole of the upper works, and also a pointer, to show the degree and nearest five minutes on the azimuth circle, while the remaining minutes and seconds are obtained by the reading-microscopes F, G, as previously explained in the description of the reading-microscope. This plate, by means of the conical part B, supported by a brace and carefully fitted, rests on the axis of the tripod, and moves concentric with it. The conical pillars H H, support the horizontal or transit axis I, which, being longer than the distance between the centres of the pillars, requires the projecting pieces *c c*, fixed to their tops to carry out the Y's *a a*, to the proper distance for the reception of the pivots of the axis. The Y's are capable of being raised or lowered for levelling, &c. the axis, by means of the milled-

* This axis is equally or more conveniently situated when it descends through the tripod.

headed screws *b b*. The weight of the axis, with the load that it carries, is prevented from pressing so heavily on its bearings as to injure the pivots by two friction-rollers on which it rests, whereof one of them is shown at *e*.

This is accomplished by a spiral spring fixed in the body of each pillar, which presses the rollers upwards with a force nearly equal to the superincumbent weight. These rollers, on receiving the axis, yield to the pressure, and allow the pivots to find their proper bearings in the Ys; while, at the same time, they relieve them from a great part of the weight which might cause them to wear rapidly and irregularly, thereby injuring the accuracy of the instrument.

The telescope K K passes through the axis I, on which, as a centre, there are fixed the two circles R R, each close against the telescope on both sides. The circles are fastened together by small brass pillars, and, in the larger classes of instruments, occasionally supported by diagonal braces. By this double circle the vertical angles are measured on graduations cut upon a ring of silver, generally on one of the sides only, which from that circumstance is called the face of the instrument—a distinction to be attended to in making observations, by placing it alternately to the right and left, when a series is being completed. The clamp for fixing, and the tangent-screw for giving slow motion to the vertical circle, are placed beneath it, between the pillars H H, and attached to them as seen at L. A similar contrivance for regulating the azimuth circle, likewise divided on silver, is represented at M. The reading-microscopes for the vertical circles are carried by two arms N N, bent upwards near their extremities, and attached towards the top of one of the pillars, one of which is shown above *e*, and the other under *o*.

A circular plate of brass, with a round hole cut in it, called a *diaphragm*,* is fixed in the principal focus of the telescope near the eye-end, across which are stretched five vertical, and five horizontal wires, at right angles with each other. The intersection of the two central ones, denoting the optical axis of the telescope, is the point by which an object ought to be bisected when only observed at one point, such as a terrestrial object when taking angles for geodesical purposes. The vertical wires are used for the same purpose as those in the transit instrument for observing the passage of a celestial body over the meridian, and the horizontal ones for taking zenith-distances, or altitudes of celestial objects, by which

* See the figures, page 468.

a mean of five observations, or rather contacts, may be readily obtained. A micrometer, having a movable wire, is sometimes attached to the eye-end of the telescope of the larger instruments, though it is not generally applied to the smaller class. This is frequently useful, but it cannot in general be so confidently relied on as an observation taken in the usual manner. The illumination of the wires necessary at night is effected by a lamp, supported near the top of one of the pillars as at *d*, and placed opposite the end of one of the pivots of the horizontal axis, which being perforated, admits the rays of light to the centre of the tube of the telescope, where, falling on a perforated diagonal reflector, they are thrown towards the eye, and illuminate the field of view so as to enable an observer to bisect a star at or close upon the intersection of the central wires.

The vertical circle is usually divided into four quadrants, especially if there be two microscopes or verniers only, each numbered from the horizontal points, when the telescope is in a vertical position, 0°, 10°, 20°, &c. as far as 90°. In this case each microscope shows zenith-distances. If verniers are used, there must be two sets of numbers on them reading in opposite directions, as shown on the figure, page 462. In reading observations, the *arrow* always indicates the degrees and every ten minutes; but if, when the *face* of the circle is on the *right*, the minutes and seconds obtained by the vernier be read *from* the arrow in the order of the upper numbers upon the vernier plate, then when the circle is reversed, thereby placing the *face* on the *left*, they must be read *towards* it in the order of the lower numbers, and *vice versa;* while care must be taken by the observer *always* to read the arc and verniers in the order of the figures in the *same* direction.

In some circles a different plan is followed. The whole vertical circle is divided into four quadrants as before, each numbered 0°, 10°, 20°, &c. as far as 90°; but, instead of the previous method, following one another in the same order of succession. Consequently, in one position of the instrument, *altitudes* are read off, but, with the face of the instrument reversed, *zenith-distances;* and with such instruments an observation is not considered complete till the object has been observed in both positions. In the latter case, the sum of the two readings will always make 90°, provided there be no error in the adjustments, the circle, or the observation.*

* This, of course, can hold good with objects perfectly at *rest* only. When celestial objects are observed, their motion will render the sum of the readings unequal to 90°. In all cases, whether for terrestrial or celestial observations, to

In cases where there are three or more microscopes, the readings will be different, according to the construction. When, however, there are but two microscopes O O, the straight line joining them should pass through the horizontal diameter of the circle, to render which perfect, a vertical motion, by means of the screws *b b*, is given to the Ys, to raise or depress them till this adjustment is accomplished.

A good spirit-level P, suspended from the arms which carry the microscopes, shows, upon turning round the circle, when the vertical axis is set perpendicular to the horizon. A scale usually showing either single seconds, or (what is more convenient for small instruments) *two seconds*, is placed along the glass-tube of the level, which exhibits either the permanency or the inclination of the vertical axis. This should be examined repeatedly, whilst making a series of observations, to ascertain whether any change has taken place in the position of the instrument after its adjustments have been completed, and, by recording its indications, to allow for any deviation if necessary. One of the points of suspension of the level is movable by means of a screw *f*, for the purpose of adjusting the bubble. A riding-level, similar to that employed to level the transit-instrument, rests upon the pivots of the axis. It ought to be carefully passed between the radial bars of the vertical circle when set up in its place, and must be removed as soon as the operation for levelling the horizontal axis is performed.

The whole instrument is supported upon three foot-screws, placed at the extremities of the three branches which form the tripod, and brass cups are placed under the ends of foot-screws when put upon its stand. A stone pedestal, set perfectly steady, is the best support for this as well as the transit-instrument; but for travellers, a strong well-made tripod of wood, firmly braced, will be the most convenient. The author has frequently used a very convenient small six-inch circle, differing a little from this, having *three* verniers, each showing 10″, and a *fixed* level, each of whose divisions indicate 2″.

The Adjustments.

1. *To make the vertical axis perpendicular to the horizontal plane.* Set up the instrument in the position where the observations are

prevent ambiguity, and the unnecessary distinction of *altitudes* and *depressions*, I prefer instruments reading *zenith-distances*.

to be made, then turn the instrument round till the spirit-level P is lengthwise in the direction of two of the foot-screws, when, by their motion, the air-bubble in the level must be made to occupy the middle of the glass-tube shown by the divisions of the scale attached to the level. When this is done, turn the instrument half round in azimuth, and if the axis is truly vertical, the bubble will again settle in the middle of the tube; but if not, the amount of the deviation will show *double* the quantity which the axis deviates from the vertical in the direction of the level. This error must be corrected —one-half by two of the foot-screws over which the level is placed, and the other half by raising or lowering one end of the spirit-level itself by the screw represented at *f*. This process of reversion and levelling should be repeated, to ascertain whether the adjustment has been accurately performed or not, since adjustments of every kind can be made perfect by successive approximations only. When this part of the adjustment is satisfactory, turn the instrument round in azimuth a quarter of a circle, so that the level P may be at right angles to its former position; and it will then be over the third foot-screw, which must be turned till the air-bubble is again central, and this adjustment will be completed. If the whole has been correctly performed, the air-bubble will remain steadily in the middle of the level, indicated by the divisions of its scale, during an entire revolution of the instrument in azimuth. If not, the operations must be repeated till it does so.

2. *To set the vertical circle at such a height that its two reading-microscopes shall be directed to two opposite points or zeros in its horizontal diameter.*

This is readily accomplished by raising or depressing the Ys by means of the screws *b b*, which carry the horizontal axis.

3. *To level the horizontal axis.*

This operation is performed by means of a riding-level. Apply this level to the pivots, bring the air-bubble to the middle of the glass-tube by observing if the extremities of the bubble stand opposite the same division on each end of the scale by means of the screws *b b*, as before. Then reverse the level by turning it end for end, and if the air-bubble still, as formerly, remain central, the axis will be horizontal; but if not, half the deviation must be corrected by the screws *b b* and the screw at one end of the level, which raises or depresses the glass-tube of the level with respect to its supports that rest upon the pivots. After performing this adjustment, the preceding must be examined to see if it be deranged by the last process. Indeed, it is preferable to set the axis horizontal first, and then, by equally raising or depressing the two ends, to bring the microscopes into a diameter, and finally to level the axis again.

4. *To adjust the line of collimation.*

This adjustment requires the middle vertical wire to describe a great circle, and the middle horizontal wire to have a certain definite position with respect to the divisions on the limb. It is usual to rectify the middle vertical wire first, the others being set parallel by the maker. Direct the telescope to some small well-defined distant object, bisect it with the intersection of the two central wires, and clamp the circle in that position. Now, turn the whole instrument half round in azimuth *exactly*, and, by the tangent-screw, elevate or depress the telescope till it cut the same object, and if it be bisected at the same point as before, the collimation adjustment is correct; if not, turn the small screws which hold the diaphragm near the eye-end of the telescope through one-half of the error, and the adjustment will be completed. But as half the deviation may not be correctly estimated in moving the wires, it is necessary to verify the adjustment by moving the telescope the other half. This operation must be repeated till, by continued approximations, the adjustment is found to be perfect. To adjust the middle horizontal wire, point the telescope to a very distant object, near the intersection of the wires, bisect it by the middle horizontal wire, and read off by the microscopes the apparent zenith-distance. Now, reverse the instrument in azimuth, and, turning the telescope again upon the same object, bisect it as before, then read the arc which they show. One of these, in this construction of the instrument, will be an altitude, and the other a zenith-distance, and, if there be no error, the sum of the two readings will be 90° exactly: if they do not make 90°, *half* the difference from 90° will be the error of collimation. If the instrument shows zenith-distances only, then half the *difference* of the arcs in opposite positions will be the index or collimation error, and its sign must be marked, whether + or —, when the face of the circle is to the right or left. This error may be either employed to correct an observation made with the instrument during its continuance in one position, or removed in the following manner. Read the zenith-distances in opposite positions of the circle—that is, with the face alternately to the right and left, of which take the mean, that will be the *true* zenith-distance: then, while the telescope bisects the object, the microscopes, by their proper screws, must be adjusted so as to read that mean. In making a series of observations, however, they are generally taken in pairs, with the face of the circle alternately to the right and left, consequently the mean of the readings gives the true zenith-distance, independent of the error of collimation,—a method commonly followed in practice.

5. *To set the central or middle wire truly vertical.*

This may be effected by directing the telescope to a well-defined distant object. If, on elevating and depressing the telescope, it is bisected by every part of the wire, that wire must be truly vertical. If not, it should be adjusted by turning the inner tube, carrying the diaphragm or wire-plate, till the preceding test of its verticality be satisfied; and, to avoid the effects of any small error on this account, care must be taken to make important observations near the centre only. The other vertical wires are, by the maker, placed equidistant from the middle one, and parallel to it, so that, when it is adjusted, the others are likewise correct. He also places the transverse wires at right angles to the middle vertical wire. These adjustments are always performed by the maker, and are little liable to derangement.

In general it may be remarked, that during a series of observations, should the instrument be found to be a small quantity out of level, (the other adjustments being perfect,) it may be restored generally by means of the foot-screws only, when they require but a slight touch to effect it. This is more especially essential when the level of the horizontal axis is the one deranged, since correcting it by moving the Ys would derange the adjustment of the vertical circle with regard to its reading microscopes,—an occurrence which must be carefully avoided. The error of the vertical axis is to be detected by the hanging level, and, by reading its scale, can be very readily allowed for in computing observations, as has already been shown in the description of the level.

General Rule. *When great accuracy is required, it is both easier and safer to correct by calculation, than to adjust by mechanical contrivance.*

Use of the Altitude and Azimuth Circle.

This is the most generally useful of all instruments, because it measures with great accuracy both horizontal and vertical angles. It does not, however, possess the power of repetition, like the circle of Borda, but the effect of any error of division on the horizontal circle may be diminished or destroyed by measuring the same angle upon different parts of the arc. For this purpose, let r be the number of repetitions required, v the number of verniers, and c the change of zero in degrees, $c=\frac{360°}{vr}$. Let, for example, $v=3$, and $r=4$, then $\frac{360°}{12}=30°$, the change. Whence the successive zeros, or rather starting points, of the vernier A are 0°, 30°, 60°, and 90°. By this process the whole circumference of the circle is equally employed, whence the small errors of excess and defect tend mutually to destroy each other. Even a *small* quantity of change, by means of the screw D, if a great one be inconvenient, will greatly diminish the chance of errors in division, reading, and pointing. A repeating-stand is frequently added to this instrument, which is a convenient appendage when great accuracy is required in the measurement of horizontal angles; and the operation is exactly similar to that explained when treating of the use of the theodolite. The vertical angles should, in all practicable cases, be taken at least twice, reversing the circle before taking the second observation, which will eliminate not only the errors of centring and division, but also those of collimation and level. In applying the instrument to astronomical purposes, this method is always employed. When the instrument is used to determine the latitude by what is termed *circum-meridian* observations—that is, several observations taken a short time before, and a like number after, the meridional passage or transit, at times nearly equidistant—observe first with the face of the instrument to the right, and then to the left, by reversion in azimuth, noting the precise time of each observation. Now if, from computation, we have the exact time of the object's transit, by a chronometer showing either true time, or with a known error and rate, the object's distance from the meridian in time, at the instant of each observation, may be found. This, with the approximate latitude of the place, and the declination of the object, afford, by the formula (6) in page 337, and the aid of Table XVII., data for computing a quantity called the reduction to the meridian, which, *subtracted* from the mean of the observed zenith-distances, when the object is on the upper meridian, will give the apparent meridional zenith-distance of the object. When the object is on the lower meridian under the pole, the reduction must be added. This reduction must be applied with a contrary sign to the altitude. The nearer the observations are taken to the meridian, the less will the accuracy of the results depend upon a true knowledge of the time. To obviate such an error as much as possible, an equal number of observations should be taken nearly equidistant from the meridian, and not extending to more than ten or twelve minutes on each side of it, when the zenith-distance is not less than twenty or thirty degrees, even when taking in quantities of the *second order*. Should the zenith-distance be less than this, in mean latitudes, the time must be limited to five or six minutes; and, when very near the zenith, this method of repetition is not to be recommended.

This instrument may also be very successfully employed to determine the time and the direction of the meridian, either by absolute altitudes and azimuths, or equal altitudes and azimuths, when corrected by the necessary equations, by Table XVIII., for those purposes. The direction of the meridian may be very accurately determined with this instrument, by means of any circumpolar star, especially by the pole-star, when referred to a mark in or near the horizon, as shown in pages 354, 355, &c.

To insure permanence of position during a course of observations, this instrument is frequently furnished with an under telescope, capable of some degrees of motion, both in a horizontal and vertical direction, till the cross-wires in its focus acurately bisect some well-defined distant object, on which it is firmly clamped at the commencement of a series of observations; and the accuracy of the bisection being examined after their termination, and found perfect, proves the steadiness of the instrument, and no relative motion has taken place during the course of the operations.

Adie and Son of Edinburgh construct a class of theodolites similar in principle to this instrument, but of smaller dimensions, the divided circles being from five to ten inches in diameter, with three verniers, reading to 20″, 15″, or 10″, according to the size.* The arms of the tripod are bent at right angles downwards, so as to raise the horizontal circle sufficiently to admit the conical axis B to descend below instead of rising above it,—a position perhaps somewhat more convenient. These instruments are, therefore, well fitted to perform all the operations of a theodolite and an astronomical circle with great precision, considering their moderate dimensions and reasonable price. The common method of placing the centre of the vertical axis accurately over a station, is by means of a plumb-line suspended from the under side of the horizontal circle; though, in some of the larger class of instruments, the axis B is hollow in the middle, with two cross-wires adapted to it, cutting each other at right angles in its centre, which, by means of a diagonal eye-piece in its top, is by a slight motion of the instrument brought to bisect the centre of the station.

* They vary in price from £25 to £50.

ZENITH-SECTOR.

In the English Trigonometrical Survey, the instrument called a Zenith-Sector has generally been used to fix the latitudes of the extremities of the arcs of the meridian. This instrument, since the time of Bradley, who by one of them discovered the aberration of the fixed stars and the nutation of the earth's axis, has always deservedly held a high place in the estimation of the astronomers of this country. It consists of a telescope firmly fixed to a trussed frame, which turns on an axis at its top in the plane of the meridian, and its lower extremity passes over a small arc of 10° or 15°. Such also was the construction of the ordnance Zenith-Sector, begun by Ramsden and completed by Berge.

It is obvious from this construction that it is merely a portion of the old astronomical quadrant, taking in only a few degrees on each side of the zenith, and possessing the property, too, of turning in azimuth, to reverse the readings on each side of zero. It might therefore have a large radius, but still all the disadvantages of the quadrant, from a want of opposite readings adapted to it, and could be reversed on *different* days only to render an observation complete.

Colonel Colby, conscious of these objections, had, after the destruction of the preceding instrument by a fire at the Tower, a new Zenith-Sector constructed by Mr Simms, on a plan suggested by Mr Airy. In this new instrument the telescope was attached to a frame supported on pivots, which gave the advantage of opposite readings and a reversal of the instrument in azimuth at each observation, and, in fact, is merely a section of the altitude and azimuth circle, having a few degrees of arc at top and bottom. It therefore occupies less room, and is more portable than a circle of equal radius.

It is, therefore, strictly analogous to the altitude and azimuth circle in its principles and application to practice.

Observations are frequently made with the aid of a basin of mercury by reflexion, and at the same time by direct vision, whence any inherent or casual discordances are corrected.

In a paper published in the *Transactions of the Royal Scottish Society of Arts*, in the year 1843, Vol. II. page 211, I ventured to recommend a transit circle of cast-iron, with the exception of the axis, from which I believed, from personal experience, much greater stability and accuracy could be obtained than from the mural circle, as then constructed, to which I had stated objections previously without producing any effect. In the year 1848, in one of the monthly notices of the *Royal Astronomical Society*, Vol. VIII., page 212, Mr Airy proposes a transit circle of cast-iron similar to what I had done, and analogous to my proposition in 1843, which, doubtless, will be shortly constructed. I have repeatedly spoken in terms of approval of the circles being cast solid, or at least of one piece, from the advantages attending the small altitude and azimuth one which I now possess consisting of solid circular plates, made by the late T. C. Robinson of London; and I have no doubt that a transit circle similarly constructed would be a decided improvement on the mural circle.

The French have hitherto chiefly used the repeating circle of Borda for geodetic purposes, which is of an ingenious construction, but its superior accuracy is quite ideal, and the observations made with it require a great deal of reductions, causing thereby much unnecessary labour.

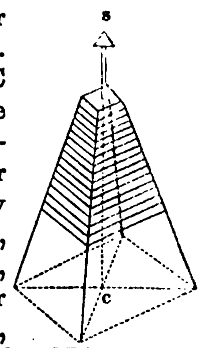
SIGNALS.

1. Signals ought to be established in such a manner as to be clearly seen at a distance. A vertical pole, with a red or white flag, makes a good signal for ordinary purposes at short distances. Spires, towers, lighthouses, &c., may occasionally serve the same purpose, but then it is necessary to reduce observations made in connexion with them to the centre of the stations, thus causing considerable additional labour in measurements and calculations. Good signals may be made of a disc of plate iron or copper painted black, perforated by a hole at its centre through which the light of day may be readily seen, especially against the sky. This disc ought to traverse round a central axis fixed in a pole for that purpose, so that its perforated surface may be successively presented to the surrounding stations. Sometimes, also, argand lamps, with parabolic reflectors, have been employed, though now generally abandoned, because at night, the time they are used, atmospheric irregularities are greatest. A signal is better seen when *projected against the sky* than against the ground or woods, and this circumstance is easily known by ascertaining from measurement the zenith-distances of any two signals. If their sum be less than 180°, they will one or both be seen against the sky. As these conditions cannot always be fulfilled, it will be advantageous to paint the signals white when they are projected against the ground, and those

black which impinge against the sky. In very distant observations, piles of stone or turf, with a vertical pole in their centre, are generally used. Their height is commonly about one ten-thousandth of their distance, so as to subtend an angle of about 20″, and the diameter of the base one-third of their height. The author has had constructed circular heliotropes of plate glass, about three or four inches in diameter, set in a brass frame having two motions at right angles to one another, so as to take all degrees of inclination. The back of the glass is painted black, with a circular hole in the middle of the frame, and the glass unpainted opposite to it, by which the proper position of the heliotrope is given, as explained previously in page 332.

The bottom of the stand is loaded with lead, so as to give it stability when placed on a proper base. There is also a brass rod prepared to screw into the bottom of this stand, so that by its means the heliotrope may be stuck into any given signal or pile when a flat steady base cannot be attained.

In remote situations, observatories must be occasionally constructed to protect the observer, when no edifice can be had to serve at the same time for a lodging and a signal. These structures have generally the form of a quadrangular pyramid truncated near the top, as in the figure. The superior prolongation S of the axis S C serves for a signal, and the observer places the centre of his instrument at the point C, the imaginary projection of S upon the ground. The four edges or corners are of beams of wood, solidly planted in the ground, and connected by traverses, properly jointed with mortises, to a central post, of which the upper extremity is S. The exterior is properly planked with deals from the top, descending within about six feet of the ground. This part is left unplanked, to enable the observer to see distant objects distinctly and take a round of angles. In coarse weather, a canvass covering may be placed *on the weather side*, to afford the necessary protection from the wind, rain, or snow. When the operations are finished, this species of signal is removed. A roughly-dressed stone is placed directly under C, upon which two straight lines are cut at right angles, whose intersection indicates the axis of the signal. At the intersection a circular hole, of about an inch in diameter, is bored with an iron cylindrical rod, like those commonly used in rock-blasting. Verifications frequently require that these posts



should be found readily and correctly, while the expense and trouble required to execute them are trifling.

One of the most portable signals is constructed of a wooden frame covered with canvass. Two circular rings or hoops of wood, about six or eight feet in diameter, having six or eight sockets of copper let into them, forming the lower and upper ends of the upright part of the signal. Into these sockets the same number of cylindrical wooden bars or posts, by ends furnished with copper bolts, are fixed. Upon this is placed a conical roof, and the whole covered with canvass, any part of which is removable at pleasure, for the purpose of facilitating observations in any given direction, while, at the same time, the remaining portion furnishes a convenient protection to the observer. After removal, the station of the signal is accurately marked as before, when necessary.

Small portable wooden houses have been lately introduced on the Ordnance Survey, which may be readily set up and taken down, when necessary, by means of appropriate joints, that are much more comfortable for the residence of the engineer officers than the usual tents.

TRANSIT INSTRUMENT.

BY TROUGHTON AND SIMMS.*

DESCRIPTION.

A transit-instrument is a telescope, properly placed in the meridian, for the purpose of observing the times at which the celestial bodies pass this circle. The telescope is fitted to an axis, of which the ends, formed into pivots, turn in notches, from their shape called Ys. This axis is made hollow, opposite one of the ends of which is placed a lamp for illuminating the wires in night observations. These wires, generally five in number, are placed in the telescope equidistant from each other, and perpendicular to the horizon, having also a horizontal wire bisecting them at right angles, near or upon which the transits are observed. When properly adjusted, the middle vertical wire coincides with the meridian, and the instant that the centre of any celestial body passes this wire is called its transit. The other parallel wires are intended to correct or verify the observation, by taking a mean between the transits over the first and last, the second and fourth, and comparing it with the

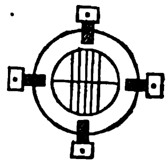
* Prices—Twenty-inch telescope, £20; thirty-inch telescope, £40.

third or meridian wire; or, what is more correct, to take a mean of the whole, called the reduction of the wires.

The figure represents this instrument when the telescope varies from eighteen inches to two feet in focal length. The telescope A A consists of two parts, connected together by a sphere B, which also receives the larger ends of the cones C C, placed at right angles to the tube of the telescope, and forming the horizontal axis. This axis terminates in two cylindrical pivots, which rest in Ys fixed at the upper ends of the vertical standards D D. One of the Ys possesses a small motion in azimuth, communicated by turning the screw *a*. But that the telescope may move in a vertical circle, the pivots must be precisely in the same level, otherwise the telescope, instead of perpendicularly, will revolve in a plane oblique to the horizon. The levelling of the axis is, therefore, one of the most important adjustments of the instrument, and is effected by means of a spirit-level E, which, for this purpose, is made to ride across the telescope, and rest on the two pivots, and must be removed as soon as the adjustment is made. The standards D D are fixed by screws upon a cast-metal or brass circle F, which rests upon three screws *b*, *c*, *d*, forming the feet of the instrument, and by the motion of which the operation of levelling is performed.

The oblique braces G G are added for the purpose of securing the supports, so that the telescope may have both a free and steady motion. On the extremity of one of the pivots, which extends beyond its Y, is fixed a circle H, which turns with the axis, while the double verniers *e e* remain stationary in a horizontal position, whereof one shows the altitude and the other the zenith-distance at which the telescope is placed. The verniers are both set horizontal by the spirit-level *f*, which is attached to them, and they are fixed in their proper position by a brass arm *g*, clamped to the supports by a screw at *h*. The whole apparatus is movable along with the telescope, and when the axis is reversed, it can be attached in the same manner to the opposite standard. The telescope of a transit belonging to the author, made by Berge, is elevated or depressed by a semicircle attached to the telescope near A. Either of these methods may be used separately or combined.

Near the eye-end of the telescope is placed a diaphragm in its principal focus, represented by the figure which, in this instrument, has five vertical wires and one or two horizontal wires close to each other, between which the observations are made. The central vertical wire ought to be fixed in the optical axis of the telescope, and perpendicular to the horizontal axis. These wires are visible in the day-time



by the light passing down the telescope to the eye; but at night, except a luminous object like the moon be observed, they cannot be seen. In this case they must be illuminated through a hole in one of the pivots of the axis, which admits the light of a lamp placed opposite to it, on the top of one of the standards as shown at I. This light is directed to the wires by a reflector placed diagonally in the sphere B; which reflector, having a large hole in its centre, admits the rays passing from the object down the telescope to the eye of the observer, who thus sees distinctly both the wires and the object at the same time. The lamp is so constructed that the light may be regulated according to the faintness of the objects, so as not to obscure its feeble rays. The telescope is also furnished with a diagonal eye-piece, by which stars near the zenith may be conveniently observed. The altitude and azimuth circle will, when well constructed and in perfect adjustment, perform the operations of a transit-instrument successfully—a circumstance very important to scientific travellers, who often have not the means of carrying a complete collection of instruments along with them.

ADJUSTMENTS.

1. *The wires should be set perfectly vertical.*

This is verified by observing that any distant vertical object, cut by a wire, does not change its position relative to that wire on moving the instrument up and down. If it does, the wires must be turned till the object is kept upon them when moved through their whole extent, and the adjustment is then complete.

2. *The telescope should have no parallax.*

When any distant object is bisected by the horizontal wire, if, on moving the eye up and down a little, the object should appear to separate from the wire, the instrument is said to have a parallax. This must be corrected by placing the object and eye-glasses at such a distance from each other that their foci may meet at the intersection of the wires. When, as is usually the case, the object-glass has been properly fixed by the maker, the observer has only to adjust the eye-glass.

3. *The line of collimation should be correct.*

This is known by bisecting any object by the meridian wire, and if, on reversing the axis, the object still remains bisected as before, the line of collimation is correct. If not, it must be adjusted by the small screws in the sides of the telescope, carrying the diaphragm near the eye-glass. This is effected by easing one screw and then tightening the other, till the error appears one-half diminished; after which the axis is again reversed, and the operation repeated till the adjustment is properly effected.

4. *To level the axis.*

This is performed by a screw under one of the Ys, which raises or depresses that end of the axis at pleasure, while the true horizontal position is ascertained by the spirit-level.

5. *To bring the telescope into the meridian.*

This is accomplished by a horizontal screw acting on one end of the axis, by which it is moved forward or backward till its proper position is obtained.

6. *To prepare the telescope for observation.*

Slide the eye-piece in or out till the wires are seen distinctly. Direct the telescope to some well-defined object, and turn the milled-head on the side of the transit till the object is seen with perfect distinctness. Place the level on the axis, and bring the bubble to the middle by the screw which elevates or depresses one of the Ys, the axis of the transit will then be parallel to the horizon.

Having brought the object to the central vertical wire by means of the screws, which act horizontally on one of the Ys, observe whether the same point of the object is covered by the wire while the telescope is elevated or depressed, and if not, correct half the apparent deviation by turning round the cell which contains the wires. Now, with the wire covering some well-defined distant object, take the instrument out of its Ys and carefully invert it, when, if the wire no longer bisects the same part of the object, correct half the error by means of the screws which act horizontally upon the wires, and the remaining half by the screws which act horizontally upon the Ys. Repeat this operation till the vertical wire covers the same part of the object in both positions of the telescope, and the line of sight will then be perpendicular to the axis.

7. *To elevate the telescope to a given object.*

This operation is performed by computing the altitude or zenith-distance, previously to any observation, and either by the circle on the extremity of the axis in small instruments, or those near the eye-end of the telescope in large ones, elevate it to the proper altitude or zenith-distance, as may be required.

8. *To compute the altitude.*

To the complement of the latitude *add* the declination, if they are of the *same* name, the sum will be the altitude; but subtract it, if of different names, and the remainder will be the altitude; when the object is between the zenith and the pole, of a contrary name to the latitude. If the object is between the zenith and the pole, of the same name with the latitude, the meridian altitude is equal to the sum of the latitude, and the polar distance of the object, when above the pole, but to their difference when below it.

9. *To take a transit.*

With the latitude of the place, and the declination of the object, compute its meridian altitude. When it is known by computation, or otherwise, to approach the meridian, elevate the telescope to the given altitude by one or other of the circles for that purpose. Now, because the telescope inverts, the object will appear to come into the field of view from the west, and move towards the east. Mark, by the clock or chronometer, the time of transit over each wire, using a dark glass to save the eye when the sun is observed, and tabulate the result in the following manner:—

EDINBURGH, 1886.

Date.	Object observed.	Wires. I.	II.	III.	IV.	V.
		s.	s.	h. m. s.	s.	s.
Jan. 15.	Sun 1 Limb	38.6	52.8	19 44 7.0	21.4	35.8
	Sun 2 Limb	58.6	12.8	46 27.3	41.7	55.9
	a Andromedæ	11.2	26.2	23 59 41.3	56.4	11.6

	h. m. s.
Reduction of sun's first limb to III. wire	19 44 7.12
Correction of instrument	+ 0.93
Correction of clock	+ 12.07
Apparent right ascension observed	19 44 20.12
In like manner the second limb	19 46 40.26
Mean or that of the centre, the observed R. A. . . .	19 45 30.19
Reduction of a Andromedæ to III. wire	23 59 41.34
Correction of instrument	+ 0.47
Correction of clock	+ 12.03
Apparent right ascension observed	23 59 55.84

TO BRING A TRANSIT-INSTRUMENT INTO THE MERIDIAN.

1. Let the time be accurately determined by absolute altitudes near the prime vertical, or by equal altitudes, as shown in the explanation of Table XVIII. Having got the error of the clock or chronometer to be used in the observation, compute the time of transit of the object to be observed either in mean solar or sidereal time, according to which the time-piece is regulated, making due allowance for error and rate, as shown in § 15, pages 339, 340, 341, &c., then bring the telescope to the celestial object, when nearly upon the meridian; and by turning the horizontal screw, make the middle wire bisect the object at the instant of its computed transit, and the instrument will be in the meridian. Should the object be the sun or moon, either limb must be observed; and, allowing for the time which the semidiameter takes to pass the meridian, that of the centre becomes known, or the limb, conversely.

TO FIND THE TIME THAT ANY STAR TAKES TO PASS FROM ONE WIRE TO ANOTHER IN A TRANSIT-INSTRUMENT, WHEN THAT ON THE EQUINOCTIAL IS KNOWN.

Rule.—To the log secant of the star's declination, add the logarithm of the time in seconds at the equinoctial, the sum will be the logarithm of the time by the star.

Ex.—On the 10th of May 1836, the declination of Capella was 45° 49′ 32″ N., what would be the time of passage of the star from one wire to another, when the time upon the equinoctial was 19ˢ.64?

Declination . . .	45° 49′ 32″ N.	secant	0.156863
Equinoctial time . .	19ˢ.64	log	1.293141
Star's time . . .	28ˢ.18	log	1.450004

This may be readily performed by a Table of Natural Secants, like that among my General Tables (XXV.;) thus, 19ˢ.64 × 1.435 = 28ˢ.19. Hence the star's expected time of approach to the other wires becomes known after the first contact is observed.

2. To place a transit-instrument in the meridian by Polaris.

On the 1st of May 1840, let a transit-instrument be placed in the meridian at Edinburgh, in latitude 55° 57′ 24″ N., longitude, in time, 12ᵐ 43ˢ.5 W.

By the *Nautical Almanac*, the right ascension of Polaris is 1ʰ 1ᵐ 16ˢ.40, and declination 88° 27′ 23″.4 N.

Whence, by § 8, page 501—

	° ′ ″		° ′ ″
Latitude, . . .	55 57 24 N.	. . .	55 57 24 N.
Star's polar distance, .	1 32 36	. . .	1 32 36
Sum, ...	57 30 0	Difference, .	54 24 48

Hence 57° 30′ 0″ is the star's altitude above the pole, or at its upper transit, and 54° 24′ 48″ at its lower transit under the pole. The complements of these will give the zenith-distances.

Now, let the clock be regulated truly, or reduced correctly by allowing for error and rate, to sidereal time, and when it shows 1ʰ 1ᵐ 16ˢ.4, make the middle wire bisect Polaris, then will the instrument be in the meridian. If the time-piece be regulated to mean solar time, the mean time of transit must be computed as shown in the explanation of Tables XXVI. and XXVII., illustrated by the example in page 342, &c.

Again, if the interval between the *inferior* and *superior* passage be *less* than the interval between the superior and inferior, the plane in which the transit moves between the zenith and the northern horizon is to the eastward of the true meridian. The quantity of deviation may be computed from the observed difference of intervals between the two passages by the following rule or formula.

RULE.—To the log secant of the latitude add the log tangent of the star's polar distance, and the logarithm of half the observed difference of the intervals, the sum will be the logarithm of the correction to be applied as directed above.

Or, if x be the correction, Δ the interval in seconds of time, l the latitude, and p the polar distance, or d the declination,

$$\text{Log } x = \text{const. log } 9.698970 + \log \sec l + \log \tan p \quad . \quad . \quad . \quad (1)$$

$$= \text{const. log } 9.698970 + \log \sec l + \log \cot d \quad . \quad . \quad . \quad (2)$$

EXAMPLE 1.—At Edinburgh, in latitude 55° 56′ 58″ N., longitude in time 12^m $43^s.5$ W., on the 4th of July 1848, the author found, from observation, the interval between the inferior and superior transit of Polaris, or the *eastern* semicircle, *less* than the interval between the superior and inferior transit, or western semicircle measured by the clock, going sidereal time, by 3^m $40^s.8$; required the correction of the transit?

Constant logarithm, ...	9.698970
$l = 55°\ 56'\ 58''$ N. secant, ...	0.251871
$d = 88\ \ 29\ \ 43$ N. cotangent, . . .	8.419434
$\Delta =$ $3^m\ 40^s.8 = 220^s.8$ log, . . .	2.343999
$x =$. $5^s.18$. log, . . .	0.714274

the correction indicating that the transit from the zenith to the *northern* horizon is to the eastward of the true meridian; and consequently between the zenith and southern horizon to the westward.

The effect may be computed for another star by the formula—

$$\text{Log } y = \log x + \log \sec d + \log \text{sine } (l \backsim d) \quad . \quad . \quad . \quad (3)$$

EXAMPLE 2.—Let the star be α Ursæ Majoris.

Log x		. . . .	0.714274
$l = 55° 56' 58'$ N.			
$d = 62\ 34\ \ 8$ N.	log secant,	. . .	0.336599
$d - l = 6\ 37\ 10$	log sine,	. . .	9.061732
$y =$ 1s.296	log,	. . . .	0·112605

the correction at the upper transit of α Ursæ Minor.

Log x		. . . .	0.714274
$l = 55° 56' 58''$ N.			
$d = 62\ 34\ \ 8$ N.	log secant,	. . .	0.336599
$l + d = 118\ 31\ \ 6$	log sine,	. . .	9.943823
y 9s.88	log,	. . . .	0.994696

the correction at the lower transit.

3. To place a transit-instrument in the meridian by a pair of circumpolar stars, differing nearly *twelve hours* in right ascension.

Let t = the time of the first star's upper transit, and t' = that of its lower; also let τ and τ' be the times of the contrary passages of the second star. Now, if δ = the polar distance of the former star, δ' that of the latter, while α is the error in azimuth, and l the latitude.

$$\alpha = \tfrac{1}{2} \{(t - \tau) - (t' - \tau')\} \sec l \sin (\delta \backsim \delta') \quad . \quad . \ (4)$$

EXAMPLE.—On the 1st of January 1828, when the right ascension of the pole-star, by the *Nautical Almanac*, was $0^h\ 59^m\ 28^s.8$, the polar distance $1°\ 36'\ 9''$; the right ascension of ζ Ursæ Majoris was $13^h\ 16^m\ 58^s.3$, the polar distance $34°\ 10'\ 44''$; when the clock of an observatory in latitude $52°\ 25'\ 50''$ N. was regulated properly, and its error and rate allowed for, the times of four passages taken by the transit-instrument placed a little out of the meridian, but otherwise well adjusted, were as follows:

	h. m. s.		h. m. s.
Pole-star above, . .	1 0 0.55 $= t$,	below,	12 58 55.47 $= t'$
ζ Ursæ Majoris below, .	1 16 55.46 $= \tau$,	above,	13 16 58.16 $= \tau'$
$t - \tau$, ...	— 16 54.91	$t' - \tau' =$	— 18 2.69

$t-\tau$	. . . . .	— 16m 54s.91
$t'-\tau'$,	. . . . .	— 18 2.69
$(t-\tau)-(t'-\tau')=$		+ 1 7.78=67s.78=Δ.

Now, as $t'-\tau'$, the second interval, exceeds $t-\tau$, the first, the deviation is towards the *east*, while the difference is 67s.78. But, in using formula (4), the error α may be computed either in time or arc, as best suits the observer, or the knowledge he has of the value of a turn of his screws, which he should previously ascertain, at least in an approximate manner. Then, if he wish the correction in time, the constant logarithm will be the arithmetical complement of the log of 2, or 9.698970; if in arcs, the log of $\frac{15}{2}=7.5$, or 0.875061 is the constant.

		In Time.		In Arc.
1. Const. log,	. .	9.698970	2. Const. log, .	0.875061
$\delta=$ 1° 36′ 9″ sin,	. .	8.446619	. . . . .	8.446619
$\delta'=$ 34 10 44 sin,	.	9.749565	. . . . .	9.749565
$\delta'-\delta=$ 32 34 35 cosec,	.	0.268876	. . . . .	0.268876
$l=$ 52 25 50 sec,	.	0.214868	. . . . .	0.214868
$\Delta=$ 67s.78 log,	.	1.831102	. . . . .	1.831102
$\alpha=$ 1s.622 log,	.	0.210000	α=24″.33 log,	1.386091

the respective deviations in time and arc towards the east.

4. To place a transit-instrument in the meridian by a pair of high and low stars.

If the difference of the right ascensions of two stars, of which the declinations are δ and δ', be $d\,\alpha$, and if a transit-instrument be placed s seconds of time out of the meridian, the interval between their transits will be $s+d\,\alpha$ seconds of time, and $d\,\alpha$ may be found from the following formula, in which l is the latitude.

$$d\,\alpha=s\cos l\sin(\delta\backsim\delta')\sec\delta\sec\delta'. \quad . \quad . \quad (5)$$

$$\text{Let}\quad p=\text{D}\sin(l-\delta)\sec\delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which δ is negative when of a contrary name to l, and D the deviation or error in the position of the transit.

$$\text{Now making}\quad n=\sin(l-\delta)\sec\delta. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

then $p=n\,\text{D}$ and $p'=n'\,\text{D}$, and hence

$$D=\frac{p-p'}{n-n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Again, let α be the calculated transit or true right ascension of the first star, and t the observed transit, consequently $p=t-t'$ and α' being the right ascension of the second star $p'=\alpha-\alpha'$, therefore,

$$D=\frac{(t-t')-(\alpha-\alpha')}{n-n'} \quad . \quad . \quad . \quad . \quad (9)$$

If the clock does not keep true time, the interval must be corrected for the rate. Then, if D be *positive*, the instrument deviates to the *west*, if negative, to the east—and the correction may be made by the divided head of the adjusting-screw, while the operation is performed as follows:

Example 1. On the 1st of April 1840, the following observations were made in latitude 51° 30′ N., and nearly on the meridian of Greenwich.

		h. m. s.		h. m. s.
Observed transit of Rigel,	$t=$	5 7 34.70	$\alpha=$	5 6 51.30
" " Capella,	$t'=$	5 5 38.08	$\alpha'=$	5 4 53.08
	$t-t'=$	1 56.62	$\alpha-\alpha'=$	1 58.22
	$\alpha-\alpha'=$	1 58.22		
	$\Delta=(t-t')-(\alpha-\alpha')=$	—1.60		

Whence, to get D, it is only necessary to compute n and n', and, by combining formulæ (7) and (9), the value of D and D′ at the two stars will be found.

$l=51°\ 30'$ N.			$l=51°\ 30'$ N.		
$\delta=45\ 50$ N.	sec	0.1569	$\delta'=8\ 24$ S.	sec.	0.0047
$l-\delta=5\ 40$	sin	8.9945	$l+\delta'=59\ 54$	sin	9.9371
$n=0.142$	log	9.1514	n' . .	log	9.9418
$n'=0.874$					
$n'-n=0.732$	a. c. l.	0.1355	. . . .		0.1355
$\Delta=-1^s.60$	log	=0.2041	. . . .		0.2041
$D=-0^s.31$	log	=9.4910	$D'=-1^s.91$	log	0.2814

Here D is *negative*, and the deviation of the telescope is toward the *east*, but when positive it is to the west.*

See my *Mathematical Tables* and *General Astronomical Tables* on this subject.

	s.		s.
Here $t-\alpha=$.	43.40,	and $t'-\alpha'=$.	45.00
D' with Cont. sign	+ 1.91,	D, with Cont. sign,	+ 0.31
Error of clock = .	45.31, fast, or	. .	45.31

Or the result may be stated thus:

	h. m. s.		h. m. s.
Capella's R. A.	5 6 51.30	Rigel's R. A.	5 4 53.08
D' . .	— 1.91	D .	— 0.31
Time by transit,	5 6 49.39	. .	5 4 52.77
Time by clock,	5 7 34.70	. .	5 5 38.08
Error of clock fast,	45.31	. .	45.31
Correction of clock,	—45.31 in sidereal time.		

If the clock is regulated according to mean time, the interval $t-t'$ must be, by Table XXVI., converted into sidereal.

Example 2.—On the 1st of March 1848, at Edinburgh, in latitude 55° 57′ 20″ N., the transits of Capella and Rigel were observed in the evening by a sidereal clock.

	h. m. s.		h. m. s.
Capella, $t'=$	5 5 30.20	Rigel, $t=$	5 7 12.80
$\alpha'=$	5 5 29.27	$\alpha=$	5 7 14.73

	m. s.		m. s.
Hence, $\alpha-\alpha'=$	1 45.46	and $t-t'=$	1 42.60

$$\text{Whence, } (\alpha-\alpha')-(t-t')=\Delta= \quad . \quad . \quad . \quad . \quad . \quad +2.86^{s}$$

* If $D=\frac{(\alpha-\alpha')-(t-t')}{n-n'}$ then the sign of D would give the correction of the observed time with its proper sign, which would be the contrary of those stated above.

$l=55°\ 57'$ N.				$55°\ 57'$ N.	
$\delta=45\ 50$ N.	sec	0.1569	$\delta'=$	8 23 S.	sec 0.0048
$l-\delta=10\ \ 7$	sin	9.2447	$l+\delta'=$	64 20	sin 9.9549
$n=$ 0.252	log	9.4016	$n'=$	0.912	log 9.9597
$n'=$ 0.912					
$n'-n=$ 0.660 *a. c. l.*		0.1805	*a. c. l.*		0.1805
$\Delta=+2^s.86$ log		0.4564			0.4564
D=+1 .093 log		0.0385+	D'=+3ˢ.95 log		0.5966+

	h. m. s.		h. m. s.
Wherefore, $t'=$	5 5 30.20	$t=$	5 7 12.80
$a'=$	5 5 29.27	$a=$	5 7 14.73
$t'-a'=$	+ 0.93	$t-a=$	— 1.93
D=	+ 1.09	D'=	+ 3.95
Cor. of clock,	+ 2.02	Cor. of clock,	+ 2.02

Example 3.—On the 24th of April 1828, the following observations were made at Paris on γ Ursæ Majoris above the pole, and on β Cephei under the pole.

	h. m. s.	h. m. s.	
γ Ursæ Maj.	$t=$11 44 13.80,	$a=$11 44 47.10,	$n'=-0.176$
β Cephei	$t'=$ 9 25 54.80,	$a'=$ 9 26 24.49,	$n=+2.542$
	$t-t'=$ 2 18 19.00,	$a-a'=$ 2 18 22.61,	$n'-n=-2.718$
	$a-a'=$ 2 18 22.61		
	$\Delta=-$ 3.61 to the right.		

Hence making $x=\dfrac{-3.61}{-2.718}=+1^s.33,$ we have

$$n\,x=2.542\times1^s.33=+3^s.38, \text{ and } n'\,x=-0.176\times1^s.33=-0^s.23$$

	h.	h. m. s.			h. m. s.
Whence to $a - 12$	=	9 26 24.49,	and a'	=	11 44 47.10
Apply $n x$	= +	3.38,	and $n'x$	= —	0.23
Transit, .		9 26 27.87,			11 44 46.87
t' .		9 25 54.80,	t		11 44 13.80
Clock slow of S. T.		33.07			33.07

Remark 1. When a circumpolar star is observed between the pole and the zenith of the upper meridian, the same formulas apply since n is then negative, because d exceeds l.

2. If the transit is taken between the pole and the horizon, the same formulas will still answer, by diminishing the right ascension of the star by 12^h and changing the sign of d. The deviation of the telescope pointing to the north is still reckoned to the right, when x is positive; but here this side is found towards the east. The contrary takes place when x is negative. When two circumpolar stars are observed, the same remark is applicable to both.

3. When the same star is observed at both passages, superior and inferior, the preceding rule is applicable to both, the right ascension of the star must be diminished by 12^h for the inferior passage, and the sign of d must be changed.

The transit instrument, besides its use in astronomy, may, when properly adjusted, be successfully applied to many purposes of engineering, such as determining the direction of tunnels in railways, and other similar operations, where great accuracy is required.

If such means had been properly applied, the errors in the line of direction which have sometimes happened could scarcely have occurred.

It may likewise be employed in determining the direction of the meridian, when required in trigonometrical surveying, and, when sufficiently powerful, is undoubtedly the instrument best calculated for that purpose.

EXPLANATION OF THE TABLES.

TABLE I.—*Depression or Dip of the Horizon.*

The dip of the horizon is the angle contained between a line perpendicular to the plumb-line, passing through the eye of the observer, elevated above the level of the sea, and a line from his eye to the visible horizon when they are in the same vertical plane. This table contains the apparent dip answering to a free unobstructed horizon, diminished by 0.08 of itself, or of the intercepted arc for the effects of refraction.

1. The numbers in the table corresponding to the height of the eye of the observer, is to be *subtracted* from the observed altitude when taken by the fore observation with Hadley's quadrant and similar instruments, but added to it in case the altitude be taken by the back observation.*

2. This has been the principal use to which analogous tables have hitherto been applied, but it may be often advantageously employed for other purposes, which has been an inducement to extend it a little beyond the usual limits. Since the *true* dip has been diminished by 0.08 or about $\frac{1}{12}$ of itself to reduce it to the *apparent*, it consequently follows that, if the apparent dip be increased by double of 0.08, or 0.16, equal to $\frac{1}{6}$ of itself nearly, the result will be the distance of the visible horizon in geographical miles.

3. If the *apparent dip* be measured with a good theodolite or astronomical circle, the corresponding height of the instrument above the sea will be found by the table with as much accuracy as the nature of the horizontal refraction will admit.

* Beyond the limits of the table, Dip to h = 2 dip to $\frac{1}{4}$ h.

EXAMPLES.—1. To the height of the eye, 16 feet in the first column, will be found 3′ 56″, the dip in the second.

2. To the height of the eye, 500 feet,

there will be found, ...	22′
To this add ⅙ of itself, ...	3⅔
Sum or distance of the horizon, .	=25⅔ miles.

3. From a point on Inchcolm the author observed the depression of the horizon of the sea down the Firth of Forth to be 8′ 21″.2; required the height of the instrument above the sea?

By the table to dip, . .	=8′ 14″	the height is	70 feet.
Proportional part to . .	7.2	.	+ 2.1 feet.
Height of instrument for .	8 21.2	=	72.1 feet.

TABLE II.—*Correction of the Apparent Altitudes of the Sun and Stars.*

In this table the altitude is found in the first column; the star's correction or *mean refraction* in the second; and the difference between the mean refraction and the sun's mean parallax, constituting the sun's correction, in the third.

EXAMPLES.—1. Required the correction of the altitude of a star which was observed to be 22° 30′?

ANSWER, ... 2′ 20″.0

2. Required the sun's correction at an altitude of 31° 20′?

ANSWER, ... 1′ 36″.3

These are the true corrections when the English barometer stands at 30 inches, and Fahrenheit's thermometer at 50°, and are always to be *subtracted* from the apparent altitude, or added to the apparent zenith-distance, to obtain the true.

TABLE III.—*To Correct the Mean Refraction.*

When the barometer differs from 30 inches, and the thermometer from 50°, the mean corrections, as above, may be reduced for the effects of pressure and temperature by this table with sufficient accuracy, when altitudes are taken with the ordinary theodolite or sextant. These corrections must be applied according to the signs in the table. Thus, in the first example to the preceding table, let the observed height of the barometer $b = 29^{in}.57$, and the temperature by Fahrenheit's thermometer $t = 84°$, then

To the mean refraction formerly found, ...	2′	20″
There must be applied for the altitude 22° 30′, and b=29^{in}.57,	—	2
For altitude 22° 30′, and t=84°, ...	—	10
True refraction, ...	2	8

or the star's correction to be subtracted.

In like manner for the second example,

To the mean correction, ...	1′	36″	.3
For altitude 31° 20′, and b=30^{in}.28, ...	+	1	.0
For altitude 31° 20′, and t=30° Fahrenheit, . .	+	4	.0
The sun's true correction, . . .	1	41	.3

and so on in similar cases.

TABLE IV.—*Correction of the Apparent Altitude of the Moon.*

This table contains the difference between the moon's parallax in altitude and the mean refraction, and must be always *added* to the apparent altitude to obtain the true. To the moon's apparent altitude in the first column on the left, and under the minutes in the moon's horizontal parallax at the top, will be found the correction for the nearest less degree of altitude and minute of parallax; and the proportional parts for minutes of altitude and seconds of parallax are found in the two adjacent right hand columns, taking care not to neglect the parts to 0′ of altitude, as for the sake of the convenience of having all the parts *additive*, the construction of the table requires.

EXAMPLE—Let the moon's apparent altitude be 32° 40′, and the equatorial horizontal parallax 58′ 32″; required the true correction when the barometer stood at 29.6 inches, and Fahrenheit's thermometer at 72°?

To app. altitude 32° and horizontal parallax 58′ correction,		+47′ 10″
To app. altitude 0° 40′ proportional parts,	. .	+ 10
To seconds of parallax 32″,	. . . . .	+ 27
True correction for b=30, and t=50°,	. .	47 47
To b=29in.6 and altitude 32° 40′ correction,	. .	+ 1
To t=72° and altitude 32° 40′ correction,	. .	+ 4
True correction,	. . . .	47 52

for real temperature and pressure, where, according to the remark at the foot of Table III., the corrections depending upon b and t have been applied with signs *contrary* to those marked in the table.

TABLE V.—*Mean Refractions.*

This table contains the *logarithms* of the mean refractions at 30 inches of the English barometer, and 50° of Fahrenheit's thermometer. It is succeeded by Tables VI., VII., VIII., IX., and X., to reduce it to any other pressure and temperature, either for the English barometer and Fahrenheit's thermometer by the first three auxiliary tables, or for the metrical barometer and centigrade thermometer by the two last, in which the logarithms for τ and t are, as is frequently the case, united in one with the argument t, a method that in general cannot sensibly affect the accuracy of the results. Logarithmic tables of refraction are used by all astronomers where extreme precision, combined with facility of calculation, are required.

EXAMPLES.—1. Let the zenith-distance θ be 68° 55′ 36″, the barometer b = 28.80 inches, and the thermometers τ and t each 65° Fahrenheit; required the refraction?

For θ	=	68° 40′	log $\delta\ \theta$,	. . .	2.17171
Prop. part for		15.6=37.1×15.6=		. . .	+ 579
b	=	28in.80	log (Table VI.)	. . .	9.98227
τ	=	65° .	log (Table VII.)	. . .	9.99935
t	=	65° .	log (Table VIII.)	. . .	9.98663
r	=	2′ 19″.87=139″ 87, .		. . log	2.14575

2. Let $\theta=87° 42' 10''$, $b=29^{in}.50$, τ and $t=35°$; required r, the refraction?

For θ	=	87° 40′ 0″	log $\delta\theta$		3.00522
Prop. parts for		2 10	. . . . .		392
b	=	$29^{in}.50$	log as before, . . .		9.99270
τ	=	35°	log, ...		0.00065
t	=	35°	log, ...		0.01379
r''	=	. . . . .		1038″.20 log	3.01628

$\frac{d\delta\theta}{d\tau}\times(35°-50°)=-0''.591\times-15° = . \quad +8\ .86$

$\frac{d\delta\theta}{dp}\times(29^{in}.5-30^{in})=+1''.04\times-0.5= . \quad -0\ .52$

r = ...	1046 .54=17′ 26″.54	
r as observed by Brinkley, . . .	17 26 .50	
Difference between theory and observation,	+ 0 .04	

3. Let $\theta=88° 24' 9''.7$, the metrical barometer $b=755$ millimetres, and τ and t each 8°.75 centigrade; required the refraction?

θ	=	88° 24′ 9″.7	log $\delta\theta$, . . .		3.08930
b	=	$755^{m.m}$	log (Table IX.) . .		9.99599
t	=	8°.75 cent.	log (Table X.) . .		0.00214
r''		. . . . .		1223″.00 log	3.08743

$\frac{d\delta\theta}{d\tau}\times(8°.75-10°)=-0''.91\times-1.25\times1.8 \quad =+2.05$

$\frac{d\delta\theta}{dp}(755^{m.m}-762^{m.m})=+1.65\times-7\times0.04\ . \quad =-0.46$

r	=1224.59=20′ 24″.59
r from observation by Plana, ...	20 24 .30
Difference of theory from observation, .	+ 0 .29

From these instances, it is evident that the table gives the value of the refraction with great accuracy. It must be added to the zenith-distance and subtracted from the altitude. The reason that the first small correction of r is multiplied by 1°.8 is, that 1° centigrade is equal to 1°.8 Fahrenheit, that to which the corrections in the table are adapted; and as $39^{in}.371$ are equal to a metre, the second must be multiplied by $\frac{1}{1000}$ of this, or even by 0.04 as sufficiently accurate.*

TABLE XI.—*Logarithms to compute the value of the Coefficient of Terrestrial Refraction.*

In the practice of Trigonometrical Levelling, it is of the utmost importance to get the value of terrestrial refraction truly. Hitherto it has generally been the practice to determine occasionally, from numerous observations, its mean value by reciprocal and simultaneous measures, and to employ it either exactly as obtained, or under assumed variations in all cases. Reflecting on the inaccuracy of this plan, I endeavoured to investigate a formula which would give, with as much precision as the nature of the case seemed to admit, the value of the coefficient of terrestrial refraction, and from which the present table has been derived.†

To employ, in the computation of n, Tables VI., VII., and VIII., it will be necessary to add the log of 30 or 1.47712 as a constant log to the log from Table XI., in such heights where the barometer does not fall below 27 inches, the under limit of Table VI. The reason why, for simplification, this constant log was not added to the logs in Table XI. when computed, was, because at considerable heights the barometer would frequently be under 27 inches.

When the ray of light, however, traverses the atmosphere very near the earth's surface, neither theory nor observation will give the coefficient of refraction with accuracy, on account of the irregularities to which it is then subjected by variations of temperature, &c.; but this is no reason why it should not be obtained as accurately as possible. No astronomer would think of applying to

* The refraction may, independent of tables, be readily computed by the following formula:—

$$r'' = 846''.4 \tan (\delta - 197'' \tan \delta) \frac{b}{T} \quad . \quad . \quad . \quad (1)$$

in which $b = b'\left(1 + \frac{50^\circ - \tau}{10000}\right)$ and $T = 385^\circ + t$

When δ does not exceed 40° or 50°,

$$r'' = 846''.4 \tan \delta \frac{b}{T} \text{ very nearly.} \quad . \quad . \quad . \quad (2)$$

Log, $-197'' = 2.29449 -$, log $846''.4 = 2.92758 +$

† See the *Edinburgh New Philosophical Journal* for April 1841.

observations the *mean* refraction, without correction from the state of the barometer and thermometer, especially at low altitudes.

EXAMPLE 1.—Let the barometer $b = 29.75$ inches, the attached thermometer $\tau = 64°$, and the detached $t = 64°$ Fahrenheit; required n the coefficient of refraction?

For b	=	$29^{in}.75$ and $t = 64°$ (Table XI.), . . .	log	7.44997
τ	=	64° log × 2 (Table VII.), . . .		9.99880
t	=	64° log (Table VIII.), . . .		9.98751
b	=	29.75 log		1.47349
n	=	0.08124 log		8.90977

EXAMPLE 2.—Let $b = 28^{in}.4$, $\tau = 52°$ Fahrenheit, $t = 45°$ Fahrenheit; required the value of n?

Constant log, or that of 30 inches, ...				1.47712
b	=	$28^{in}.4$ and $t = 45°$ gives by Table XI., .	log	7.45175
b	=	$28^{in}.4$ gives by Table VI., . . .	log	9.97620
τ	=	52° gives by Table VII., . . .	2 log	9.99982
t	=	45° gives by Table VIII., . . .	log	0.00455
n	=	0.08118, ...	log	8.90944

TABLE XII.—*Parallax of the Sun in Altitude or Zenith-distance.*

The parallax of the sun on the first day of each month at the top, and to every third degree of altitude in the left hand column, or zenith-distance on the right, will be obtained from this table by inspection; and for any intermediate day or degree it may be readily found by interpolation, as will be subsequently exemplified.

TABLE XIII.—*Parallax of the Planets in Altitude or Zenith-distance.*

This is precisely similar to the last, and is used in the same manner.

EXAMPLE.—Required the parallax of Venus at an altitude of 30° on the first of December 1840, when the horizontal parallax was 12″.6?

	°		″	″
To altitude	30	and parallax,	10.	8.7
...	30	...	2.	1.7
...	30	...	0.6	0.5
To altitude	30	and horizontal parallax,	12.6	10.9

Hence the parallax in altitude is found to be 10″.9.

TABLE XIV.—*Augmentation of the Moon's Semidiameter in Altitude or Zenith-distance.*

With the moon's semidiameter at the top, and altitude on the left, or zenith-distance on the right-hand column, will be found the augmentation to be added to the semidiameter on that account.

TABLE XV.—*Reduction of the Moon's Parallax on the Spheroid.*

With the moon's equatorial horizontal parallax at the top, and the latitude on the left-hand column, the reduction to be *subtracted* from the moon's equatorial horizontal parallax, to reduce it to the given latitude, will be found.

TABLE XVI.—*Reduction of the Latitude on the Spheroid.*

With the observed latitude on the left, the reduction will be found on the right, to be subtracted from the observed latitude, to get the reduced latitude, or that referred to the centre of the earth considered as a spheroid of $\frac{1}{300}$ of compression.

TABLE XVII.—*Reduction to the Meridian.*

The method of determining the latitude by repeated observations near the meridian, makes the smaller classes of instruments much more efficient than they otherwise would be. Indeed it renders them much more nearly equal to the larger classes of instruments, such as the mural circles, than could have been anticipated. There are various methods of accomplishing this. In many cases the numbers in the table are given in seconds of arc and decimals, in others they are merely versines. The late Dr Thomas Young first gave, I believe, a small table of versines for this purpose similar to ours. I have, however, extended the numbers in the column titled V to one place more than his, which includes quantities to the first order only. To this I have added another column v, entirely omitted in Young's, embracing quantities to the second order in the formula, which cannot be dispensed with when the zenith-distance is small, not greater than about 10°; and the time extending to about ten minutes from the meridian, &c.

In this way the numbers are all integers, which, by those not very familiar with decimal fractions, render them more easily manageable; while, by means of the logarithms corresponding to the number of the observations at the end of the table, the results will be readily converted into seconds of arc. The observations are generally taken in pairs, and therefore logarithms of the even numbers will generally be enough, though the logarithm for one observation, which may occasionally be required, is also given.

EXAMPLE 1. Required the value of V and v for 12^m 36^s from the meridian?

By the table, under 12^m at the top, and opposite 36^s in the left-hand column, will be found $V = 15109$, and $v = 2283$.

2. Let the time from the meridian exceed the limits of the table, then the value of V to *half* the time being *quadrupled*, will be the value to the whole time nearly; and the value of v to half the time multiplied by 16 will give the value of v also for the whole time nearly.

Thus, let the time be 20^m 40^s, then to one-half of this, or 10^m 20^s, $V' = 10163$ and $v' = 1033$, whence $V = 10163 \times 4 = 40652$, and $v = 1033 \times 16 = 16528$ nearly. The true values of these, by direct calculation, being $V = 40630$ and $v = 16508$. The differences would not materially affect the accuracy of the final result in any ordinary case, especially when combined with a number of other values near the meridian within the limits of the table. It would not be desirable, however, to extend observations beyond the limits of the table; and it will be conducive to accuracy to take always an equal number of observations nearly equidistant, on each side of the meridian, to avoid, as far as possible, the effects of any little uncertainty in the time.

If the sun be the object, the observations should be taken by a watch showing mean solar time—if a star, by a watch showing sidereal time.

If the sun be the object, and the watch regulated to sidereal time, V must be multiplied by 0.9945466, the square of the number to convert sidereal into mean solar time, of which the logarithm is 9.997625; and if the watch be regulated to mean solar time when a star is observed, V must be multiplied by 1.0054833, of which the log is 0.002375, to convert the effect from mean solar into that from sidereal time.

$$\text{Cor.} = \frac{55}{10000} = \frac{11}{2000} = \frac{1}{200} + \frac{1}{10} \text{ of } \frac{1}{200} \text{ nearly.}$$

When the watch does not go accurately to either times, the value of V must be further multiplied by 1 + 0.00002315 r, whose log is 0.000010053 r, in which r is the rate of the watch, reckoned PLUS when LOSING, and *minus* when *gaining*. When r is negative, the arithmetical complement of the log denoted by 0.000010053 r must be taken.*

TABLE.

CORRECTION OF THE REDUCTION TO THE MERIDIAN FOR THE RATE OF THE CHRONOMETER.

Rate.	Logarithms for		Rate.	Logarithms for	
	Gaining.	Losing.		Gaining.	Losing.
s.			s.		
1	9.999990	0.000010	11	9.999889	0.000111
2	999980	000020	12	999879	000121
3	999970	000030	13	999869	000131
4	999960	000040	14	999859	000141
5	999950	000050	15	999849	000151
6	999940	000060	16	999839	000161
7	999930	000070	17	999829	000171
8	999920	000080	18	999819	000181
9	999910	000090	19	999809	000191
10	999899	000101	20	999799	000201

For proportional parts,—*Subtract* a unit for each *tenth* of a second in the last place of decimals from log for *gaining*, and add a unit for each tenth of a second for losing.

1. To convert the effect from sidereal to mean solar time, C. L. +9.997625
2. To convert the effect from mean solar to sidereal time, C. L. +0.002375

* There has been added to Shortrede's very extensive and accurate Logarithmic Tables, first impression in 1844, a table for this express purpose by the Editor of this work, from which the logarithmic correction of the rate, as well as the logarithms, for adapting the chronometer to mean solar, or sidereal time, as the case may require, are readily taken by inspection. See page 343.

If these rates be small, and the distance from the meridian moderate, their effects will hardly be sensible.

When the zenith-distance exceeds 30°, the *first term* of formula (6), page 337, will be sufficient; and if the object is below the pole, the reduction must be applied with a contrary sign. The most eligible zenith-distance for this mode is from about 10° to 60°.

On applying this table in formulæ (11) and (12), page 355, $d\,l$ is *positive* when the star is *above* the pole, negative when below it; $d\,m$ is *positive* when the star is in the semicircle *farthest* from the referring lamp or staff, negative when nearest. Consequently, in page 356, line 12 from the top, the reduction to the centre was really — 33".18, but to render $d\,m$ positive, so that all the columns might be *added*, the double of — 1".26, the mean of these, or — 2".52, was added to — 33".18, making it — 35".70, which artifices are admissible, at the option of the computer, when they conduce to facility or convenience.

Table XVIII.—*Logarithms to compute the Equation to Equal Altitudes and Equal Azimuths.**

The first column in this table contains the elapsed time, E. T. between the observations, and is the common argument to the other three columns A, B, C. The two first, A and B, are employed to compute the equation to equal altitudes in *seconds of time*, and C to compute the equation to equal azimuths in *seconds of arc*.

The computation of the equation to equal altitudes is performed by the following rules.

1. To the log A, from Table XVIII., add the log tangent of the latitude, the log of the hourly variation of the sun's declination from the *Nautical Almanac*, to be marked positive, or +, when the polar distance is increasing, and negative, or —, when decreasing; the sum of these three logarithms will be the log of part first of the equation for *noon*. The signs must be reversed for midnight.

* These tables were formed by the author to enable computers to employ the hourly variations of the sun's declination directly from the *Nautical Almanac*.

When equal altitudes are observed on the *forenoon* and *afternoon* of the given day, it would contribute to greater accuracy to take half the sum of the hourly variations for the given and preceding days. If the equal altitudes are observed on the evening and succeeding morning, half the sum of the horary variations for the given and following days should be taken. The log μ from the Berlin Ephemeris may be employed when the constant log 8.31876 is added.

In the application of these tables, great care must be taken of the *signs* of the different quantities, as the *first* part has, in one case, by Baron Zach, in his *Nouvelles Tables d'Aberration*, &c., and copied by Woodhouse in his Astronomy, been taken with a wrong sign, making the equation +4ˢ.75, instead of —5ˢ.19.

2. To log B add the log tangent of the declination to be reckoned *positive*, if the polar distance is *less* than 90°, but negative if greater; and the log of the sun's hourly variation reckoned positive, if the polar distance is decreasing, but negative if increasing; the sum will be the log of the second part of the equation for noon or midnight.

3. To the log C, from Table XVIII., add the log secant of the latitude, and the log of the sun's hourly motion from the *Nautical Almanac*, the sum will be the equation to equal azimuths, in seconds of arc, to be allowed to the left of the meridian indicated on the horizontal circle for the noon of the same day, when the polar distance is decreasing, but to the right if increasing. The signs must be reversed for midnight, or the correction for the meridian must be allowed to the right when the polar distance is decreasing, but to the left when increasing.

EXAMPLE 1.—At Madeira, in latitude 32° 38′ 25″ N., longitude $1^h\ 7^m.35^s$ W., on the 8th of August 1840, at $9^h\ 55^m\ 34^s.2$ forenoon, and $4^h\ 26^m\ 26^s.5$ afternoon, by chronometer, the sun had equal altitudes; required the time of apparent noon?

$$\tfrac{1}{2}\,(\overset{h.}{16}\ \overset{m.}{26}\ \overset{s.}{26.5} + \overset{h.}{9}\ \overset{m.}{55}\ \overset{s.}{34.2}) = \text{Approx. noon} = \overset{h.}{13}\ \overset{m.}{11}\ \overset{s.}{0.35}$$
$$\text{E. T.} = (16\ 26\ 26.5 - 9\ 55\ 34.2) = \quad . \quad . \quad 6\ 30\ 52.30$$

Log A . . .	+ 9.4599	Log B . .	+ 9.2779
Lat. 32° 38′.4 N. tan	+ 9.8065	Dec. 16° 4′ N. tan	+ 9.4594
$\delta d = +\ 43''.17$ log	+ 1.6352	. . . .	− 1.6352
1st part + $7^s.97$ log	+ 0.9016	2d part = − $2^s.36$ log	− 0.3725
2d part − 2 .36			

E. E. A. + 5.61 + $13^h\ 11^m\ 0^s.35$ − 12^h (App. noon) $1^h\ 11^m\ 5^s.99$

2.—At the same place, in August 1840.

August 8th, P. M. and August 9th, A. M.

$$\tfrac{1}{2}\,(\overset{h.}{4}\ \overset{m.}{29}\ \overset{s.}{56.5} + \overset{h.}{21}\ \overset{m.}{59}\ \overset{s.}{37.5}) = \text{App. midnight} = \overset{h.}{13}\ \overset{m.}{14}\ \overset{s.}{47.00}$$
$$\text{E. T.} = (21\ 59\ 37.5 - 4\ 29\ 56.5) = \quad . \quad . \quad 17\ 29\ 41.00$$

Equation to equal altitudes for Midnight.

Log A . . .	+ 9.8895	Log B . .	− 9.7082
Lat. 32° 38′.4 N. tan	+ 9.8065	Dec. 15° 56′ N. .	+ 9.4556
$\delta d = -\ 43''.45$ log	− 1.6380	. . . .	− 1.6380
1st term − $21^s.58$ log	− 1.3340	2d term + $6^s.34$ log	+ 0.8018
2d term + 6 .34			

E. E. A. − 15 .24 + $13^h\ 14^m\ 47^s.00$ − 12^h (midnight) = $1^h\ 11^m\ 31^s.76$

3.—On the 28th of February 1840, in latitude 55° 57′ N., longitude $12^m\ 43^s.5$ W., in an interval of $5^h\ 30^m\ 0^s$, the sun had equal altitudes when the azimuth circle read 130° 10′ 15″ and 32° 36′ 15″, and consequently the middle point = ½ (130° 10′ 15″ + 32° 36′ 15″) = 81° 23′ 15″; required the true meridian point?

To interval $5^h\ 30^m\ 0^s$ log C			0.6202
Latitude 55° 57′ N. log secant			0.2519
δd = 56″.69 log			1.7535
E. E. Az.	7′ 2″.3	= 422″.3	log 2.6256
			° ′ ″
Hence if from			81 23 15.0
There be subtracted			— 7 2.3
The remainder			81 16 12.7

is the reading of the instrument when set to the true meridian.

4. In latitude 55° 56′ 58″ N., on the 10th of July 1848, the sun had equal altitudes after an interval of $8^h\ 10^m$; required the equation to equal azimuths?

Interval,	$8^h\ 10^m$, log C		0.6682
Latitude,	55° 56′ 58″ N. secant		0.2519
Horary variation,	— 19″.75 log		1.2957 —
	— 164″.4 log		2.2158
Correction,	+2′ 44″.4,		

or to the right, since the polar distance is increasing.

TABLE XIX.—*Logarithms to convert Feet on the Surface of the Terrestrial Spheroid into Seconds of Arc, and conversely.*

This Table, of great use in Trigonometrical Surveying, contains the logarithms of the reciprocals of the radii of curvature in any given direction multiplied by the arc, equal to the radius in seconds. Log M are those on the meridian, log P those on an arc perpendicular to the meridian, and log O those in any oblique direction indicated by the azimuth α or Z. The differences for each degree in M and P are given, to interpolate more easily for minutes of latitude.

EXAMPLES.—Required the log M for latitude 51° 13′.5, the log P for 50° 58′.3, and the log O for latitude 56° 4′.5, and α or Z = S. 106° 46′.4 W.

1. To latitude 51° 0′ log M	7.9940850
Prop. parts for 13.5	— 167
Log M to latitude 51° 13′.5	7.9340683
2. To latitude 50° 0′ log P	7.9929588
Prop. part to 58′.3	— 241
Log P to latitude 50° 58′.3	7.9929347
3. To lat. 56° and Z = 100° log O	7.9928405
Prop. parts to 4′.5 of latitude	— 19
Prop. parts to 6°.8 of azimuth	+ 536
Log O to lat. 56° 4′.5 and Z 106° 46′.4 .	7.9928922

The numbers from Tables XX. and XXI. are taken out in the same manner.*

TABLE XXII.—*Reduction of λ to l.*

This table, computed from the formula $p''^2 \frac{1}{2} \sin 1'' \tan \lambda$, in which λ is the latitude of the foot of the perpendicular arc from the given station on the meridian passing through that required, and p'' the length of that arc itself in seconds of arc, gives to λ at the top of the page, and p'' in the left-hand column, in minutes, a small correction within its limits to be subtracted from λ to give *l*, the true latitude of the required point, derived trigonometrically from the first. If the quantity is not got at sight, it may be easily found by interpolation.

TABLE XXIII.

This table is the same in principle as the last, but extended to every degree through the British Islands, for the purpose of facilitating calculations made within its range.

* These tables were computed, from the equator to the pole, by the author a considerable number of years ago, to facilitate computations in Trigonometrical Surveying. He first published an abstract from latitude 50° to 60°, in 1838, to every 10′ of latitude and 10° of azimuth, extending over the British Isles. For his own practice, he has interpolated them to every minute of latitude, so that the principal numbers, log M and log P, can be taken out by inspection. In 1847, there has been published in the Ordnance Survey similar tables, on slightly different data, extending from latitude 49° to 61° to every 10′ of latitude, and every 5° of azimuth, which will be very useful within their limits.

EXAMPLE.—Required the reduction of λ to l, when $\lambda = 57° 51' 4''.5$, and $p'' = 24' 36''$?

$\lambda = 57°$ and $p'' = 24'$ give	— 7″.73
Prop. parts for 51′ of λ	— 0 .27
... for 36″ of p''	— 0 .40
Sum	— 8 .40
λ	57° 51′ 4 .50
l the true altitude	57 50 56 .10

TABLE XXIV.—*To reduce a Base at the level of the Sea to any height above it, or from any height above the Sea to its level.*

EXAMPLE 1.—Required the length of the chord K, when the arc a is 164045 feet, and height above the sea $h = 6562$ feet?

Log a	+	5.2149630
For h = 6000 feet, into	+	0.0001246
500	+	104
62	+	13
For a = 100000 feet, pa^2	—	4
64000 ft. $\Delta_{,}$ + 0.64 — Eq. Δ_2 = —13 + 0.64 — 1 =	—	7
Log K at the height h		5.2150982

2. General Roy's scale was in the imperial standard, 1.0000244, and Ramsden's scale was also the imperial standard, 1.0000691,

Whence Roy's base was at 62° Fahrenheit, .	27404.0843 feet.
Reduction, 27404 × 0.0000244 = . . .	+ 0.6699
Roy's base in the imperial standard = . .	27404.7542 (1)
on Hounslow Heath, 100 feet above the sea.	
General Mudge's base, ...	27404.3155 feet.
Reduction, 27404 × 0.0000691 = . . .	+ 1.8936
Mudge's base in the imperial standard = . .	27406.2091 (2)
Mean of these two 100 feet above the sea, . .	27405.4816
Reduction for 100 feet to sea, ...	— 0.1310
Mean base at the level of the sea, . . .	= 27405.3506
Reduction of the arc to the chord, . . .	= — 0.0020
Base or K correctly at the sea level, . .	27405.3486
Logarithm of K, ...	4.4378354

3. Log of base on the Heath, 27405.4816 . .	= 4.4378375
Log reduction for 100 feet, ...	— 21
Log reduction of arc to chord, ...	— 0
Logarithm of K by Table XXIV., at the mean level of the sea. . . .	= 4.4378354

This table, therefore, serves to reduce bases from the level of the sea to great heights, for the purpose of accurate trigonometrical levelling, or for reducing a measured base to the level of the sea, in order to extend a series of triangles at that level over a tract of country.

TABLE XXV.—*The measure of one Minute of Arc in feet at each degree of latitude.*

As the latitudes and longitudes of a number of places throughout the British Isles will shortly be made known in the volumes of the Trigonometrical Survey, then, by taking a few angles, and either measuring a base carefully, or, if possible, selecting a distance from the survey, the position of any particular point at a moderate distance may be readily fixed by means of this table.*

EXAMPLE.—In the island of Iona, Carn Cul ri Eirn is south of Carn Dunii 9955 feet, and west of it 8111 feet; required the latitude and longitude of Carn Cul ri Eirn, those of Dunii being 56° 20′ 33″ N., longitude 6° 23′ 36″ W.?

By the table, 1′ of latitude at 56° 20′ is 6087.2 feet, therefore $9955 \div 6087.2 = 1'.65 = 1'\ 39''$ S. Hence $56°\ 20'\ 33'' - 1'\ 39'' = 56°\ 18'\ 54''$ N., the latitude of Carn Cul ri Eirn.

In like manner, the length of a minute of longitude is 3381.3 feet; hence $8111 \div 3381.3 = 2'.4 - 2'\ 24''$, therefore $6°\ 23'\ 36'' + 2'\ 24'' = 6°\ 26'\ 0''$ W., the longitude of Carn Cul ri Eirn.

Formulæ to compute the length of a degree of latitude and longitude at any given latitude.

1. Let d' = the length of a degree of latitude at the equator,
   d = that at any given latitude,
   l = the given latitude, and
   e = the eccentricity; then

* In some tables, such as in Captain Frome's Surveying, and in Simms *On Mathematical Instruments*, the column titled one second of latitude is computed on the hypothesis that a degree of latitude is equal to a degree of longitude on the equator, though they differ about 2430 feet from one another. It is therefore entirely erroneous.

Thus, D' = ...	365169.1
d' = ...	362738.7
$D' - d'$ = ...	2430.4

$$d = d' (1 + \tfrac{3}{4} e^2 \sin^2 l + \tfrac{15}{8} e^4 \sin^4 l \quad \ldots \} \quad \ldots \quad (1)$$

$$= d' + f \sin^2 l + g \sin^4 l \quad \ldots \quad (2)$$

$d' = 362738.7$ feet,	log . . .	5.5595937
$f = 3621.34$ feet,	log . . .	3.5588686
$g = 30.13$ feet,	log . . .	1.4789619

2. Let D′ = the length of a degree of longitude at the equator,
D = that at any given latitude; then

$$D = D' \cos l\ (1 + \tfrac{1}{2} e^2 \sin^2 l + \tfrac{3}{8} e^4 \sin^4 l \ \ldots\ ) \quad . \quad (3)$$

$$= D' \cos l + m \sin^2 l + n \sin^4 l \quad \ldots \quad (4)$$

Now, making $a : b :: 300 : 299$

$D' = 365169.1$ feet,	log . . .	5.5624940
$D' \cos l\, \tfrac{1}{2} e^2 = m$,	$\log \tfrac{1}{2} e^2$ . . .	$\bar{3}.5221536$
$D' \cos l\, \tfrac{3}{8} e^4 = n$,	$\log \tfrac{3}{8} e^4$ . . .	$\bar{5}.2203985$

EXAMPLE 1.—Required the length of a degree of latitude at latitude 56°?

$l = 56° 0' 0''$	$\sin^2 l$ . . .	9.8371484
$d' = 362738.70$	$\log f$ . . .	3.5588686
1st term = +2488.96	log	3.3960170
	$\sin^4 l$ . . .	9.6742968
	$\log g$ . . .	1.47896
2d term = + 14.23		1.15326

1° = 365241.89, and 1′ = 6087.365 feet.

EXAMPLE 2.—Required the length of a degree of longitude in latitude 56°?

$l = 56° 0' 0''$	cos	9.7475617
$D' = 365169.10$	log	5.5624940
1st term = 204200.00	log, $D' \cos l$ . . .	5.3100557
	$\sin^2 l$ . . .	9.8371484
	$\log \tfrac{1}{2} e^2$ . .	$\bar{3}.5221536$
2d term = + 467.04	log . . .	2.6693577
	$D' \cos l$ . .	5.31006
	$\tfrac{3}{8} e^4 \log$ . .	$\bar{5}.22040$
	$\sin^4 l$ . . .	9.67430
3d term = + 1.60	log . . .	0.20476

1° = 204668.64, and 1′ = 3411.144 feet.

The formula to compute the length of a degree perpendicular to meridian, or

$$\Delta = D' (1 + \tfrac{1}{2} e^2 \sin^2 l + \tfrac{3}{8} e^4 \sin^4 l) \quad . \quad . \quad . \quad (5)$$
$$= D' + p \sin^2 l + q \sin^4 l \quad . \quad . \quad . \quad . \quad (6)$$

EXAMPLE 3.—Required the length of a degree perpendicular to the meridian in latitude 56°?

Now the value of D′ is the same as in formula (3) and (4), and

	$\log p =$		3.0846476
	$\log q =$		0.7828925
	$\sin^2 l$ . . .	log	9.8371484
D′ = 365169.10	feet, . . .	log p	3.0846476
1st term = + 835.21	. . . .	log	2.9217960
	$\sin^4 l$ . . .	log	9.6742968
		log q	0.7828925
2d term = + 2.87	. . . .	log	0.4571893

1° = 366006.18 and 1′ = 6100.12 feet.

TABLE XXVI.—*To convert Mean Solar into Sidereal Time.*

This table gives the quantity to be ADDED, as expressed at the top of the table, or the acceleration, as it is generally called, at the bottom, to be added to any quantity of mean time, to reduce it to sidereal.

TABLE XXVII.—*To convert Sidereal into Mean Solar Time.*

This table gives a portion of time to be subtracted (and therefore called retardation) from a known portion of sidereal time, to reduce it to mean time.*

The conversion of the time of any phenomenon recorded in side-

* This Table, XXVII., will therefore enable an observer to rate a chronometer by observing the time of a star's attaining a constant altitude on different days. Comparing the observed acceleration of the star with the table will give the chronometer's rate during 35 days, the extent of the table, which will generally be more than sufficient.

real time, into that by mean solar time, and conversely, may be performed in the following manner :—

Let m be the mean solar time at the place of observation.

s the corresponding sidereal time.

σ the sidereal time at mean noon on the meridian of the place of observation, deduced from the *Nautical Almanac*, page II. of each month, by Table XXVI., the reduction from which is + in west longitude, and — in east.

a the acceleration for mean time, m, by Table XXVI.

α the acceleration of the fixed stars for the sidereal time, $s - \sigma$, from Table XXVII., then

$$m = (s - \sigma) - a \quad . \quad . \quad . \quad . \quad (A)$$

$$s = \sigma + m + a \quad . \quad . \quad . \quad . \quad (B)$$

EXAMPLES.—1. At what mean solar time did α Aquilæ pass the meridian of Inchkeith on the 21st of August 1840?

	h. m. s.
σ, the sidereal time at Greenwich mean noon, August 21, 1840, by *Nautical Almanac*, .	9 59 29.28
Reduction for longitude $12^m\ 32^s$ W., (Table XXVI.)	+ 2.06
σ at Inchkeith mean noon, ...	9 59 31.34
s the R. A. or Sid. T. of transit of star,.	19 43 2.08
$s - \sigma$, or difference, ...	9 43 30.74
a to this difference, (Table XXVII.,) . .	— 1 35.60
$m = (s - \sigma) - \alpha$ = mean solar time, ...	9 41 35.14

2. On the 14th of August 1840, on the meridian of Paris, in longitude $9^m\ 21^s.33$ E., at $22^h\ 22^m\ 13^s.4$ mean solar time, what was the sidereal time?

	h. m. s.
σ at Greenwich mean noon, *Nautical Almanac*, .	9 31 53.41
Reduction to $9^m\ 21^s.3$ E. (Table XXVI.) .	— 1.54
σ at Paris mean noon, ...	9 31 51.87
m	22 22 13.40
a to $22^h\ 22^m\ 13^s.4$ (Table XXVI.) . .	+ 3 40.49
$s = \sigma + m + a$ = sidereal time, ...	7 57 45.76

3. The mean time of a star's transit may be found by means of the column in the *Nautical Almanac* titled, Mean Time of Transit of the first point of Aries, or of the mean time of transit of the vernal equinox.

Rule 1. Reduce the star's right ascension in sidereal time to mean solar time, by Table XXVII.

2. Reduce the mean time of transit of the first point of Aries to the place of observation, by applying the reduction answering to the longitude from Table XXVII., reckoned *additive* in *east* longitude, but subtractive in west.

3. If the sum of the star's right ascension and transit of Aries exceed 24h, take the transit of Aries for the day preceding that given.

Example. Required the mean time of transit of α Aquilæ at Broddick, in Arran, in longitude 20m 37s W., on the 16th day of August 1843.

	h. m. s.	h. m. s.
Star's right ascension in sidereal time, . .		19 43 11.40
Reduction of this to mean time, (Table XXVII.)		— 3 13.84
Star's right ascension in mean time, . .		19 39 57.56
Transit of Aries at Greenwich on 15th,	14 24 39.40	
Reduction to 20m 37s of long. W., (T. 27.)—	3.38	
Transit of Aries at Broddick, .	14 24 36.02,	+14 24 36.02
Mean time of transit of α Aquilæ on the 16th,		= 10 4 33.58

TABLE XXVIII.—*To convert Degrees, Minutes, and Seconds of Arc on the Equator into Sidereal Time.*

EXAMPLE.—What is the sidereal time corresponding to 56° 38′ 40″?

	° ′ ″	h. m. s.
To	55 0 0 sidereal time . . .	3 40 0
	1 0 0	4 0
	38 0	2 32
	40	2.667
To	56 38 40 sidereal time . . .	3 46 34.667

TABLE XXIX.—*To convert Sidereal Time into Degrees, Minutes, and Seconds of the Equator.*

EXAMPLE.—Required the arc of the Equator corresponding to $5^h\ 48^m\ 36^s.48$ of sidereal time?

	h.	m.	s.		°	′	″
To	5	0	0	the arc is	75	0	0
		48	0	. . . . . .	12	0	0
			36	. . . . . .		9	0
			0.4	. . . . . .			6.0
			0.08	. . . . . .			1.2
To	5	48	36.48	the arc is	87	9	7.2

TABLE XXX.—*Diurnal Variations.*

As in the *Nautical Almanac*, and other Ephemerides, the places of many of the celestial bodies are given for 24^h or 12^h, this table will serve to reduce them to any intermediate time very readily.

EXAMPLE.—What was the sun's longitude at Edinburgh on the 21st of August 1840, at $9^h\ 41^m\ 35^s$, or at $9^h\ 54^m\ 18^s$ on the meridian of Greenwich?

	°	′	″
Longitude 21st, at mean noon, ...	148	25	19.0
22d, ...	149	23	11.1
Variation in 24^h . .	+0	57	52.1

	°	′	″
Now to longitude 21st, . . .	148	25	19.0
Prop. parts for $9^h\ 54^m\ 18^s$ . . .	+	23	52.5
Longitude required, .	148	49	11.5

In those cases where there are differences given in the *Nautical Almanac* for one hour, ten minutes, &c., the reduction by this table is then unnecessary, because, when the time of observation is known, the proportional parts may be obtained by multiplying the variation by the hours and parts of an hour, &c. Thus at Lamlash, in the island of Arran, in longitude $20^m\ 30^s$ W., on the 11th of August 1836, at $6^h\ 21^m\ 30^s$ of Lamlash time, or adding the longitude ($20^m\ 30^s$), because it is west, and the sum $6^h\ 42^m = 6^h.7$ is the

Greenwich time, at which a series of observations on the sun were made to determine the true time and error of the chronometer. For this time, then, the sun's declination, equation of time, &c. are required by the *Nautical Almanac*.

	°	′	″	
August 11, 1836, at Greenwich mean noon, the sun's declination is, ...	15	12	51.8	N.
Reduction $= -45''.02 \times 6^h.7 = -301''.6 =$.	—	5	1.6	
True declination for Lamlash, .	15	7	50.2	N.
	90	0	0.0	
North polar distance, ...	74	52	9.8	

When the latitude and declination are of the same name, the declination must be subtracted from 90° to get the polar distance, but must be added to it when they are of contrary names.

In the same way, the equation of time, proportional parts of the daily rates of chronometers, &c., may be found.

TABLE XXXI.—*Showing the lengths of horizontal lines equivalent to the several ascending and descending planes, the length of the plane being unity; in reference to the different classes of engines, including the gross weight with engine and tender.*

The first part of this table was drawn up, I believe, by Mr Barlow of Woolwich, for the Railway Commission appointed to examine the different railways submitted to Parliament, and its use has been shown in the article on Railways in a preceding part of the work.

In the second part are also given similar results from experiments to which I had access, and the velocities in different slopes from experiments lately made by Dr Lardner, the value of which rests on his authority.

TABLE XXXII.

This table gives the content in cubic yards of any cutting for one imperial chain of 100 links, or 66 feet, or 22 yards in length, and varying in depth from 1 to 50 feet, on a base or formation-level of 30 feet, with the different slopes 1 to 1, 1½ to 1, and 2 to 1, that is, 1 horizontal to 1 perpendicular, 1½ horizontal to 1 perpendicular, and 2 horizontal to 1 perpendicular, which include most of the slopes generally required. Thus, clay, chalk, &c., will stand on the sides of cuttings at 1 to 1, gravel 1½ to 1, sand, &c., 2 to 1, and the cuttings must be made accordingly. To this formation-level of 30 feet will likewise be found half the width at the top or surface, when the cutting varies from 1 to 50 feet at the different slopes mentioned in the table. There is also added another column giving the effect of a change of 1 perpendicular foot in breadth, in order to adapt the table to different bases, either above or below 30 feet. If the base exceed 30, the number of yards in this column, multiplied by the number of feet *greater* than 30, gives a correction to be *added* to the content from the preceding column, but to be subtracted if less. The half-width must also be corrected by increasing or diminishing the change made on the base, in the ratio of the slopes.

If the length of the cutting differ from one chain, the number from the table must be multiplied by the number of chains considered an integer, and the links a decimal: the product will be the content in cubic yards. This table is computed on the supposition that the *depth* is uniform, or nearly so, in each portion for which the calculation is made. If *it* varies rapidly, the portions to which it is applied must be diminished to a few links. In this manner, the table will suit most ordinary cases likely to occur. If not, then Sir John Macneill's tables must be applied, which are well adapted to all sorts of cuttings, but are unfortunately rather expensive for common use.*

Though the slopes in the table are those most commonly used, yet they may sometimes fall between or beyond them. Then to the width at the base in feet, add the horizontal length of the side of the triangle formed by the slope; multiply the sum by the depth of the cutting, and also by the length, all in feet: the product, divided by 27, will give the content in cubic yards.

It is to be remarked that the depth, multiplied by the slope, gives the side of the triangle to be added to the base, to give the mean breadth, which, multiplied by the depth, gives the area of the section, and this by the length, to give the content of the cutting.

* These tables are founded on the prismoidal formula, which, however, does not give correct results when the slope at right angles to the line of railway is considerable. Indeed, no mathematical formula will, without judicious application, do so. In such a case, the best plan is to determine the *area of the section*, which, multiplied by such a length as may be judged proper to retain the necessary accuracy, will give the content in the same *cubic* measure as that in which the lineal measures were taken.

EXAMPLES.—1. Let the length of a cutting be 3.75 chains, the depth 40 feet, the base or formation-level 30 feet, with slopes $1\frac{1}{2}$ to 1, there will be found in the table 8800 cubic yards for 1 chain.

Therefore $8800 \times 3.75 = 33000$ cubic yards, the quantity of cutting required.

2. For a height or depth of 40, and a base likewise of 40 feet, multiply the number under content for 1 perpendicular foot in breadth by 10, the product will be the number of cubic yards to be added to the number for 30 in the table, to give that for 40 feet of base: thus,—

$$8800.00+10 \times 97.77=8800.00+977.7=9777.7$$

cubic yards for 1 chain.

This last, multiplied by the length 3.75 chains, will give

$$9777.7 \times 3.75=36666.37 \text{ cubic yards.}$$

3. To compute the content for 1 chain in length, for slopes not given in the table, suppose we have a cutting with a width of base or formation-level of 28 feet, and a depth of 16 feet, the sides of which have a slope of $1\frac{1}{4}$ to 1; then by the directions previously given,

$$(16 \times 1\tfrac{1}{4}+28) \times 16=(20+28) \times 16=768$$

square feet, the area of the section. Then this area, multiplied by the length in feet, and the product divided by 27, will give the content of the cutting in cubic yards. For one chain of 66 feet this will be

$$\frac{768 \times 66}{27}=\frac{256 \times 22}{3}=1877\tfrac{1}{3} \text{ cubic yards.}$$

The same process may be followed for any section, long or short, which may be made to vary according to the change of the configuration of the ground.

For these purposes, the tables of Sibley and Rutherford will generally be found the most convenient for practical men.

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GEODETICAL TABLES

FOR

TRIGONOMETRICAL SURVEYING & LEVELLING

STEREOTYPED BY NEILL AND COMPANY, EDINBURGH.

Table I. Depression or Dip of the Horizon.

Ht.	Dip.	Ht.	Dip.
Feet	′ ″	Feet	′ ″
1	0 59	70	8 14
2	1 24	80	8 48
3	1 43	90	9 20
4	1 58	100	9 50
5	2 12	110	10 19
6	2 25	120	10 46
7	2 36	130	11 13
8	2 47	140	11 39
9	2 57	150	12 3
10	3 7	160	12 26
11	3 16	170	12 49
12	3 24	180	13 12
13	3 33	190	13 34
14	3 40	200	13 55
15	3 49	210	14 16
16	3 56	220	14 36
17	4 3	230	14 55
18	4 10	240	15 15
19	4 17	250	15 34
20	4 24	260	15 52
21	4 31	270	16 10
22	4 37	280	16 28
23	4 43	290	16 46
24	4 49	300	17 3
25	4 55	310	17 20
26	5 1	320	17 36
27	5 7	330	17 53
28	5 13	340	18 9
29	5 18	350	18 25
30	5 23	360	18 40
31	5 29	370	18 56
32	5 34	380	19 11
33	5 39	390	19 26
34	5 44	400	19 41
35	5 49	410	19 56
36	5 54	420	20 10
37	5 59	430	20 25
38	6 4	440	20 39
39	6 9	450	20 53
40	6 14	460	21 7
41	6 18	470	21 20
42	6 23	480	21 34
43	6 28	490	21 47
44	6 32	500	22 0
45	6 36	510	22 13
46	6 41	520	22 26
47	6 45	530	22 39
48	6 49	540	22 52
49	6 53	550	23 5
50	6 58	560	23 17
51	7 2	570	23 30
52	7 6	580	23 42
53	7 10	590	23 54
54	7 14	600	24 6
55	7 18	610	24 18
56	7 22	620	24 30
57	7 26	630	24 42
58	7 30	640	24 54
59	7 34	650	25 6
60	7 38	660	25 17

Table II. Correction of the Apparent Altitudes of the Sun and Stars.

Alt.	Star.	Sun.	Alt.	Star.	Sun.	Alt.	Star.	Sun.
° ′	′ ″	′ ″	° ′	′ ″	′ ″	°	′ ″	′ ″
0 0	34 32	34 23	10 0	5 20	5 11	30	1 41	1 33
10	32 25	32 16	20	5 10	5 1	31	1 37	1 29
20	30 33	30 24	40	5 1	4 52	32	1 33	1 26
30	28 50	28 41	11 0	4 52	4 43	33	1 30	1 22
40	27 14	27 5	20	4 44	4 35	34	1 26	1 19
50	25 47	25 38	40	4 36	4 27	35	1 23	1 16
1 0	24 27	24 18	12 0	4 28	4 19	36	1 20	1 13
10	23 13	23 4	20	4 21	4 12	37	1 17	1 10
20	22 5	21 56	40	4 14	4 5	38	1 15	1 8
30	21 3	20 54	13 0	4 8	3 59	39	1 12	1 5
40	20 5	19 56	20	4 2	3 53	40	1 10	1 2
50	19 11	19 2	40	3 56	3 47	41	1 7	1 0
2 0	18 21	18 12	14 0	3 50	3 41	42	1 5	0 59
10	17 35	17 26	20	3 45	3 36	43	1 3	0 56
20	16 52	16 43	40	3 40	3 31	44	1 0	0 54
30	16 12	16 3	15 0	3 35	3 26	45	0 58	0 52
40	15 35	15 26	20	3 30	3 21	46	0 56	0 50
50	15 0	14 51	40	3 25	3 16	47	0 54	0 48
3 0	14 27	14 18	16 0	3 21	3 12	48	0 53	0 46
10	13 56	13 47	20	3 17	3 8	49	0 51	0 45
20	13 27	13 18	40	3 13	3 4	50	0 49	0 43
30	13 0	12 51	17 0	3 9	3 0	51	0 47	0 42
40	12 34	12 25	20	3 5	2 56	52	0 46	0 40
50	12 10	12 1	40	3 1	2 52	53	0 44	0 39
4 0	11 47	11 38	18 0	2 58	2 49	54	0 42	0 37
10	11 26	11 17	20	2 54	2 45	55	0 41	0 36
20	11 6	10 56	40	2 51	2 42	56	0 39	0 34
30	10 46	10 38	19 0	2 48	2 39	57	0 38	0 33
40	10 28	10 19	20	2 45	2 37	58	0 36	0 32
50	10 11	10 2	40	2 42	2 34	59	0 35	0 31
5 0	9 54	9 45	20 0	2 39	2 31	60	0 34	0 30
10	9 38	9 29	20	2 36	2 28	61	0 32	0 23
20	9 23	9 14	40	2 34	2 25	62	0 31	0 27
30	9 9	9 0	21 0	2 31	2 22	63	0 30	0 26
40	8 55	8 47	20	2 28	2 19	64	0 28	0 25
50	8 42	8 34	40	2 26	2 17	65	0 27	0 24
6 0	8 30	8 21	22 0	2 24	2 15	66	0 26	0 23
10	8 18	8 9	20	2 21	2 13	67	0 25	0 22
20	8 7	7 58	40	2 19	2 11	68	0 24	0 21
30	7 56	7 47	23 0	2 17	2 9	69	0 22	0 19
40	7 45	7 36	20	2 15	2 7	70	0 21	0 18
50	7 35	7 26	40	2 13	2 5	71	0 20	0 17
7 0	7 25	7 17	24 0	2 10	2 3	72	0 19	0 16
10	7 16	7 7	20	2 8	2 1	73	0 18	0 15
20	7 7	6 59	40	2 7	1 59	74	0 17	0 14
30	6 59	6 50	25 0	2 5	1 57	75	0 16	0 13
40	6 50	6 42	20	2 3	1 55	76	0 15	0 12
50	6 42	6 34	40	2 1	1 54	77	0 13	0 11
8 0	6 35	6 26	26 0	1 59	1 52	78	0 12	0 10
10	6 27	6 19	20	1 57	1 50	79	0 11	0 9
20	6 20	6 11	40	1 56	1 49	80	0 10	0 8
30	6 13	6 5	27 0	1 54	1 47	81	0 9	0 8
40	6 7	5 58	20	1 53	1 45	82	0 8	0 7
50	6 0	5 51	40	1 51	1 43	83	0 7	0 6
9 0	5 54	5 45	28 0	1 49	1 41	84	0 6	0 5
10	5 48	5 39	20	1 48	1 39	85	0 5	0 4
20	5 42	5 33	40	1 46	1 38	86	0 4	0 4
30	5 36	5 28	29 0	1 45	1 36	87	0 3	0 3
40	5 31	5 22	20	1 44	1 35	88	0 2	0 2
50	5 25	5 17	40	1 42	1 34	89	0 1	0 1

Note 1. If the dip be increased by one-sixth of itself, the sum will be the distance of the visible horizon in geographical minutes and seconds.

2. If the dip be determined by observation, the height of the instrument above the sea will be found.

TABLE III. To correct the Mean Refraction.

Fahrenheit's Thermometer.

+	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50° +
−	90°	88°	86°	84°	82°	80°	78°	76°	74°	72°	70°	68°	66°	64°	62°	60°	58°	56°	54°	52°	50° −
Alt. °	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
4	68	65	61	58	54	51	48	44	41	37	34	31	27	24	20	17	14	10	7	3	0
5	55	52	50	47	44	41	39	36	33	30	28	25	22	19	17	14	11	8	6	3	0
6	46	44	41	39	37	35	32	30	28	25	23	21	18	16	14	12	9	7	5	2	0
7	39	37	35	33	31	29	27	25	24	22	20	18	16	14	12	10	8	6	4	2	0
8	34	32	31	29	28	26	24	22	20	19	17	15	14	12	10	8	7	5	3	2	0
10	28	26	25	23	22	21	19	18	17	15	14	12	11	10	8	7	6	4	3	1	0
12	22	21	20	19	18	17	16	14	13	12	11	10	9	8	7	6	4	3	2	1	0
15	18	17	16	15	14	13	12	11	11	10	9	8	7	6	5	4	4	3	2	1	0
18	14	14	13	12	12	11	10	9	9	8	7	7	6	5	4	4	3	2	1	1	0
22	12	11	10	10	9	9	8	8	7	6	6	5	5	4	3	3	2	2	1	1	0
30	8	8	7	7	6	6	6	5	5	4	4	4	3	3	2	2	2	1	1	0	0
50	4	4	4	3	3	3	3	3	2	2	2	2	2	1	1	1	1	1	0	0	0
60	3	3	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	0	0	0	0
70	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
80	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
−	27.16	27.30	27.44	27.59	27.73	27.87	28.01	28.15	28.30	28.44	28.58	28.72	28.86	29.01	29.15	29.29	29.43	29.57	29.72	29.86	30.00 −
+											31.42	31.28	31.14	30.99	30.85	30.71	30.57	30.43	30.28	30.14	30.00 +

Height of the English Barometer in inches and decimals.

The signs must be changed when the numbers from this Table are applied to the correction of the Moon's Altitude.

TABLE IV. Correction of the Apparent Altitude of the Moon.

☽'s Alt	Moon's Horizontal Parallax. 54′	55′	56′	57′	58′	59′	60′	61′	′ ″	P. P. for Alt. + 0′	2′	4′	6′	8′	P. P. for Par. + 0″	2″	4″	6″	8″
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″			″	″	″	″	″	″	″	″	″
0	19 28	20 28	21 28	22 28	23 28	24 28	25 28	26 28	0						0	2	4	6	8
1	29 33	30 33	31 33	32 33	33 33	34 33	35 33	36 33	10						10	12	14	16	18
2	35 37	36 37	37 37	38 37	39 37	40 37	41 37	42 37	20						20	22	24	26	28
3	39 30	40 30	41 30	42 29	43 29	44 29	45 29	46 29	30						30	32	34	36	38
4	42 6	43 6	44 6	45 6	46 5	47 5	48 5	49 5	40						40	42	44	46	48
5	43 55	44 55	45 54	46 54	47 54	48 54	49 53	50 53	50						50	52	54	56	58
6	45 13	46 12	47 12	48 11	49 11	50 10	51 10	52 10	0						0	2	4	6	8
7	46 10	47 10	48 9	49 9	50 9	51 8	52 7	53 7	10						10	12	14	16	18
8	46 54	47 53	48 52	49 52	50 51	51 51	52 50	53 50	20						20	22	24	26	28
9	47 26	48 26	49 25	50 24	51 23	52 23	53 22	54 21	30						30	32	34	36	37
10	47 51	48 50	49 48	50 48	51 47	52 46	53 44	54 45	40						39	41	43	45	47
11	48 10	49 9	50 7	51 6	52 5	53 4	54 3	55 2	50						49	51	53	55	57
12	48 22	49 21	50 19	51 18	52 17	53 15	54 14	55 13	0	0	0	0	1	1	0	2	4	6	8
13	48 30	49 29	50 27	51 26	52 24	53 23	54 21	55 20	10	1	1	1	1	2	10	12	14	15	17
14	48 35	49 33	50 31	51 30	52 28	53 26	54 24	55 22	20	2	2	2	2	2	19	21	23	25	27
15	48 36	49 34	50 32	51 30	52 28	53 26	54 24	55 22	30	3	3	3	3	3	29	31	33	35	37
16	48 35	49 32	50 30	51 28	52 26	53 23	54 21	55 20	40	3	3	4	4	4	39	41	43	45	46
17	48 31	49 28	50 26	51 23	52 20	53 18	54 15	55 13	50	4	4	5	5	5	48	50	52	54	56
18	48 17	49 13	50 10	51 7	52 4	53 0	53 57	54 54	0	14	13	13	13	12	0	2	4	6	7
19	48 7	49 3	49 59	50 56	51 52	52 49	53 45	54 41	10	12	11	11	10	10	9	11	13	15	17
20	47 55	48 51	49 47	50 43	51 39	52 35	53 31	54 27	20	9	9	8	8	7	19	21	22	24	26
21	47 42	48 37	49 33	50 29	51 24	52 20	53 16	54 11	30	7	6	6	6	5	28	30	32	34	36
22	47 27	48 22	49 17	50 12	51 8	52 3	52 58	53 53	40	5	4	4	3	3	37	39	41	43	45
23	47 11	48 5	49 0	49 55	50 50	51 45	52 39	53 34	50	2	2	1	1	0	47	49	51	52	54
24	46 53	47 47	48 42	49 36	50 31	51 25	52 19	53 14	0	23	22	21	21	20	0	2	4	5	7
25	46 34	47 28	48 22	49 16	50 10	51 4	51 58	52 52	10	19	18	18	17	16	9	11	13	14	16
26	46 14	47 7	48 1	48 54	49 48	50 41	51 35	52 28	20	15	15	14	13	12	18	20	22	23	25
27	45 52	46 45	47 38	48 31	49 24	50 17	51 10	52 3	30	11	11	10	9	8	27	29	30	32	34
28	45 30	46 22	47 15	48 7	49 0	49 52	50 45	51 37	40	8	7	6	5	5	36	38	39	41	43
29	45 6	45 58	46 50	47 42	48 34	49 26	50 18	50 10	50	4	3	2	1	1	45	47	48	50	52

Table IV. Correction of the Apparent Altitude of the Moon.

☽'s Alt	Moon's Horizontal Parallax.								P. P. for Alt. +						P. P. for Par. +				
	54′	55′	56′	57′	58′	59′	60′	61′	′ ″	0′	2′	4′	6′	8′	0″	2″	4″	6″	8″
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″		″	″	″	″	″	″	″	″	″	″
30	44 41	45 33	46 24	47 16	48 7	48 58	49 50	50 41	0	30	29	28	27	26	0	2	3	5	7
31	44 15	45 6	45 57	46 48	47 39	48 30	49 21	50 12	10	25	24	23	22	21	8	10	12	13	15
32	43 48	44 39	45 29	46 19	47 10	48 0	48 50	49 41	20	20	19	18	17	16	17	19	20	22	23
33	43 21	44 10	45 0	45 50	46 40	47 29	48 19	49 9	30	15	14	13	12	11	25	27	28	30	32
34	42 52	43 41	44 30	45 19	46 8	46 57	47 47	48 36	40	10	9	8	7	6	34	35	37	38	40
35	42 22	43 10	43 59	44 48	45 36	46 25	47 13	48 2	50	5	4	3	2	1	42	44	45	47	48
36	41 51	42 39	43 27	44 15	45 3	45 51	46 39	47 27	0	36	35	34	32	31	0	2	3	5	6
37	41 19	42 7	42 54	43 41	44 29	45 16	46 3	46 51	10	30	29	28	26	25	8	9	11	12	14
38	40 47	41 33	42 20	43 7	43 53	44 40	45 27	46 13	20	24	23	22	20	19	16	17	19	20	22
39	40 13	40 59	41 45	42 31	43 17	44 3	44 49	45 35	30	18	17	16	14	13	23	25	27	28	30
40	39 39	40 24	41 10	41 55	42 40	43 25	44 11	44 56	40	12	11	10	8	7	31	33	35	36	38
41	39 4	39 48	40 33	41 18	42 2	42 47	43 31	44 16	50	6	5	4	2	1	39	41	42	44	45
42	38 28	39 12	39 56	40 40	41 23	42 7	42 51	43 35	0	41	40	38	37	35	0	1	3	4	6
43	37 51	38 34	39 17	40 1	40 44	41 27	42 10	42 53	10	34	33	31	30	29	7	9	10	11	13
44	37 13	37 56	38 38	39 21	40 3	40 46	41 28	42 11	20	27	26	25	23	22	14	16	17	19	21
45	36 35	37 17	37 59	38 40	39 22	40 4	40 45	41 27	30	20	19	18	16	15	22	23	24	26	27
46	35 56	36 37	37 18	37 59	38 40	39 21	40 2	40 43	40	14	12	11	10	8	29	30	31	33	34
47	35 16	35 56	36 36	37 17	37 57	38 37	39 17	39 57	50	7	5	4	3	1	36	37	39	40	41
48	34 36	35 15	35 54	36 34	37 13	37 52	38 32	39 11	0	45	43	42	40	39	0	1	3	4	5
49	33 54	34 33	35 11	35 50	36 29	37 7	37 46	38 24	10	37	36	34	33	31	6	7	9	10	11
50	33 12	33 50	34 28	35 6	35 43	36 21	36 59	37 37	20	30	28	27	25	24	13	14	15	17	18
51	32 30	33 7	33 44	34 21	34 58	35 35	36 11	36 48	30	22	21	19	18	16	19	20	22	23	24
52	31 47	32 23	32 59	33 35	34 11	34 47	35 23	35 59	40	15	13	12	10	9	25	27	28	29	31
53	31 3	31 38	32 13	32 48	33 24	33 59	34 34	35 10	50	7	6	4	3	1	32	33	34	36	37
54	30 18	30 53	31 27	32 2	32 36	33 10	33 45	34 19	0	49	47	46	44	42	0	1	2	3	4
55	29 33	30 7	30 40	31 14	31 47	32 21	32 54	33 28	10	41	39	38	36	34	6	7	8	9	10
56	28 47	29 20	29 53	30 25	30 58	31 31	32 3	32 36	20	33	31	29	28	26	11	12	13	14	15
57	28 1	28 33	29 5	29 36	30 8	30 40	31 12	31 44	30	24	23	21	20	18	17	18	19	20	21
58	27 14	27 45	28 16	28 47	29 18	29 49	30 20	30 51	40	16	15	13	11	10	22	23	24	25	26
59	26 27	26 57	27 27	27 57	28 27	28 57	29 27	29 57	50	8	6	5	3	2	28	29	30	31	32
60	25 39	26 8	26 37	27 6	27 35	28 4	28 34	29 3	0	52	50	48	47	45	0	1	2	3	4
61	24 51	25 19	25 47	26 15	26 43	27 11	27 40	28 8	10	43	42	40	38	36	5	6	6	7	8
62	24 2	24 29	24 56	25 23	25 51	26 18	26 45	27 12	20	35	33	31	29	28	9	10	11	12	13
63	23 12	23 39	24 5	24 31	24 58	25 24	25 50	26 16	30	26	24	22	21	19	14	15	16	17	18
64	22 22	22 48	23 13	23 39	24 4	24 29	24 55	25 20	40	17	16	14	12	10	18	19	20	21	22
65	21 32	21 57	22 21	22 46	23 10	23 34	23 59	24 23	50	9	7	5	3	2	23	24	25	26	27
66	20 42	21 5	21 29	21 52	22 15	22 39	23 2	23 26	0	55	53	51	49	48	0	1	2	2	3
67	19 51	20 13	20 36	20 58	21 21	21 43	22 5	22 28	10	46	44	42	40	38	4	4	5	6	7
68	18 59	19 21	19 42	20 4	20 25	20 47	21 8	21 30	20	37	35	33	31	29	7	8	9	10	10
69	18 7	18 28	18 48	19 9	19 29	19 50	20 10	20 31	30	27	25	24	22	20	11	12	12	13	14
70	17 15	17 35	17 54	18 14	18 33	18 53	19 12	19 32	40	18	16	15	13	11	15	15	16	17	17
71	16 23	16 41	17 0	17 18	17 37	17 55	18 14	18 32	50	9	7	5	4	2	18	19	20	20	21
72	15 30	15 47	16 5	16 22	16 40	16 57	17 15	17 33	0	57	55	53	51	49	0	1	1	2	2
73	14 37	14 53	15 10	15 26	15 43	15 59	16 16	16 32	10	47	46	44	42	40	3	3	4	4	5
74	13 43	13 59	14 14	14 30	14 45	15 1	15 16	15 32	20	38	36	34	32	30	5	6	6	7	7
75	12 50	13 4	13 19	13 33	13 48	14 2	14 17	14 31	30	28	27	25	23	21	8	9	9	10	10
76	11 56	12 9	12 23	12 36	12 50	13 3	13 17	13 30	40	19	17	15	13	11	11	11	12	12	13
77	11 1	11 14	11 26	11 39	11 51	12 4	12 16	12 29	50	9	8	6	4	2	13	14	14	15	15
78	10 7	10 19	10 30	10 41	10 53	11 4	11 16	11 27	0	58	56	54	52	50	0	0	1	1	1
79	9 12	9 23	9 33	9 44	9 54	10 5	10 15	10 25	10	48	46	44	42	41	2	2	2	3	3
80	8 18	8 27	8 37	8 46	8 55	9 5	9 14	9 24	20	39	37	35	33	31	3	4	4	4	5
81	7 23	7 31	7 40	7 48	7 56	8 5	8 13	8 21	30	29	27	25	23	21	5	5	6	6	6
82	6 28	6 35	6 42	6 50	6 57	7 4	7 12	7 19	40	19	17	15	13	12	7	7	7	8	8
83	5 33	5 39	5 45	5 51	5 58	6 4	6 10	6 17	50	10	8	6	4	2	8	9	9	9	10
84	4 37	4 43	4 48	4 53	4 58	5 4	5 9	5 14	0	59	57	55	53	51	0	0	0	0	0
85	3 42	3 46	3 50	3 55	3 59	4 3	4 7	4 11	10	49	47	45	43	41	1	1	1	1	1
86	2 47	2 50	2 53	2 56	2 59	3 2	3 5	3 8	20	39	37	35	33	31	1	1	1	2	2
87	1 51	1 53	1 55	1 57	1 59	2 2	2 4	2 6	30	29	27	26	24	22	2	2	2	2	2
88	0 56	0 57	0 58	0 59	1 0	1 1	1 2	1 3	40	20	18	16	14	12	2	3	3	3	3
89	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	50	10	8	6	4	2	3	3	3	3	4
	54′	55′	56′	57′	58′	59′	60′	61′		0′	2′	4′	6′	8′	0″	2″	4″	6″	8″

TABLE V. Mean Refractions.

English Barometer 30 inches. Fahrenheit's Thermometer 50°.

Z. D.	Log δt.	Diff. to 1′	Z. D.	Log δt.	Diff. to 1′	Z. D.	Log δt.	Diff. to 1′	$\frac{d\delta t}{d\tau}$	$\frac{d\delta t}{d\gamma}$
°			° ′			° ′			—	
0			60 0	2.00868	29.1	80 0	2.50541	69.6	0.030	0.04
1	0.0085	50.2	20	2.00949	29.3	10	2.51237	70.7	0.031	0.04
2	0.3097	29.4	40	2.01535	29.5	20	2.51944	71.6	0.033	0.05
3	0.4860	20.8	61 0	2.02124	29.8	30	2.52660	72.7	0.034	0.05
4	0.6112	16.2	20	2.02718	30.0	40	2.53387	73.8	0.036	0.05
5	0.7086	13.3	40	2.03316	30.1	50	2.54125	74.9	0.038	0.05
6	0.7882	11.2	62 0	2.03918	30.4	81 0	2.54874	75.9	0.040	0.05
7	0.8557	9.8	20	2.04525	30.7	10	2.55635	77.2	0.042	0.06
8	0.9144	8.7	40	2.05137	30.9	20	2.56407	78.5	0.044	0.06
9	0.9663	7.7	63 0	2.05754	31.2	30	2.57192	79.7	0.046	0.07
10	1.0129	7.0	20	2.06376	31.5	40	2.57989	81.1	0.049	0.07
11	1.0552	6.4	40	2.07003	31.7	50	2.58800	82.4	0.051	0.08
12	1.0941	6.0	64 0	2.07635	32.0	82 0	2.59624	83.8	0.053	0.08
13	1.1300	5.6	20	2.08273	32.3	10	2.60462	85.1	0.057	0.09
14	1.1634	5.4	40	2.08917	32.6	20	2.61313	86.6	0.060	0.09
15	1.1947	4.9	65 0	2.09567	33.0	30	2.62179	88.3	0.063	0.10
16	1.2241	4.6	20	2.10224	33.2	40	2.63062	89.9	0.067	0.10
17	1.2519	4.4	40	2.10886	33.5	50	2.63961	91.6	0.069	0.11
18	1.2784	4.2	66 0	2.11555	33.9	83 0	2.64877	93.3	0.070	0.11
19	1.3036	4.0	20	2.12231	34.2	10	2.65810	94.8	0.074	0.12
20	1.3277	3.9	40	2.12913	34.5	20	2.66758	97.0	0.079	0.12
21	1.3507	3.7	67 0	2.13603	34.9	30	2.67728	98.5	0.085	0.13
22	1.3729	3.6	20	2.14300	35.4	40	2.68713	100.5	0.089	0.14
23	1.3945	3.4	40	2.15006	35.8	50	2.69718	102.8	0.095	0.15
24	1.4151	3.3	68 0	2.15719	36.2	84 0	2.70746	104.7	0.100	0.16
25	1.4352	3.2	20	2.16440	36.6	10	2.71793	106.9	0.107	0.17
26	1.4547	3.2	40	2.17171	37.1	20	2.72862	109.2	0.114	0.18
27	1.4736	3.1	69 0	2.17910	37.5	30	2.73954	111.6	0.122	0.19
28	1.4921	3.0	20	2.18658	38.1	40	2.75070	114.0	0.131	0.20
29	1.5102	2.9	40	2.19417	38.5	50	2.76210	116.6	0.141	0.22
30	1.5279	2.8	70 0	2.20185	39.0	85 0	2.77376	119.4	0.150	0.24
31	1.5452	2.8	20	2.20963	39.6	10	2.78570	121.9	0.161	0.25
32	1.5622	2.7	40	2.21752	40.2	20	2.79789	124.8	0.174	0.27
33	1.5790	2.7	71 0	2.22552	40.7	30	2.81037	128.0	0.189	0.30
34	1.5954	2.7	20	2.23363	41.3	40	2.82317	131.1	0.205	0.33
35	1.6116	2.6	40	2.24186	41.9	50	2.83628	134.1	0.222	0.36
36	1.6276	2.6	72 0	2.25022	42.5	86 0	2.84969	137.5	0.240	0.39
37	1.6435	2.6	20	2.25870	43.3	10	2.86344	141.3	0.260	0.43
38	1.6591	2.6	40	2.26732	44.0	20	2.87757	144.8	0.284	0.47
39	1.6746	2.6	73 0	2.27608	44.7	30	2.89205	148.8	0.310	0.51
40	1.6901	2.6	20	2.28498	45.4	40	2.90693	152.7	0.336	0.56
41	1.7055	2.5	40	2.29402	46.2	50	2.92220	157.0	0.362	0.61
42	1.7207	2.5	74 0	2.30322	47.0	87 0	2.93790	161.2	0.390	0.67
43	1.7358	2.5	20	2.31259	47.9	10	2.95402	165.8	0.430	0.75
44	1.7510	2.5	40	2.32213	48.8	20	2.97060	170.4	0.470	0.83
45	1.7661	2.5	75 0	2.33184	49.7	30	2.98764	175.8	0.520	0.91
46	1.7812	2.5	20	2.34174	50.7	40	3.00522	180.8	0.580	1.01
47	1.7964	2.5	40	2.35183	51.7	50	3.02330	186.2	0.630	1.13
48	1.8116	2.5	76 0	2.36212	52.8	88 0	3.04192	191.8	0.690	1.24
49	1.8268	2.5	20	2.37263	53.8	10	3.06110	197.7	0.780	1.41
50	1.8421	2.5	40	2.38334	55.1	20	3.08087	204.0	0.870	1.58
51	1.8575	2.6	77 0	2.39430	56.3	30	3.10127	210.2	0.960	1.75
52	1.8730	2.6	20	2.40550	57.6	40	3.12229	216.9	1.070	2.00
53	1.8886	2.6	40	2.41695	58.9	50	3.14398	223.9	1.190	2.24
54	1.9044	2.6	78 0	2.42867	60.3	89 0	3.16637	231.6	1.320	2.48
55	1.9204	2.6	20	2.44066	61.8	10	3.18943	238.8	1.520	2.91
56	1.9365	2.7	40	2.45295	63.5	20	3.21331	246.1	1.720	3.34
57	1.9529	2.7	79 0	2.46556	65.0	30	3.23792	252.9	1.920	3.77
58	1.9696	2.8	20	2.47848	66.9	40	3.26321	257.3	2.200	4.34
59	1.9865	2.8	40	2.49176	68.8	50	3.28894	275.5	2.480	5.00
60	2.0037	2.9	80 0	2.50541	69.6	90 0	3.31649		2.760	5.70

This Table contains the last edition of Ivory's Refractions, in a paper forming the Bakerian Lecture, printed in the Philosophical Transactions for 1838. The first three Tables on the opposite page correct the refractions for the English Barometer and Fahrenheit's Thermometer, as commonly used in Britain, and last two for the Metrical Barometer and Centigrade Thermometer, as employed on the Continent.

Table VI. Barometer.

P. P.	b	Log.
	In.	
+	27.0	9.95424
16	1	9.95585
32	2	9.95745
47	3	9.95904
63	4	9.96063
79	5	9.96221
95	6	9.96379
111	7	9.96536
126	8	9.96692
142	9	9.96848
	28.0	9.97004
15	1	9.97158
30	2	9.97313
46	3	9.97466
61	4	9.97620
76	5	9.97772
91	6	9.97924
106	7	9.98076
122	8	9.98227
137	9	9.98378
	29.0	9.98528
15	1	9.98677
29	2	9.98826
44	3	9.98975
59	4	9.99123
73	5	9.99270
88	6	9.99417
103	7	9.99563
118	8	9.99709
132	9	9.99855
	30.0	0.00000
14	1	0.00145
29	2	0.00289
43	3	0.00432
57	4	0.00575
71	5	0.00718
86	6	0.00860
100	7	0.01002
114	8	0.01143
129	9	0.01284
	31.0	0.01424

Table VII. Interior Thermometer.

τ	Log.	τ	Log.
°		°	
10	0.00173	50	0.00000
11	0.00169	51	9.99996
12	0.00164	52	9.99991
13	0.00160	53	9.99987
14	0.00156	54	9.99983
15	0.00151	55	9.99978
16	0.00147	56	9.99974
17	0.00143	57	9.99970
18	0.00138	58	9.99965
19	0.00134	59	9.99961
20	0.00130	60	9.99957
21	0.00126	61	9.99953
22	0.00121	62	9.99948
23	0.00117	63	9.99944
24	0.00113	64	9.99940
25	0.00108	65	9.99935
26	0.00104	66	9.99931
27	0.00100	67	9.99927
28	0.00095	68	9.99922
29	0.00091	69	9.99918
30	0.00087	70	9.99913
31	0.00083	71	9.99909
32	0.00078	72	9.99904
33	0.00074	73	9.99900
34	0.00070	74	9.99896
35	0.00065	75	9.99891
36	0.00061	76	9.99887
37	0.00057	77	9.99883
38	0.00052	78	9.99878
39	0.00048	79	9.99874
40	0.00043	80	9.99870
41	0.00039	81	9.99866
42	0.00034	82	9.99861
43	0.00030	83	9.99857
44	0.00026	84	9.99853
45	0.00021	85	9.99848
46	0.00017	86	9.99844
47	0.00013	87	9.99840
48	0.00008	88	9.99835
49	0.00004	89	9.99831
50	0.00000	90	9.99827

Table VIII. Exterior Thermometer.

P. P.	t	Log.	P. P.	t	Log.
	°			°	
—	10	0.03779	—	50	0.00000
10	1	0.03680	9	1	9.99910
20	2	0.03582	18	2	9.99820
29	3	0.03484	27	3	9.99730
39	4	0.03386	36	4	9.99640
49	5	0.03288	45	5	9.99550
59	6	0.03191	54	6	9.99460
69	7	0.03094	63	7	9.99371
78	8	0.02997	72	8	9.99282
88	9	0.02900	81	9	9.99193
	20	0.02803		60	9.99104
10	1	0.02706	9	1	9.99016
19	2	0.02609	18	2	9.98927
29	3	0.02514	26	3	9.98839
38	4	0.02418	35	4	9.98751
48	5	0.02323	44	5	9.98663
58	6	0.02227	53	6	9.98575
67	7	0.02132	62	7	9.98488
77	8	0.02037	70	8	9.98401
86	9	0.01942	79	9	9.98314
	30	0.01848		70	9.98227
9	1	0.01754	9	1	9.98140
19	2	0.01660	17	2	9.98054
28	3	0.01566	26	3	9.97967
38	4	0.01472	34	4	9.97881
47	5	0.01379	43	5	9.97795
56	6	0.01285	52	6	9.97709
66	7	0.01192	60	7	9.97623
75	8	0.01099	69	8	9.97537
85	9	0.01006	77	9	9.97452
	40	0.00914		80	9.97367
9	1	0.00822	8	1	9.97282
18	2	0.00730	17	2	9.97197
28	3	0.00638	25	3	9.97112
37	4	0.00546	34	4	9.97027
46	5	0.00455	42	5	9.96943
55	6	0.00363	50	6	9.96859
64	7	0.00272	59	7	9.96775
74	8	0.00181	67	8	9.96691
83	9	0.00090	76	9	9.96607
	50	0.00000		90	9.96524

Table IX. Metrical Barometer.

b	Log.	b	Log.
m. m.			
730	9.98137	750	9.99311
731	9.98196	751	9.99368
732	9.98256	752	9.99426
733	9.98315	753	9.99484
734	9.98374	754	9.99542
735	9.98433	755	9.99599
736	9.98492	756	9.99657
737	9.98551	757	9.99714
738	9.98610	758	9.99771
739	9.98669	759	9.99829
740	9.98728	760	9.99886
741	9.98786	761	9.99943
742	9.98845	762	0.00000
743	9.98903	763	0.00057
744	9.98962	764	0.00114
745	9.99020	765	0.00171
746	9.99078	766	0.00227
747	9.99137	767	0.00284
748	9.99195	768	0.00341
749	9.99253	769	0.00397

P. P.	1	2	3	4	5	6	7	8	9
+	6	12	17	23	29	35	41	46	52

Table X. Centigrade Thermometer.

t	Log.	t	Log.
°		°	
− 10	0.03542	+ 10	0.00000
9	0.03358	11	9.99829
8	0.03175	12	9.99659
7	0.02994	13	9.99491
6	0.02812	14	9.99322
5	0.02631	15	9.99154
4	0.02451	16	9.98987
3	0.02272	17	9.98820
2	0.02094	18	9.98654
− 1	0.01915	19	9.98488
0	0.01738	20	9.98323
+ 1	0.01563	21	9.98158
2	0.01385	22	9.97994
3	0.01210	23	9.97832
4	0.01035	24	9.97669
5	0.00861	25	9.97506
6	0.00687	26	9.97344
7	0.00515	27	9.97183
8	0.00343	28	9.97023
9	0.00171	29	9.96863

P.P.	1	2	3	4	5	6	7	8	9
−	17	34	51	68	85	102	119	136	153

TABLE XI. Logs to compute the Terrestrial Refraction.

Fahr. Ther. t	English Barometer, b.							Diff.
	24 in.	25 in.	26 in.	27 in.	28 in.	29 in.	30 in.	
°								+
30	7.45244	7.45249	7.45253	7.45258	7.45263	7.45267	7.45272	5
31	7.45239	7.45244	7.45248	7.45253	7.45258	7.45262	7.45267	5
32	7.45233	7.45238	7.45243	7.45248	7.45253	7.45258	7.45263	5
33	7.45227	7.45232	7.45238	7.45243	7.45248	7.45253	7.45259	5
34	7.45221	7.45226	7.45232	7.45237	7.45243	7.45248	7.45254	5
35	7.45215	7.45221	7.45226	7.45232	7.45238	7.45243	7.45249	6
36	7.45209	7.45215	7.45221	7.45226	7.45232	7.45238	7.45244	6
37	7.45202	7.45208	7.45214	7.45220	7.45226	7.45232	7.45238	6
38	7.45195	7.45201	7.45208	7.45214	7.45220	7.45226	7.45233	6
39	7.45188	7.45194	7.45201	7.45207	7.45214	7.45220	7.45227	7
40	7.45181	7.45188	7.45194	7.45201	7.45208	7.45214	7.45221	7
41	7.45173	7.45180	7.45187	7.45194	7.45201	7.45208	7.45215	7
42	7.45165	7.45172	7.45180	7.45187	7.45194	7.45201	7.45209	7
43	7.45157	7.45164	7.45172	7.45179	7.45187	7.45194	7.45202	7
44	7.45148	7.45156	7.45164	7.45171	7.45179	7.45187	7.45195	8
45	7.45140	7.45148	7.45156	7.45164	7.45172	7.45180	7.45188	8
46	7.45131	7.45139	7.45148	7.45156	7.45164	7.45172	7.45181	8
47	7.45121	7.45130	7.45139	7.45147	7.45156	7.45165	7.45174	9
48	7.45111	7.45120	7.45129	7.45139	7.45148	7.45157	7.45166	9
49	7.45101	7.45110	7.45120	7.45129	7.45139	7.45148	7.45158	9
50	7.45091	7.45101	7.45111	7.45120	7.45130	7.45140	7.45150	10
51	7.45080	7.45090	7.45100	7.45110	7.45121	7.45131	7.45141	10
52	7.45069	7.45079	7.45090	7.45100	7.45111	7.45121	7.45132	10
53	7.45058	7.45069	7.45080	7.45090	7.45101	7.45112	7.45123	11
54	7.45046	7.45057	7.45068	7.45080	7.45091	7.45102	7.45113	11
55	7.45034	7.45046	7.45057	7.45069	7.45081	7.45092	7.45104	12
56	7.45021	7.45033	7.45045	7.45058	7.45070	7.45082	7.45094	12
57	7.45008	7.45021	7.45033	7.45046	7.45059	7.45071	7.45084	13
58	7.44994	7.45007	7.45020	7.45034	7.45047	7.45060	7.45073	13
59	7.44981	7.44994	7.45008	7.45021	7.45035	7.45048	7.45062	14
60	7.44967	7.44981	7.44995	7.45009	7.45023	7.45037	7.45051	14
61	7.44952	7.44966	7.44981	7.44995	7.45010	7.45024	7.45039	15
62	7.44937	7.44952	7.44967	7.44982	7.44997	7.45012	7.45027	15
63	7.44921	7.44936	7.44952	7.44967	7.44983	7.44998	7.45014	16
64	7.44905	7.44921	7.44937	7.44953	7.44969	7.44985	7.45001	16
65	7.44889	7.44905	7.44922	7.44938	7.44955	7.44971	7.44988	17
66	7.44872	7.44889	7.44906	7.44922	7.44939	7.44956	7.44973	17
67	7.44854	7.44871	7.44889	7.44906	7.44923	7.44941	7.44958	17
68	7.44835	7.44853	7.44871	7.44890	7.44908	7.44926	7.44944	18
69	7.44817	7.44836	7.44855	7.44873	7.44892	7.44911	7.44930	18
70	7.44798	7.44817	7.44837	7.44856	7.44876	7.44896	7.44915	19
71	7.44778	7.44798	7.44818	7.44839	7.44859	7.44879	7.44899	20
72	7.44757	7.44778	7.44799	7.44820	7.44841	7.44862	7.44883	21
73	7.44735	7.44757	7.44779	7.44800	7.44822	7.44844	7.44866	22
74	7.44713	7.44735	7.44758	7.44780	7.44803	7.44825	7.44848	23
75	7.44691	7.44714	7.44737	7.44761	7.44784	7.44807	7.44830	24
76	7.44668	7.44692	7.44716	7.44740	7.44763	7.44787	7.44811	24
77	7.44644	7.44669	7.44693	7.44718	7.44742	7.44767	7.44792	25
78	7.44619	7.44645	7.44670	7.44695	7.44721	7.44746	7.44772	26
79	7.44593	7.44619	7.44646	7.44672	7.44698	7.44724	7.44751	26
80	7.44567	7.44594	7.44622	7.44649	7.44676	7.44704	7.44731	27
81	7.44542	7.44570	7.44598	7.44626	7.44654	7.44682	7.44710	28
82	7.44515	7.44544	7.44573	7.44601	7.44630	7.44659	7.44688	29
83	7.44486	7.44516	7.44546	7.44575	7.44605	7.44635	7.44665	30
84	7.44455	7.44486	7.44517	7.44549	7.44580	7.44611	7.44642	31
85	7.44424	7.44456	7.44489	7.44521	7.44553	7.44586	7.44618	32
86	7.44393	7.44426	7.44460	7.44493	7.44526	7.44560	7.44593	33
87	7.44361	7.44395	7.44430	7.44464	7.44498	7.44533	7.44567	34
88	7.44328	7.44363	7.44399	7.44434	7.44470	7.44506	7.44541	35
89	7.44294	7.44331	7.44367	7.44404	7.44441	7.44477	7.44514	36
90	7.44258	7.44296	7.44334	7.44372	7.44410	7.44448	7.44486	38
°								°
30	— 6	— 6	— 5	— 5	— 5	— 5	— 5	30
40	8	8	7	7	7	6	6	40
50	10	10	10	9	9	8	8	50
60	14	14	14	14	13	12	11	60
70	19	19	18	18	17	16	15	70
80	26	26	24	23	22	21	20	80
90	35	35	33	32	30	29	28	90

Table XII. Parallax of the Sun in Altitude or Z. D.

Alt.	Jan. 1	Feb. Dec. 1	Mar. Nov. 1	Apr. Oct. 1	May Sept. 1	June Aug. 1	July 1
°	"	"	"	"	"	"	"
0	8.8	8.7	8.7	8.6	8.5	8.5	8.5
3	8.7	8.7	8.7	8.6	8.5	8.5	8.4
6	8.7	8.7	8.6	8.5	8.5	8.4	8.4
9	8.6	8.6	8.6	8.5	8.4	8.4	8.3
12	8.5	8.5	8.5	8.4	8.3	8.3	8.3
15	8.4	8.4	8.4	8.3	8.2	8.2	8.2
18	8.3	8.3	8.2	8.2	8.1	8.1	8.0
21	8.2	8.2	8.1	8.0	8.0	7.9	7.9
24	8.0	8.0	7.9	7.8	7.8	7.7	7.7
27	7.8	7.8	7.7	7.7	7.6	7.5	7.5
30	7.6	7.6	7.5	7.4	7.4	7.3	7.3
33	7.3	7.3	7.3	7.2	7.1	7.1	7.1
36	7.1	7.1	7.0	7.0	6.9	6.9	6.8
39	6.8	6.8	6.7	6.7	6.6	6.6	6.6
42	6.5	6.5	6.4	6.4	6.3	6.3	6.3
45	6.2	6.2	6.1	6.1	6.0	6.0	6.0
48	5.8	5.8	5.8	5.7	5.7	5.7	5.7
51	5.5	5.5	5.5	5.4	5.4	5.3	5.3
54	5.1	5.1	5.1	5.0	5.0	5.0	5.0
57	4.8	4.7	4.7	4.7	4.6	4.6	4.6
60	4.4	4.4	4.3	4.3	4.2	4.2	4.2
63	4.0	4.0	3.9	3.9	3.9	3.8	3.8
66	3.6	3.5	3.5	3.5	3.5	3.4	3.4
69	3.1	3.1	3.1	3.1	3.1	3.0	3.0
72	2.7	2.7	2.7	2.7	2.6	2.6	2.6
75	2.3	2.3	2.2	2.2	2.2	2.2	2.2
78	1.8	1.8	1.8	1.8	1.8	1.8	1.8
81	1.4	1.4	1.4	1.3	1.3	1.3	1.3
84	0.9	0.9	0.9	0.9	0.9	0.9	0.9
87	0.5	0.5	0.5	0.4	0.4	0.4	0.4
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table XIII. Parallax of the Planets in Altitude or Zenith-Distance.

Alt.	Horizontal Parallax. 10"	20"	30"	1"	2"	3"	4"	5"	6"	7"	8"	9"	Z.D.
°	"	"	"	"	"	"	"	"	"	"	"	"	°
0	10.0	20.0	30.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	90
3	10.0	20.0	30.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	87
6	9.9	19.9	29.8	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	84
9	9.9	19.8	29.6	1.0	2.0	3.0	4.0	4.9	5.9	6.9	7.9	8.9	81
12	9.8	19.6	29.3	1.0	2.0	2.9	3.9	4.9	5.9	6.8	7.8	8.8	78
15	9.7	19.3	29.0	1.0	1.9	2.9	3.9	4.8	5.8	6.8	7.7	8.7	75
18	9.5	19.0	28.5	1.0	1.9	2.9	3.8	4.8	5.7	6.7	7.6	8.6	72
21	9.3	18.7	28.0	0.9	1.9	2.8	3.7	4.7	5.6	6.5	7.5	8.4	69
24	9.1	18.3	27.4	0.9	1.8	2.7	3.7	4.6	5.5	6.4	7.3	8.2	66
27	8.9	17.8	26.7	0.9	1.8	2.7	3.6	4.5	5.3	6.2	7.1	8.0	63
30	8.7	17.3	26.0	0.9	1.7	2.6	3.5	4.3	5.2	6.1	6.9	7.8	60
33	8.4	16.8	25.2	0.8	1.7	2.5	3.4	4.2	5.0	5.9	6.7	7.5	57
36	8.1	16.2	24.3	0.8	1.6	2.4	3.2	4.0	4.9	5.7	6.5	7.3	54
39	7.8	15.5	23.3	0.8	1.6	2.3	3.1	3.9	4.7	5.4	6.2	7.0	51
42	7.4	14.9	22.3	0.7	1.5	2.2	3.0	3.7	4.5	5.2	5.9	6.7	48
45	7.1	14.1	21.2	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.7	6.4	45
48	6.7	13.4	20.1	0.7	1.3	2.0	2.7	3.3	4.0	4.7	5.4	6.0	42
51	6.3	12.6	18.9	0.6	1.3	1.9	2.5	3.1	3.8	4.4	5.0	5.7	39
54	5.9	11.8	17.6	0.6	1.2	1.8	2.4	2.9	3.5	4.1	4.7	5.3	36
57	5.4	10.9	16.3	0.5	1.1	1.6	2.2	2.7	3.3	3.8	4.4	4.9	33
60	5.0	10.0	15.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	30
63	4.5	9.1	13.6	0.5	0.9	1.4	1.8	2.3	2.7	3.2	3.6	4.1	27
66	4.1	8.1	12.2	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.7	24
69	3.6	7.2	10.8	0.4	0.7	1.1	1.4	1.8	2.2	2.5	2.9	3.2	21
72	3.1	6.2	9.3	0.3	0.6	0.9	1.2	1.5	1.9	2.2	2.5	2.8	18
75	2.6	5.2	7.8	0.3	0.5	0.8	1.0	1.3	1.6	1.8	2.1	2.3	15
78	2.1	4.2	6.2	0.2	0.4	0.6	0.8	1.0	1.2	1.5	1.7	1.9	12
81	1.6	3.1	4.7	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.3	1.4	9
84	1.0	2.1	3.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	6
87	0.5	1.0	1.6	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5	3
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0

Table XIV. Augmentation of the Moon's semidiameter in Altitude or Z. D. +

Alt.	14′ 30″	15′ 0″	15′ 30″	16′ 0″	16′ 30″	17′ 0″	Z.D.
°	"	"	"	"	"	"	°
0	0.1	0.1	0.1	0.1	0.1	0.2	90
3	0.8	0.9	0.9	1.0	1.1	1.1	87
6	1.5	1.6	1.7	1.8	2.0	2.1	84
9	2.2	2.4	2.5	2.7	2.9	3.1	81
12	2.9	3.1	3.4	3.5	3.8	4.0	78
15	3.6	3.9	4.1	4.4	4.7	5.0	75
18	4.3	4.6	4.9	5.2	5.5	5.9	72
21	4.9	5.3	5.6	6.0	6.4	6.8	69
24	5.6	6.0	6.4	6.8	7.3	7.7	66
27	6.2	6.7	7.1	7.6	8.1	8.6	63
30	6.9	7.3	7.8	8.3	8.9	9.4	60
33	7.5	8.0	8.5	9.1	9.7	10.3	57
36	8.0	8.6	9.2	9.8	10.4	11.1	54
39	8.6	9.2	9.8	10.5	11.2	11.8	51
42	9.1	9.8	10.5	11.2	11.9	12.6	48
45	9.7	10.3	11.1	11.8	12.5	13.3	45
48	10.2	10.9	11.6	12.4	13.2	14.0	42
51	10.6	11.4	12.1	12.9	13.8	14.6	39
54	11.1	11.8	12.6	13.5	14.3	15.2	36
57	11.5	12.3	13.1	14.0	14.9	15.8	33
60	11.8	12.7	13.5	14.4	15.3	16.3	30
63	12.2	13.0	13.9	14.8	15.8	16.8	27
66	12.5	13.4	14.3	15.2	16.2	17.2	24
69	12.8	13.6	14.6	15.5	16.5	17.6	21
72	13.0	13.9	14.9	15.8	16.8	17.9	18
75	13.2	14.1	15.1	16.1	17.1	18.2	15
78	13.4	14.3	15.3	16.3	17.3	18.4	12
81	13.5	14.4	15.4	16.4	17.5	18.6	9
84	13.6	14.5	15.5	16.5	17.6	18.7	6
87	13.6	14.6	15.6	16.6	17.7	18.8	3
90	13.7	14.6	15.6	16.7	17.7	18.8	0

Table XV. Reduction of the Moon's Parallax in the Spheroid —

Lat	54′	55′	56′	57′	58′	59′	60′	61′
°	"	"	"	"	"	"	"	"
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2
9	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
12	0.4	0.5	0.5	0.5	0.5	0.5	0.5	0.6
15	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9
18	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.2
21	1.4	1.5	1.5	1.5	1.5	1.5	1.5	1.6
24	1.8	1.8	1.9	1.9	1.9	1.9	2.0	2.0
27	2.2	2.3	2.3	2.4	2.4	2.4	2.4	2.5
30	2.7	2.7	2.8	2.8	2.9	2.9	3.0	3.1
33	3.2	3.3	3.3	3.4	3.4	3.5	3.5	3.6
36	3.7	3.8	3.8	3.9	4.0	4.0	4.0	4.1
39	4.3	4.3	4.4	4.5	4.5	4.6	4.7	4.7
42	4.9	4.9	4.9	5.0	5.1	5.2	5.3	5.4
45	5.4	5.4	5.5	5.5	5.6	5.8	5.9	6.0
48	6.0	6.0	6.1	6.2	6.3	6.4	6.5	6.6
51	6.5	6.5	6.6	6.7	6.8	6.9	7.1	7.2
54	7.0	7.1	7.2	7.3	7.4	7.6	7.7	7.8
57	7.5	7.6	7.7	7.8	7.9	8.1	8.2	8.4
60	8.0	8.1	8.2	8.4	8.6	8.7	8.8	9.0
63	8.5	8.5	8.6	8.8	9.0	9.2	9.4	9.5
66	9.0	9.0	9.2	9.3	9.5	9.7	9.8	10.0
69	9.4	9.4	9.6	9.7	9.9	10.2	10.3	10.5
72	9.7	9.8	10.1	10.2	10.3	10.5	10.8	10.9
75	10.0	10.2	10.3	10.4	10.6	10.8	11.1	11.4
78	10.2	10.3	10.5	10.7	10.9	11.2	11.4	11.6
81	10.4	10.5	10.7	10.9	11.1	11.4	11.6	11.7
84	10.6	10.8	11.0	11.2	11.4	11.6	11.7	11.9
87	10.7	10.8	11.2	11.3	11.5	11.7	11.8	12.1
90	10.8	10.9	11.2	11.4	11.6	11.7	11.9	12.2

Table XVI. Reduction of the Lat.

Lat	Reduction —	
°	′	″
0	0	0.0
3	1	11.8
6	2	22.7
9	3	32.1
12	4	39.3
15	5	43.4
18	6	43.7
21	7	39.7
24	8	30.7
27	9	16.1
30	9	55.4
33	10	28.3
36	10	54.3
39	11	13.2
42	11	24.7
45	11	28.7
48	11	25.2
51	11	14.1
54	10	55.7
57	10	30.0
60	9	57.4
63	9	18.3
66	8	32.9
69	7	42.0
72	6	45.9
75	5	45.4
78	4	41.0
81	3	38.5
84	2	23.7
87	1	12.3
90	0	0.0

Table XVII. Reduction to the Meridian. Versines.

Time from the Meridian.

s	0 m.		1 m.		2 m.		3 m.		4 m.		5 m.		6 m.		7 m.	
	V	v	V	v	V	v	V	v	V	v	V	v	V	v	V	v
0	0	0	95	0	381	1	857	7	1523	23	2380	57	3427	117	4664	218
1	0	0	98	0	387	1	866	7	1536	24	2396	57	3446	118	4686	220
2	0	0	101	0	394	1	876	8	1548	24	2412	58	3465	120	4708	222
3	0	0	105	0	400	1	885	8	1561	24	2428	59	3484	121	4731	224
4	0	0	108	0	407	1	895	8	1574	25	2444	60	3503	123	4753	226
5	0	0	111	0	413	1	905	8	1587	25	2460	61	3522	124	4775	228
6	1	0	115	0	420	2	915	8	1600	26	2476	61	3542	125	4798	230
7	1	0	118	0	426	2	925	9	1613	26	2492	62	3561	127	4820	232
8	2	0	122	0	433	2	935	9	1626	26	2508	63	3581	128	4843	235
9	2	0	126	0	440	2	945	9	1639	27	2524	64	3600	130	4866	237
10	3	0	130	0	447	2	955	9	1652	27	2541	65	3620	131	4888	239
11	3	0	133	0	454	2	965	9	1666	28	2557	65	3639	132	4911	241
12	4	0	137	0	461	2	975	10	1679	28	2574	66	3659	133	4934	243
13	5	0	141	0	468	2	985	10	1692	29	2590	67	3679	135	4957	246
14	6	0	145	0	475	2	995	10	1706	29	2607	68	3698	137	4980	248
15	6	0	149	0	482	2	1006	10	1719	30	2623	69	3718	138	5003	250
16	7	0	153	0	489	3	1016	10	1733	30	2640	70	3738	140	5026	253
17	8	0	157	0	496	3	1026	11	1746	30	2657	71	3758	141	5049	255
18	9	0	161	0	503	3	1037	11	1760	31	2674	71	3778	143	5072	257
19	10	0	165	0	510	3	1047	11	1773	31	2691	72	3798	144	5096	260
20	11	0	169	0	518	3	1058	11	1787	32	2708	73	3818	146	5119	262
21	12	0	174	0	525	3	1068	12	1801	32	2725	74	3838	147	5142	264
22	13	0	178	0	533	3	1079	12	1815	33	2742	75	3858	149	5165	267
23	14	0	183	0	540	3	1089	12	1829	33	2759	76	3879	150	5189	269
24	15	0	187	0	548	3	1100	12	1843	34	2776	77	3899	152	5212	272
25	16	0	192	0	556	3	1111	12	1857	34	2793	78	3919	154	5236	274
26	18	0	196	0	563	4	1122	13	1871	35	2810	79	3939	155	5259	277
27	19	0	201	0	571	4	1133	13	1885	35	2827	80	3960	157	5283	279
28	21	0	205	0	579	4	1144	13	1899	36	2844	81	3980	158	5307	282
29	22	0	210	0	587	4	1155	13	1913	36	2861	82	4000	160	5331	284
30	24	0	214	0	595	4	1166	14	1927	37	2879	83	4021	162	5354	287
31	25	0	219	0	603	4	1177	14	1942	38	2896	84	4042	163	5378	289
32	27	0	224	0	611	4	1188	14	1956	38	2914	85	4063	165	5402	291
33	29	0	229	0	619	4	1200	14	1970	39	2932	86	4084	167	5426	294
34	31	0	234	0	627	4	1211	15	1985	39	2949	87	4104	168	5450	297
35	32	0	239	0	635	4	1222	15	1999	40	2967	88	4125	170	5474	300
36	34	0	244	0	643	5	1234	15	2014	40	2985	89	4146	172	5498	302
37	36	0	249	0	651	5	1245	16	2028	41	3003	90	4167	174	5522	305
38	38	0	254	0	660	5	1257	16	2043	42	3021	91	4188	175	5546	308
39	40	0	259	0	668	5	1268	16	2058	42	3039	92	4209	177	5571	310
40	42	0	264	1	677	5	1280	16	2073	43	3057	93	4230	179	5595	313
41	44	0	269	1	685	5	1292	17	2088	44	3075	94	4251	181	5619	316
42	46	0	275	1	694	5	1303	17	2103	44	3093	96	4273	183	5643	318
43	49	0	280	1	703	5	1315	17	2118	45	3111	97	4294	184	5668	321
44	51	0	286	1	711	5	1327	18	2133	45	3129	98	4316	186	5692	324
45	53	0	291	1	719	5	1339	18	2148	46	3147	99	4337	188	5717	327
46	56	0	297	1	728	6	1351	18	2163	47	3165	100	4358	190	5741	330
47	58	0	302	1	737	6	1363	19	2178	47	3184	101	4380	192	5766	332
48	61	0	308	1	746	6	1375	19	2193	48	3202	103	4401	194	5791	335
49	64	0	314	1	755	6	1387	19	2208	48	3220	104	4423	196	5816	338
50	67	0	320	1	764	6	1399	20	2224	49	3239	105	4444	197	5840	341
51	69	0	326	1	773	6	1411	20	2239	50	3257	106	4466	199	5865	344
52	72	0	332	1	782	6	1423	20	2255	51	3276	107	4488	201	5890	347
53	75	0	338	1	791	6	1435	21	2270	52	3295	109	4510	203	5915	350
54	78	0	344	1	800	6	1448	21	2286	52	3313	110	4532	205	5940	353
55	80	0	350	1	810	6	1460	21	2301	53	3332	111	4554	207	5966	356
56	83	0	356	1	819	7	1473	22	2317	54	3351	112	4576	209	5991	359
57	86	0	362	1	828	7	1485	22	2333	54	3370	114	4598	211	6016	362
58	89	0	369	1	838	7	1498	22	2348	55	3389	115	4620	213	6041	365
59	92	0	375	1	847	7	1510	23	2364	56	3408	116	4642	215	6067	368
s																
0.1	0		0		1		1		1		2		2		2	
0.2	0		1		2		2		3		4		4		5	
0.3	1		1		2		3		4		5		6		7	
0.4	1		2		3		4		6		7		8		10	
0.5	1		2		4		5		7		9		10		12	
0.6	1		3		5		7		8		11		13		14	
0.7	1		3		6		8		10		13		15		17	
0.8	1		4		6		9		11		14		17		19	
0.9	2		4		7		10		13		16		19		22	

Table XVII. Reduction to the Meridian. Versines.

Time from the Meridian.

s	8 m. V	8 m. v	9 m. V	9 m. v	10 m. V	10 m. v	11 m. V	11 m. v	12 m. V	12 m. v	13 m. V	13 m. v
0	6092	371	7710	594	9518	906	11516	1326	13705	1878	16083	2587
1	6117	374	7738	599	9550	912	11551	1334	13743	1889	16124	2600
2	6143	377	7767	603	9581	918	11586	1342	13781	1899	16166	2613
3	6168	380	7795	608	9613	924	11621	1350	13819	1910	16207	2627
4	6194	384	7824	612	9645	930	11656	1359	13857	1920	16249	2640
5	6219	387	7853	617	9677	936	11691	1367	13895	1931	16290	2654
6	6245	390	7882	621	9709	942	11726	1375	13934	1942	16332	2667
7	6270	393	7911	626	9741	949	11762	1383	13972	1953	16373	2681
8	6296	396	7940	630	9773	955	11797	1392	14011	1963	16415	2695
9	6322	400	7969	635	9805	961	11832	1400	14049	1974	16457	2708
10	6348	403	7998	640	9837	968	11868	1408	14088	1984	16498	2722
11	6374	406	8027	644	9870	974	11903	1417	14126	1995	16540	2736
12	6400	410	8056	649	9902	980	11939	1425	14165	2006	16582	2750
13	6426	413	8085	654	9934	987	11974	1434	14204	2017	16624	2764
14	6452	416	8114	658	9967	993	12010	1442	14242	2028	16666	2778
15	6479	420	8144	663	9999	1000	12045	1451	14281	2039	16708	2792
16	6505	423	8173	668	10032	1006	12081	1460	14320	2051	16750	2806
17	6531	427	8202	673	10065	1013	12117	1468	14359	2062	16792	2820
18	6557	430	8232	678	10097	1020	12153	1477	14398	2073	16834	2834
19	6584	433	8261	682	10130	1026	12189	1486	14437	2084	16876	2848
20	6610	437	8291	687	10163	1033	12225	1495	14476	2095	16918	2862
21	6636	440	8321	692	10196	1039	12261	1503	14515	2107	16960	2876
22	6663	444	8350	697	10228	1046	12297	1512	14554	2118	17003	2891
23	6689	447	8380	702	10261	1053	12333	1521	14594	2130	17045	2905
24	6716	451	8410	707	10294	1060	12369	1530	14633	2141	17088	2920
25	6743	455	8440	712	10327	1066	12405	1539	14672	2153	17130	2934
26	6769	458	8470	717	10360	1073	12441	1548	14712	2164	17173	2949
27	6796	462	8500	722	10394	1080	12478	1557	14751	2176	17215	2964
28	6823	466	8530	728	10427	1087	12514	1566	14791	2187	17258	2978
29	6850	469	8560	733	10460	1094	12550	1575	14831	2199	17301	2993
30	6877	473	8590	738	10493	1101	12587	1584	14870	2211	17344	3008
31	6904	477	8620	743	10527	1108	12623	1593	14910	2223	17387	3023
32	6931	480	8650	748	10560	1115	12660	1603	14950	2235	17430	3038
33	6958	484	8680	753	10593	1122	12696	1612	14990	2247	17473	3053
34	6985	488	8711	758	10627	1129	12733	1621	15029	2259	17516	3068
35	7013	492	8741	764	10660	1136	12769	1630	15069	2271	17559	3083
36	7040	496	8772	769	10694	1144	12806	1640	15109	2283	17602	3098
37	7067	499	8802	775	10728	1151	12843	1649	15149	2295	17645	3113
38	7094	503	8833	780	10761	1158	12880	1659	15189	2307	17688	3129
39	7122	507	8863	786	10795	1165	12917	1668	15229	2319	17732	3144
40	7149	511	8894	791	10829	1172	12954	1678	15269	2331		
41	7177	515	8925	797	10863	1180	12991	1687	15309	2344		
42	7204	519	8955	802	10897	1187	13028	1697	15349	2356		
43	7232	523	8986	807	10931	1195	13065	1707	15390	2369		
44	7260	527	9017	813	10965	1203	13102	1717	15430	2381		
45	7288	531	9048	819	10999	1210	13139	1726	15470	2393		
46	7315	535	9079	824	11033	1217	13177	1736	15511	2406		
47	7343	539	9110	830	11067	1225	13214	1746	15551	2418		
48	7371	543	9141	836	11101	1232	13252	1756	15592	2431		
49	7399	547	9172	841	11135	1240	13289	1766	15633	2444		
50	7427	552	9203	847	11170	1248	13327	1776	15673	2456		
51	7455	556	9235	853	11204	1255	13364	1786	15714	2469		
52	7483	560	9266	859	11239	1263	13402	1796	15755	2482		
53	7511	564	9297	864	11273	1271	13440	1806	15796	2495		
54	7539	568	9328	870	11308	1279	13477	1816	15837	2508		
55	7568	573	9360	876	11342	1286	13515	1826	15878	2521		
56	7596	577	9391	882	11377	1294	13553	1837	15919	2534		
57	7624	581	9423	888	11412	1302	13591	1847	15960	2547		
58	7653	586	9454	894	11446	1310	13629	1857	16001	2560		
59	7681	590	9486	900	11481	1318	13667	1861	16042	2573		
s												
0.1	3	0	3	0	3	1	4	1	4	1		
0.2	5	1	6	1	7	1	7	2	8	2		
0.3	8	1	9	1	10	2	11	3	12	4		
0.4	11	2	12	2	13	3	15	4	16	5		
0.5	13	2	15	2	16	3	18	4	20	6		
0.6	16	2	18	3	20	4	22	5	24	7		
0.7	19	3	21	3	23	5	26	6	28	8		
0.8	22	3	24	4	26	6	29	7	32	10		
0.9	24	4	27	4	29	6	33	8	36	11		

Logarithms for V′ +

No.	Logs.
1	8.314425
2	8.013395
4	7.712365
6	7.536274
8	7.411335
10	7.314425
12	7.235244
14	7.168297
16	7.110305
18	7.059152
20	7.013395

Logarithms for v′ +

No.	Logs.
1	6.013395
2	5.712365
4	5.411335
6	5.235244
8	5.110305
10	5.013395
12	4.934214
14	4.867267
16	4.809275
18	4.758122
20	4.712365

TABLE XVIII. To compute the Equation to Equal Altitudes and Equal Azimuths.

E. T.	Log A.	Log B	Log C	E. T.	Log A	Log B	Log C
h. m.				h. m.			
2 0	+9.4109	+9.3958	0.5870	13 0	+9.6405	—8.7562	0.8166
10	4117	3940	5879	10	6474	8.8296	8235
20	4127	3921	5888	20	6544	8.8941	8306
30	4137	3900	5898	30	6616	8.9518	8378
40	4148	3877	5909	40	6689	9.0043	8451
50	4159	3853	5920	50	6764	9.0524	8525
3 0	+9.4171	+9.3827	0.5933	14 0	+9.6840	—9.0970	0.8601
10	4184	3800	5945	10	6918	1387	8679
20	4198	3770	5959	20	6998	1779	8759
30	4212	3739	5973	30	7079	2150	8840
40	4227	3706	5988	40	7162	2502	8923
50	4243	3671	6004	50	7247	2839	9008
4 0	+9.4259	+9.3635	0.6021	15 0	+9.7333	—9.3162	0.9094
10	4276	3596	6038	10	7422	3472	9182
20	4294	3555	6056	20	7512	3771	9273
30	4313	3512	6074	30	7604	4061	9366
40	4333	3466	6094	40	7699	4343	9460
50	4353	3416	6114	50	7795	4617	9557
5 0	+9.4374	+9.3368	0.6135	16 0	+9.7894	—9.4884	0.9656
10	4396	3316	6155	10	7995	5145	9757
20	4418	3260	6179	20	8099	5401	9860
30	4441	3202	6202	30	8205	5652	0.9966
40	4465	3141	6226	40	8313	5899	1.0075
50	4490	3077	6251	50	8424	6142	1.0186
6 0	+9.4515	+9.3010	0.6276	17 0	+9.8538	—9.6382	1.0300
10	4541	2939	6303	10	8655	6620	0416
20	4568	2865	6330	20	8775	6855	0536
30	4596	2787	6358	30	8898	7089	0659
40	4625	2706	6386	40	9024	7320	0785
50	4654	2620	6416	50	9153	7551	0915
7 0	+9.4685	+9.2529	0.6446	18 0	+9.9286	—9.7781	1.1048
10	4716	2434	6477	10	9423	8011	1184
20	4748	2334	6509	20	9564	8240	1325
30	4781	2228	6542	30	9709	8470	1470
40	4814	2116	6575	40	9.9858	8701	1620
50	4849	1998	6610	50	0.0012	8933	1774
8 0	+9.4884	+9.1874	0.6645	19 0	+0.0171	—9.9166	1.1933
10	4920	1742	6682	10	0336	9401	2097
20	4957	1601	6719	20	0506	9639	2267
30	4995	1452	6757	30	0681	—9.9880	2443
40	5034	1294	6795	40	0864	—0.0124	2625
50	5074	1124	6834	50	1053	—0.0372	2814
9 0	+9.5114	+9.0943	0.6876	20 0	+0.1249	—0.0624	1.3010
10	5156	0749	6918	10	1453	0882	3215
20	5199	0540	6960	20	1666	1146	3428
30	5242	0313	7004	30	1889	1416	3650
40	5287	9.0068	7048	40	2122	1694	3883
50	5332	8.9801	7094	50	2366	1981	4127
10 0	+9.5379	+8.9509	0.7140	21 0	+0.2622	—0.2278	1.4383
10	5426	9186	7188	10	2893	2587	4654
20	5475	8828	7236	20	3178	2908	4940
30	5525	8427	7286	30	3482	3245	5243
40	5575	7972	7336	40	3805	3600	5566
50	5627	7449	7388	50	4151	3974	5912
11 0	+9.5680	+8.6837	0.7441	22 0	+0.4523	—0.4372	1.6284
10	5734	6102	7495	10	4925	4799	6687
20	5789	5191	7550	20	5365	5260	7125
30	5845	4001	7606	30	5848	5764	7609
40	5902	8.2299	7663	40	6386	6319	8147
50	5960	+7.9348	7722	50	6992	6941	8753
12 0	+9.6020		0.7782	23 0	+0.7689	—0.7651	1.9450
10	6081	—7.9469	7842	10	0.8508	8482	2.0269
20	6143	8.2540	7905	20	0.9505	9489	1267
30	6207	4363	7968	30	1.0783	—1.0774	2544
40	6272	5675	8033	40	1.2573	1.2569	4334
50	6338	6707	8099	50	1.5613	1.5612	7374

Table XIX. To convert feet on the Terrestrial Spheroid into seconds of Arc, and conversely.

Lat.	Log M.		Azimuth from the Meridian, or Z, Log O.								Log P.	
	0° 360°	Diff.	10° 350°	20° 340°	30° 330°	40° 320°	50° 310°	60° 300°	70° 290°	80° 280°	90° 270°	D.
0	7.9967088	13	66216	63706	59856	55129	50092	45353	41468	38965	38086	5
1	67075	40	66203	63693	59844	55120	50085	45347	41481	38960	38081	13
2	67035	66	66166	63656	59812	55091	50059	45327	41466	38945	38068	22
3	66969	93	66101	63595	59755	55042	50020	45295	41438	38922	38046	31
4	66876	118	66008	63510	59679	54974	49963	45248	41402	38889	38015	39
5	66758	144	65893	63400	59579	54889	49892	45185	41353	38848	37976	48
6	66614	170	65753	63268	59460	54789	49803	45117	41293	38796	37928	57
7	66444	196	65585	63111	59318	54662	49700	45031	41223	38736	37871	65
8	66248	221	65394	62930	59156	54519	49581	44934	41143	38667	37806	74
9	66027	244	65177	62727	58970	54359	49445	44821	41051	38588	37732	81
10	7.9965783	270	64938	62502	58767	54182	49296	44701	40960	38503	37651	90
11	65513	297	64675	62251	58541	53985	49133	44564	40839	38407	37561	99
12	65216	321	64382	61978	58294	53770	48952	44416	40718	38303	37462	107
13	64895	345	64068	61682	58026	53539	48756	44255	40586	38188	37355	115
14	64550	368	63730	61363	57738	53286	48545	44083	40444	38036	37240	122
15	64182	391	63369	61024	57432	53021	48321	43900	40293	37937	37118	130
16	63791	411	62985	60663	57105	52736	48082	43704	40131	37800	36988	138
17	63380	432	62582	60283	56762	52438	47832	43498	39961	37653	36850	144
18	62948	456	62160	59886	56402	52125	47568	43282	39784	37501	36706	152
19	62492	479	61714	59465	56022	51793	47290	43053	39596	37339	36554	160
20	7.9962013	497	61244	59024	55622	51447	46998	42811	39399	37168	36394	166
21	61516	514	60758	58566	55207	51086	46694	42562	39194	36993	36228	172
22	61002	535	60254	58090	54778	50712	46381	42304	38980	36810	36056	178
23	60467	555	59728	57597	54334	50325	46053	42038	38761	36622	35878	184
24	59912	573	59184	57086	53871	49923	45717	41761	38534	36426	35694	191
25	59339	589	58623	56556	53391	49505	45371	41474	38298	36226	35503	196
26	58750	605	58046	56014	52901	49080	45009	41180	38055	36016	35307	202
27	58145	621	57453	55457	52396	48640	44639	40876	37806	35802	35105	207
28	57524	636	56843	54882	51878	48190	44261	40565	37550	35584	34898	212
29	56888	650	56221	54297	51348	47727	43874	40246	37288	35358	34686	217
30	7.9956238	662	55584	53653	50819	47257	43478	39920	37008	35128	34469	221
31	55576	674	54935	53089	50253	46775	43074	39589	36748	34893	34248	225
32	54902	687	54274	52465	49691	46288	42662	39253	36470	34655	34023	228
33	54215	698	53601	51831	49119	45793	42244	38907	36189	34412	33795	233
34	53517	711	52917	51183	48536	45283	41819	38559	35909	34167	33562	237
35	52806	717	52220	50532	47943	44768	41385	38202	35606	33915	33325	239
36	52089	724	51518	49869	47346	44248	40949	37844	35313	33661	33086	241
37	51365	732	50809	49202	46742	43723	40508	37482	35015	33405	32845	244
38	50633	740	50090	48527	46132	43193	40060	37116	34715	33147	32601	247
39	49893	744	49365	47845	45515	42655	39610	36745	34409	32884	32354	248
40	7.9949149	749	48637	47146	44893	42115	39156	36372	34099	32652	32106	250
41	48400	754	47902	46468	44269	41572	38700	35997	33793	32357	31856	251
42	47646	757	47164	45772	43640	41026	38241	35620	33484	32090	31606	252
43	46889	756	46422	45074	43009	40477	37779	35242	33173	31823	31353	253
44	46133	761	45681	44377	42378	39928	37319	34862	32862	31556	31100	253
45	45372	757	44934	43676	41744	39377	36856	34483	32547	31287	30847	252
46	44615	759	44193	42977	41114	38828	35394	34102	32236	31019	30595	253
47	43856	758	43449	42276	40483	38278	35931	33723	31925	30750	30342	253
48	43098	754	42707	41578	39849	37727	35468	33344	31612	30483	30089	251
49	42344	748	41968	40883	39221	37181	35011	32967	31302	30217	29838	250
50	7.9941596	746	41232	40192	38596	36639	34555	32593	30995	29952	29588	248
51	40850	742	40503	39505	37975	35100	34101	32220	30687	29688	29340	247
52	40108	734	39777	38820	37357	35560	33648	31848	30383	29427	29093	245
53	39374	726	39057	38143	36745	35028	33201	31481	30081	29166	28848	242
54	38648	720	38347	37474	36139	34504	32759	31118	29782	28910	28606	240
55	37928	708	37641	36810	35539	33980	32320	30758	29485	28655	28356	236
56	37220	701	36947	36158	34938	33466	31888	30403	29193	28405	28130	235
57	36519	691	36258	35508	34362	32953	31459	30051	28905	28155	27896	232
58	35828	676	35579	34869	33784	32455	31036	29700	28618	27910	27663	223
59	35152	665	34918	34251	33226	31969	30629	29368	28344	27674	27440	221
	180° 180°		170° 190°	160° 200°	150° 210°	140° 220°	130° 230°	120° 240°	110° 250°	100° 260°	90° 270°	

Table XIX. To convert feet on the Terrestrial Spheroid into Seconds of Arc, and conversely.

Lat.	Log. M. 0° 360°	Diff.	Azimuth from the Meridian, or Z, Log O 10° 350°	20° 340°	30° 330°	40° 320°	50° 310°	60° 300°	70° 290	80° 280°	Log P. 90° 270	D.
60°	7.9934487	652	34268	33638	32669	31468	30223	29037	28070	27439	27219	217
61	33835	638	33631	33036	32127	31013	29826	28711	27802	27209	27002	213
62	33197	623	33004	32447	31596	30551	29438	28391	27539	26984	26789	207
63	32574	608	32394	31874	31076	30099	29059	28080	27283	26764	26582	203
64	31966	591	31797	31312	30569	29658	28689	27776	27033	26549	26379	197
65	31375	572	31221	30769	30071	29231	28329	27481	26790	26339	26182	191
66	30803	557	30659	30240	29600	28816	27980	27194	26553	26136	25991	186
67	30246	539	30111	29726	29135	28412	27640	26917	26325	25939	25805	179
68	29707	520	29584	29230	28687	28022	27316	26646	26103	25749	25626	174
69	29187	499	29074	28750	28252	27644	26996	26386	25889	25565	25452	166
70	7.9928688	478	28587	28290	27832	27283	26693	26135	25684	25389	25286	159
71	28210	458	28118	27851	27440	26939	26403	25898	25489	25222	25127	154
72	27752	437	27669	27426	27057	26604	26121	25668	25297	25057	24973	145
73	27315	417	27240	27024	26693	26288	25856	25450	25118	24904	24828	139
74	26898	392	26831	26640	26345	25985	25602	25241	24946	24756	24689	130
75	26506	368	26448	26278	26020	25703	25363	25045	24786	24619	24559	123
76	26138	344	26087	25939	25712	25436	25140	24861	24635	24488	24436	115
77	25794	322	25750	25622	25425	25186	24930	24688	24494	24366	24321	107
78	25472	296	25435	25325	25158	24953	24734	24528	24362	24254	24214	99
79	25176	273	25145	25052	24910	24739	24553	24379	24239	24147	24115	91
80	24903	250	24877	24799	24682	24541	24387	24244	24127	24048	24024	83
	180° 180°		170° 190°	160° 200°	150° 210°	140° 220°	130° 230°	120° 240°	110° 250°	100° 260°	90° 270°	

Table XX. To find the Seconds in the Intercepted Arc reduced for the effect of refraction, as used in the computation of Heights.

Lat	Log M′ 0°	Diff. Lat.	Azimuth from the Meridian Z, Log O′ 10°	20°	30°	40°	50°	60°	70°	80°	Log P′ 90°	Diff. Lat.	Co-lat.
0°	7.619958	131	9871	9620	9235	8762	8258	7784	7398	7145	7058	44	90°
10	9827	377	9743	9499	9126	8667	8179	7719	7345	7099	7014	126	80
20	9450	577	9373	9151	8811	8394	7949	7530	7189	6966	6888	192	70
30	8873	709	8807	8614	8331	7975	7597	7241	6950	6762	6696	236	60
40	8164	755	8113	7964	7738	7461	7165	6886	6659	6514	6460	252	50
50	7409	711	7372	7268	7109	6913	6705	6508	6348	6244	6208	237	40
60	6698	580	6676	6613	6516	6398	6271	6153	6056	5993	5971	193	30
70	6118	379	6108	6078	6032	5977	5918	5863	5817	5788	5778	127	20
80	5739	132	5738	5729	5717	5703	5688	5673	5662	5654	5651	44	10
90	5607		5607	5607	5607	5607	5607	5607	5607	5607	5607		0

Table XXI. To compute the Height of the place of Observation by the depression of the horizon of the sea.

Lat.	Log M″ 0°	Azimuth from the Meridian, or Z, Log O″ 10°	20°	30°	40°	50°	60°	70°	80°	Log P″ 90°	Colat
0°	6.454684	4771	5022	5407	5890	6384	6858	7244	7497	7584	90°
10	4815	4899	5143	5516	5975	6433	6923	7297	7543	7628	80
20	5192	5269	5491	5831	6248	6693	7112	7453	7676	7754	70
30	5769	5835	6028	6311	6667	7045	7401	7692	7880	7946	60
40	6478	6529	6678	6904	7181	7477	7756	7983	8128	8182	50
50	7233	7270	7374	7533	7729	7937	8134	8294	8398	8434	40
60	7944	7963	8029	8126	8244	8371	8489	8586	8649	8671	30
70	8524	8534	8564	8610	8665	8724	8779	8825	8854	8864	20
80	8903	8904	8913	8925	8939	8954	8969	8980	8988	8991	10
90	9035	9035	9035	9035	9035	9035	9035	9035	9035	9035	0
Lat.	180°	170°	160°	150°	140°	130°	120°	110°	100°	90°	Colat.

Table XXII. Reduction of λ to *l*. Subtractive.

p''	λ 0°	10°	20°	3)°	40°	50°	60°	70°	80°	90°
0 0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0 0 0.00
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0 1 0.00
2	0.00	0.01	0.02	0.03	0.04	0.06	0.08	0.13	0.27	0 2 0.00
3	0.00	0.02	0.03	0.05	0.08	0.11	0.16	0.26	0.54	0 3 0.00
4	0.00	0.03	0.05	0.08	0.12	0.17	0.25	0.39	0.81	0 4 0.00
5	0.00	0.04	0.09	0.14	0.20	0.28	0.41	0.65	1.35	0 5 0.00
6	0.00	0.06	0.12	0.19	0.28	0.40	0.58	0.91	1.89	0 6 0.00
7	0.00	0.08	0.16	0.25	0.36	0.54	0.73	1.17	2.43	0 7 0.00
8	0.00	0.10	0.21	0.33	0.48	0.68	0.99	1.57	3.24	0 8 0.00
9	0.00	0.13	0.26	0.41	0.60	0.85	1.23	1.96	4.05	0 9 0.00
0 10	0.00	0.15	0.31	0.49	0.72	1.00	1.48	2.38	4.85	0 10 0.00
11	0.00	0.18	0.38	0.60	0.88	1.20	1.80	2.90	5.80	0 11 0.00
12	0.00	0.22	0.45	0.72	1.05	1.46	2.20	3.40	7.00	0 12 0.00
13	0.00	0.26	0.53	0.86	1.23	1.72	2.60	4.00	8.25	0 13 0.00
14	0.00	0.30	0.62	1.00	1.44	2.00	3.00	4.70	9.55	0 14 0.00
15	0.00	0.34	0.72	1.14	1.66	2.30	3.40	5.40	11.00	0 15 0.00
16	0.00	0.39	0.82	1.30	1.88	2.64	3.87	6.20	12.60	0 16 0.00
17	0.00	0.44	0.92	1.46	2.13	3.00	4.44	7.05	14.25	0 17 0.00
18	0.00	0.50	1.03	1.64	2.38	3.37	5.00	7.90	16.00	0 18 0.00
19	0.00	0.55	1.14	1.82	2.64	3.75	5.50	8.70	17.70	0 19 0.00
0 20	0.00	0.61	1.26	2.01	2.92	4.11	6.00	9.50	19.54	0 20 0.00
21	0.00	0.67	1.40	2.22	3.23	4.55	6.68	10.60	21.70	0 21 0.00
22	0.00	0.74	1.53	2.43	3.56	5.00	7.30	11.70	23.70	0 22 0.00
23	0.00	0.81	1.67	2.65	3.88	5.45	8.00	12.80	26.00	0 23 0.00
24	0.00	0.88	1.83	2.90	4.23	6.00	8.74	13.90	28.40	0 24 0.00
25	0.00	0.96	1.98	3.15	4.58	6.50	9.50	15.00	30.85	0 25 0.00
26	0.00	1.05	2.14	3.40	4.94	7.00	10.28	16.20	33.40	0 26 0.00
27	0.00	1.14	2.32	3.68	5.34	7.55	11.10	17.50	36.00	0 27 0.00
28	0.00	1.22	2.49	3.95	5.74	8.12	11.90	18.80	38.65	0 28 0.00
29	0.00	1.30	2.67	4.25	6.14	8.73	12.70	20.20	41.35	0 29 0.00
0 30	0.00	1.38	2.85	4.53	6.58	9.34	13.60	21.50	44.20	0 30 0.00
31	0.00	1.48	3.04	4.84	7.03	10.00	14.60	23.10	47.50	0 31 0.00
32	0.00	1.58	3.23	5.16	7.47	10.60	15.50	24.50	50.50	0 32 0.00
33	0.00	1.68	3.43	5.50	7.94	11.30	16.50	26.10	53.75	0 33 0.00
34	0.00	1.78	3.65	5.83	8.44	12.00	17.50	27.70	57.00	0 34 0.00
35	0.00	1.88	3.88	6.17	8.94	12.73	18.50	29.40	1 0.50	0 35 0.00
36	0.00	1.99	4.11	6.53	9.48	13.45	19.60	31.10	1 4.00	0 36 0.00
37	0.00	2.11	4.34	6.90	10.00	14.20	20.70	32.90	1 7.75	0 37 0.00
38	0.00	2.23	4.57	7.28	10.55	15.00	21.80	34.60	1 11.25	0 38 0.00
39	0.00	2.35	4.82	7.66	11.12	15.80	23.00	36.40	1 15.00	0 39 0.00
0 40	0.00	2.46	5.09	8.06	11.72	16.62	24.25	38.39	1 19.00	0 40 0.00
41	0.00	2.58	5.35	8.45	12.30	17.47	25.40	40.30	1 23.00	0 41 0.00
42	0.00	2.71	5.60	8.85	12.90	18.34	26.69	42.30	1 27.00	0 42 0.00
43	0.00	2.84	5.88	9.28	13.54	19.20	28.00	44.40	1 31.30	0 43 0.00
44	0.00	2.98	6.15	9.73	14.20	20.10	29.32	46.50	1 35.60	0 44 0.00
45	0.00	3.12	6.43	10.20	14.84	21.05	30.67	48.60	1 40.00	0 45 0.00
46	0.00	3.26	6.72	10.67	15.50	22.00	32.03	50.80	1 44.50	0 46 0.00
47	0.00	3.40	7.02	11.13	16.14	22.95	33.40	53.00	1 49.00	0 47 0.00
48	0.00	3.54	7.32	11.60	16.84	23.94	34.80	55.20	1 53.70	0 48 0.00
49	0.00	3.69	7.63	12.08	17.54	24.96	36.23	57.50	1 58.60	0 49 0.00
0 50	0.00	3.84	7.95	12.58	18.27	26.06	37.65	59.90	2 3.30	0 50 0.00
51	0.00	3.99	8.26	13.10	19.04	27.06	39.30	1 2.40	2 8.40	0 51 0.00
52	0.00	4.17	8.58	13.61	19.80	28.10	40.90	1 4.80	2 13.50	0 52 0.00
53	0.00	4.32	8.92	14.13	20.54	29.17	42.50	1 7.30	2 18.60	0 53 0.00
54	0.00	4.49	9.27	14.67	21.34	30.35	44.10	1 9.90	2 23.80	0 54 0.00
55	0.00	4.66	9.62	15.23	22.14	31.45	45.70	1 12.50	2 29.20	0 55 0.00
56	0.00	4.82	9.97	15.80	22.94	32.58	47.40	1 15.20	2 34.80	0 56 0.00
57	0.00	5.00	10.32	16.35	23.80	33.75	49.10	1 17.90	2 40.40	0 57 0.00
58	0.00	5.18	10.67	16.98	24.64	34.95	50.83	1 20.70	2 46.00	0 58 0.00
59	0.00	5.36	11.05	17.51	25.50	36.20	52.64	1 23.60	2 51.80	0 59 0.00
1 0	0.00	5.54	11.43	18.15	26.34	37.45	54.30	1 26.40	2 57.80	1 0 0.00
1 10	0.00	7.54	15.55	24.68	35.88	50.90	1 13.90	1 57.36	4 1.54	1 10 0.00
1 20	0.00	9.85	20.33	32.25	46.84	1 6.50	1 36.74	2 33.26	5 15.27	1 20 0.00

Log Cor = log p'' · ½ sin 1″ + log tan λ, log ½ sin 1″ = 4.384545.

TABLE XXIII. Reduction of λ to *l*. Subtractive.

p''	λ 50°	51°	52°	53°	54°	55°	56°	57°	58°	59°
° ′	″	″	″	″	″	″	″	″	″	″
0 0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.08	0.08
3	0.11	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16
4	0.16	0.17	0.18	0.19	0.20	0.20	0.21	0.22	0.23	0.24
5	0.28	0.30	0.30	0.32	0.33	0.34	0.35	0.37	0.38	0.39
6	0.40	0.42	0.42	0.44	0.46	0.48	0.49	0.51	0.53	0.55
7	0.54	0.55	0.55	0.57	0.59	0.61	0.63	0.66	0.68	0.71
8	0.68	0.71	0.73	0.76	0.78	0.82	0.84	0.88	0.91	0.94
9	0.85	0.88	0.91	0.95	0.98	1.04	1.07	1.10	1.14	1.18
0 10	1.00	1.08	1.12	1.13	1.20	1.24	1.28	1.32	1.38	1.43
11	1.20	1.30	1.33	1.35	1.46	1.48	1.56	1.59	1.66	1.71
12	1.46	1.54	1.58	1.63	1.75	1.75	1.86	1.90	1.98	2.03
13	1.72	1.81	1.88	1.93	2.06	2.10	2.20	2.26	2.34	2.40
14	2.00	2.10	2.22	2.25	2.38	2.46	2.56	2.64	2.72	2.80
15	2.30	2.41	2.58	2.60	2.72	2.82	2.94	3.03	3.12	3.25
16	2.64	2.75	2.95	2.98	3.08	3.20	3.35	3.45	3.55	3.71
17	3.00	3.12	3.32	3.37	3.49	3.62	3.77	3.90	4.00	4.19
18	3.37	3.50	3.70	3.78	3.91	4.05	4.20	4.36	4.48	4.70
19	3.75	3.88	4.12	4.20	4.34	4.48	4.63	4.83	5.00	5.24
0 20	4.11	4.30	4.52	4.64	4.80	4.95	5.14	5.33	5.53	5.79
21	4.55	4.74	4.97	5.09	5.30	5.48	5.68	5.87	6.11	6.39
22	5.00	5.20	5.42	5.58	5.82	6.05	6.25	6.46	6.73	7.00
23	5.45	5.72	5.91	6.10	6.35	6.62	6.85	7.08	7.38	7.63
24	6.00	6.24	6.48	6.66	6.90	7.20	7.48	7.73	8.06	8.33
25	6.50	6.78	7.00	7.25	7.50	7.80	8.12	8.40	8.76	9.05
26	7.00	7.32	7.54	7.83	8.13	8.43	8.77	9.09	9.48	9.79
27	7.55	7.88	8.15	8.44	8.78	9.12	9.44	9.80	10.23	10.58
28	8.12	8.46	8.78	9.07	9.45	9.82	10.16	10.53	10.99	11.39
29	8.73	9.06	9.42	9.74	10.13	10.54	10.92	11.30	11.76	12.22
0 30	9.34	9.70	10.06	10.40	10.85	11.26	11.69	12.09	12.55	13.07
31	10.00	10.36	10.74	11.13	11.59	12.00	12.48	12.91	13.38	13.95
32	10.60	11.03	11.45	11.88	12.35	12.75	13.28	13.77	14.24	14.87
33	11.30	11.70	12.18	12.63	13.13	13.57	14.09	14.66	15.13	15.81
34	12.00	12.40	12.93	13.38	13.92	14.41	14.93	15.56	16.05	16.79
35	12.73	13.14	13.73	14.16	14.72	15.27	15.80	16.48	17.02	17.85
36	13.45	13.93	14.54	15.00	15.57	16.14	16.71	17.43	18.04	18.93
37	14.20	14.73	15.36	15.85	16.44	17.07	17.67	18.40	19.09	19.95
38	15.00	15.55	16.18	16.70	17.34	18.00	18.65	19.41	20.15	21.00
39	15.80	16.40	17.00	17.57	18.24	18.95	19.63	20.44	21.23	22.08
0 40	16.62	17.25	17.89	18.51	19.20	20.00	20.70	21.51	22.33	23.24
41	17.47	18.12	18.79	19.47	20.18	21.00	21.77	22.60	23.46	24.41
42	18.34	19.00	19.73	20.45	21.20	22.00	22.86	23.73	24.61	25.63
43	19.20	19.93	20.69	21.44	22.24	23.07	23.96	24.88	25.81	26.87
44	20.10	20.87	21.66	22.45	23.30	24.14	25.08	26.04	27.03	28.12
45	21.05	21.82	22.64	23.47	24.37	25.27	26.23	27.22	28.27	29.39
46	22.00	22.82	23.62	24.50	25.46	26.42	27.39	28.42	29.53	30.70
47	22.95	23.83	24.68	25.58	26.56	27.60	28.57	29.67	30.82	32.05
48	23.94	24.84	25.76	26.70	27.68	28.78	29.79	30.94	32.15	33.43
49	24.96	25.85	26.85	27.88	28.83	29.96	31.04	32.22	33.51	34.85
0 50	26.06	26.92	27.93	28.98	30.03	31.20	32.33	33.57	34.88	36.30
51	27.06	28.01	29.06	30.13	31.26	32.46	33.65	34.95	36.30	37.78
52	28.10	29.12	30.19	31.32	32.51	33.75	35.00	36.34	37.76	39.27
53	29.17	30.24	31.36	32.53	33.78	35.05	36.37	37.75	39.23	40.79
54	30.35	31.41	32.56	33.76	35.07	36.35	37.75	39.20	40.72	42.32
55	31.45	32.59	33.79	35.03	36.37	37.70	39.14	40.66	42.23	43.90
56	32.58	33.76	35.04	36.32	37.69	39.07	40.56	42.15	43.76	45.52
57	33.75	35.00	36.30	37.63	39.04	40.48	42.02	43.66	45.32	47.16
58	34.95	36.24	37.58	38.96	40.43	41.93	43.50	45.22	46.92	48.84
59	36.20	37.53	38.90	40.31	41.85	43.41	45.03	46.81	48.57	50.55
1 0	37.45	38.82	40.24	41.76	43.33	44.88	46.63	48.44	50.30	52.32
1 10	50.90	51.82	54.73	56.69	58.85	1′ 1.07	1′ 3.35	1′ 5.80	1′ 8.38	1′ 11.08
1 20	1′ 6.50	1′ 9.06	1′ 11.48	1′ 14.13	1′ 16.85	1 19.75	1 22.77	1 26.00	1 29.36	1 32.87

Table XXIV. To reduce a base at the level of the sea to any height above it, and conversely, &c.

h	$m\,h$ +	a	$p\,a^2$ —	S +	$\Delta_{,}$	Arg.	Eq. Δ_2
Feet.	Correction.	Feet.	Correction.	Reduction.	+		
1000	0.0000208	100000	0.0000004	0.0000008	26	1	0.8
2000	0.0000415	200000	0.0000017	0.0000034	44	2	1.4
3000	0.0000623	300000	0.0000037	0.0000078	60	3	1.8
4000	0.0000830	400000	0.0000065	0.0000138	78	4	2.0
5000	0.0001038	500000	0.0000103	0.0000216	94	5	2.1
6000	0.0001246	600000	0.0000149	0.0000310	112	6	2.0
7000	0.0001453	700000	0.0000203	0.0000422	130	7	1.8
8000	0.0001661	800000	0.0000265	0.0000552	147	8	1.4
9000	0.0001868	900000	0.0000335	0.0000699		9	0.8

To facilitate the calculation of arcs on the terrestrial spheroid, as well as various operations in Geodesy, the following table to $\frac{1}{305}$ of compression has been formed.

Table XXV. The measure of one minute of arc at each degree of latitude in English feet.

Latitude.	Minute of Latitude.	Minute of Longitude.	Minute of Perpendic.	Latitude.	Minute of Latitude.	Minute of Longitude.	Minute of Perpendic.
°	Feet.	Feet.	Feet.	°	Feet.	Feet.	Feet.
0	6045.9	6085.7	6085.7	45	6075.7	4310.3	6095.7
1	6045.9	6084.8	6085.7	46	6076.7	4234.7	6096.0
2	6046.0	6082.0	6085.7	47	6077.8	4157.7	6096.4
3	6046.0	6077.4	6085.8	48	6078.8	4079.5	6096.7
4	6046.1	6071.0	6085.8	49	6079.8	4000.0	6097.1
5	6046.3	6062.7	6085.9	50	6080.9	3919.3	6097.4
6	6046.5	6052.6	6085.9	51	6081.9	3837.4	6097.8
7	6046.7	6040.6	6086.0	52	6082.9	3754.4	6098.1
8	6047.0	6026.9	6086.1	53	6083.9	3670.1	6098.4
9	6047.3	6011.3	6086.2	54	6084.9	3584.8	6098.8
10	6047.6	5993.8	6086.3	55	6085.9	3498.3	6099.1
11	6048.0	5974.6	6086.4	56	6086.9	3410.8	6099.4
12	6048.4	5953.6	6086.6	57	6087.9	3322.2	6099.8
13	6048.9	5930.7	6086.7	58	6088.8	3232.5	6100.1
14	6049.3	5906.1	6086.9	59	6089.7	3141.9	6100.4
15	6049.8	5879.6	6087.0	60	6090.7	3050.3	6100.7
16	6050.4	5851.4	6087.2	61	6091.6	2957.8	6101.0
17	6050.9	5821.4	6087.4	62	6092.4	2864.4	6101.3
18	6051.5	5789.7	6087.6	63	6093.3	2770.1	6101.6
19	6052.2	5756.1	6087.8	64	6094.1	2674.9	6101.9
20	6052.8	5720.9	6088.0	65	6095.0	2578.9	6102.1
21	6053.5	5683.9	6088.3	66	6095.7	2482.1	6102.4
22	6054.2	5645.2	6088.5	67	6096.5	2384.5	6102.7
23	6054.9	5604.7	6088.7	68	6097.3	2286.2	6102.9
24	6055.7	5562.6	6089.0	69	6098.0	2187.2	6103.1
25	6056.5	5518.7	6089.3	70	6098.7	2087.5	6103.4
26	6057.3	5473.2	6089.5	71	6099.3	1987.1	6103.6
27	6058.1	5426.1	6089.8	72	6100.0	1886.2	6103.8
28	6059.0	5377.2	6090.1	73	6100.6	1784.6	6104.0
29	6059.8	5326.8	6090.3	74	6101.1	1682.5	6104.2
30	6060.7	5274.7	6090.7	75	6101.7	1579.9	6104.4
31	6061.6	5221.0	6091.0	76	6102.2	1476.8	6104.5
32	6062.6	5165.7	6091.3	77	6102.7	1373.3	6104.7
33	6063.5	5108.9	6091.6	78	6103.1	1269.3	6104.8
34	6064.5	5050.4	6091.9	79	6103.5	1164.9	6105.0
35	6065.4	4990.5	6092.3	80	6103.9	1060.1	6105.1
36	6066.4	4929.0	6092.6	81	6104.2	955.1	6105.2
37	6067.4	4866.0	6092.9	82	6104.6	849.7	6105.3
38	6068.4	4801.6	6093.3	83	6104.8	744.1	6105.4
39	6069.5	4735.5	6093.6	84	6105.1	638.2	6105.5
40	6070.5	4668.2	6093.9	85	6105.3	532.1	6105.6
41	6071.5	4599.4	6094.3	86	6105.4	425.9	6105.6
42	6072.5	4529.2	6094.6	87	6105.6	319.5	6105.7
43	6073.6	4457.6	6095.0	88	6105.6	213.1	6105.7
44	6074.6	4384.6	6095.3	89	6105.7	106.6	6105.7
45	6075.7	4310.3	6095.7	90	6105.7	0.0	6105.7

TABLE XXVI.
hange mean Solar into Sidereal Time.

TABLE XXVII.
To change Sidereal into mean Solar Time.

Add (h. m. s.)	Solar Min.	Add Seconds (s.)	Solar Sec.	Add Parts of a Sec. (s.)	Sidereal Days.	Subtract (h. m. s.)	Sider. Min.	Subtract Seconds (s.)	Sider. Sec.	Subtr. Pts. of a Sec.
0 3 56.555	1	0.164	1	0.003	1	0 3 55.909	1	0.164	1	0.003
0 7 53.111	2	0.329	2	0.006	2	0 7 51.819	2	0.328	2	0.005
0 11 49.666	3	0.493	3	0.008	3	0 11 47.728	3	0.491	3	0.008
0 15 46.221	4	0.658	4	0.011	4	0 15 43.638	4	0.655	4	0.011
0 19 42.777	5	0.822	5	0.014	5	0 19 39.547	5	0.819	5	0.014
0 23 39.332	6	0.986	6	0.017	6	0 23 35.457	6	0.983	6	0.016
0 27 35.887	7	1.150	7	0.019	7	0 27 31.366	7	1.147	7	0.019
0 31 32.443	8	1.315	8	0.022	8	0 31 27.276	8	1.311	8	0.022
0 35 28.998	9	1.479	9	0.025	9	0 35 23.185	9	1.474	9	0.025
0 39 25.553	10	1.643	10	0.027	10	0 39 19.094	10	1.638	10	0.027
0 43 22.109	11	1.807	11	0.030	11	0 43 15.004	11	1.802	11	0.030
0 47 18.664	12	1.972	12	0.033	12	0 47 10.913	12	1.966	12	0.032
0 51 15.220	13	2.136	13	0.036	13	0 51 6.823	13	2.130	13	0.035
0 55 11.775	14	2.300	14	0.038	14	0 55 2.732	14	2.294	14	0.038
0 59 8.330	15	2.464	15	0.041	15	0 58 58.642	15	2.457	15	0.041
1 3 4.886	16	2.629	16	0.044	16	1 2 54.551	16	2.621	16	0.044
1 7 1.441	17	2.793	17	0.047	17	1 6 50.461	17	2.785	17	0.046
1 10 57.996	18	2.957	18	0.050	18	1 10 46.370	18	2.949	18	0.049
1 14 54.552	19	3.121	19	0.053	19	1 14 42.280	19	3.113	19	0.052
1 18 51.107	20	3.286	20	0.055	20	1 18 38.189	20	3.277	20	0.055
1 22 46.662	21	3.450	21	0.058	21	1 22 34.098	21	3.440	21	0.057
1 26 44.218	22	3.614	22	0.061	22	1 26 30.008	22	3.604	22	0.060
1 30 40.773	23	3.779	23	0.064	23	1 30 25.917	23	3.768	23	0.063
1 34 37.328	24	3.943	24	0.066	24	1 34 21.827	24	3.932	24	0.066
1 38 33.884	25	4.108	25	0.069	25	1 38 17.736	25	4.096	25	0.068
1 42 30.439	23	4.272	26	0.072	26	1 42 13.646	26	4.259	26	0.071
1 46 26.994	27	4.436	27	0.075	27	1 46 9.555	27	4.423	27	0.074
1 50 23.550	28	4.600	28	0.077	28	1 50 5.465	28	4.587	28	0.076
1 54 20.105	29	4.764	29	0.080	29	1 54 1.374	29	4.751	29	0.079
1 58 16.660	30	4.928	30	0.082	30	1 57 57.283	30	4.915	30	0.082
2 2 13.216	31	5.092	31	0.085	31	2 1 53.193	31	5.079	31	0.085
2 6 9.771	32	5.257	32	0.088	32	2 5 49.102	32	5.242	32	0.087
2 10 6.326	33	5.421	33	0.091	33	2 9 45.012	33	5.406	33	0.090
2 14 2.882	34	5.585	34	0.094	34	2 13 40.921	34	5.570	34	0.093
2 17 59.437	35	5.750	35	0.097	35	2 17 36.831	35	5.734	35	0.096
m. s.	33	5.914	36	0.100	Sid. Hrs	m. s.	36	5.898	36	0.098
0 9.8565	37	6.078	37	0.102	1	0 9.829	37	6.062	37	0.101
0 19.713	38	6.242	38	0.105	2	0 19.659	38	6.225	38	0.104
0 29.569	39	6.407	39	0.107	3	0 29.489	39	6.389	39	0.106
0 39.426	40	6.571	40	0.110	4	0 39.318	40	6.553	40	0.109
0 49.282	41	6.735	41	0.113	5	0 49.148	41	6.717	41	0.112
0 59.139	42	6.900	42	0.116	6	0 58.977	42	6.881	42	0.115
1 8.995	43	7.064	43	0.119	7	1 8.807	43	7.044	43	0.117
1 18.852	44	7.228	44	0.121	8	1 18.636	44	7.208	44	0.120
1 28.708	45	7.393	45	0.124	9	1 28.466	45	7.372	45	0.123
1 38.565	46	7.557	46	0.127	10	1 38.296	46	7.536	46	0.126
1 48.421	47	7.722	47	0.129	11	1 48.125	47	7.699	47	0.128
1 58.278	48	7.886	48	0.132	12	1 57.955	48	7.864	48	0.131
2 8.134	49	8.050	49	0.136	13	2 7.784	49	8.027	49	0.134
2 17.991	50	8.214	50	0.138	14	2 17.614	50	8.191	50	0.137
2 27.847	51	8.378	51	0.141	15	2 27.442	51	8.355	51	0.139
2 37.704	52	8.543	52	0.143	16	2 37.272	52	8.519	52	0.142
2 47.560	53	8.707	53	0.146	17	2 47.103	53	8.683	53	0.145
2 57.416	54	8.872	54	0.149	18	2 56.932	54	8.846	54	0.147
3 7.273	55	9.036	55	0.151	19	3 6.762	55	9.010	55	0.150
3 17.129	56	9.200	56	0.154	20	3 16.591	56	9.174	56	0.153
3 26.986	57	9.364	57	0.157	21	3 26.421	57	9.338	57	0.156
3 36.841	58	9.528	58	0.159	22	3 36.249	58	9.502	58	0.158
3 46.700	59	9.692	59	0.162	23	3 46.080	59	9.666	59	0.161
3 56.555	60	9.856	60	0.164	24	3 55.909	60	9.829	60	0.164

Acceleration. | Retardation.

TABLE XXVIII. Space into Time.
To convert Degrees and parts of the Equator into Sidereal Time.

°	h. m.	′	m. s.	″	s.
1	0 4	1	0 4	1	0.066
2	0 8	2	0 8	2	0.133
3	0 12	3	0 12	3	0.200
4	0 16	4	0 16	4	0.266
5	0 20	5	0 20	5	0.333
6	0 24	6	0 24	6	0.400
7	0 28	7	0 28	7	0.466
8	0 32	8	0 32	8	0.533
9	0 36	9	0 36	9	0.600
10	0 40	10	0 40	10	0.666
11	0 44	11	0 44	11	0.733
12	0 48	12	0 48	12	0.799
13	0 52	13	0 52	13	0.866
14	0 56	14	0 56	14	0.933
15	1 0	15	1 0	15	1.000
16	1 4	16	1 4	16	1.066
17	1 8	17	1 8	17	1.133
18	1 12	18	1 12	18	1.200
19	1 16	19	1 16	19	1.266
20	1 20	20	1 20	20	1.333
25	1 40	21	1 24	21	1.400
30	2 0	22	1 28	22	1.466
35	2 20	23	1 32	23	1.533
40	2 40	24	1 36	24	1.600
45	3 0	25	1 40	25	1.666
50	3 20	26	1 44	26	1.733
55	3 40	27	1 48	27	1.799
60	4 0	28	1 52	28	1.866
65	4 20	29	1 56	29	1.933
70	4 40	30	2 0	30	2.000
75	5 0	31	2 4	31	2.066
80	5 20	32	2 8	32	2.133
90	6 0	33	2 12	33	2.200
100	6 40	34	2 16	34	2.266
110	7 20	35	2 20	35	2.333
120	8 0	36	2 24	36	2.400
130	8 40	37	2 28	37	2.466
140	9 20	38	2 32	38	2.533
150	10 0	39	2 36	39	2.600
160	10 40	40	2 40	40	2.666
170	11 20	41	2 44	41	2.733
180	12 0	42	2 48	42	2.799
190	12 40	43	2 52	43	2.866
200	13 20	44	2 56	44	2.933
210	14 0	45	3 0	45	3.000
220	14 40	46	3 4	46	3.066
230	15 20	47	3 8	47	3.133
240	16 0	48	3 12	48	3.200
250	16 40	49	3 16	49	3.266
260	17 20	50	3 20	50	3.333
270	18 0	51	3 24	51	3.400
280	18 40	52	3 28	52	3.466
290	19 20	53	3 32	53	3.533
300	20 0	54	3 36	54	3.600
310	20 40	55	3 40	55	3.666
320	21 20	56	3 44	56	3.733
330	22 0	57	3 48	57	3.799
340	22 40	58	3 52	58	3.866
350	23 20	59	3 56	59	3.933
360	24 0	60	4 0	60	4.000

Or to convert Degrees and parts of Terrestrial Longitude into Time.

TABLE XXIX. Time into Space.
To convert Sidereal Time into Degrees and Parts of the Equator.

h.	°
1	15
2	30
3	45
4	60
5	75
6	90
7	105
8	120
9	135
10	150
11	165
12	180
13	195
14	210
15	225
16	240
17	255
18	270
19	285
20	300
21	315
22	330
23	345
24	360

Tenths.

s.	″
0.1	1.5
0.2	3.0
0.3	4.5
0.4	6.0
0.5	7.5
0.6	9.0
0.7	10.5
0.8	12.0
0.9	13.5
1.0	15.0

Hundredths.

s.	″
0.01	0.15
0.02	0.30
0.03	0.45
0.04	0.60
0.05	0.75
0.06	0.90
0.07	1.05
0.08	1.20
0.09	1.35
0.10	1.50

Thousandths.

s.	″
0.001	0.015
0.002	0.030
0.003	0.045
0.004	0.060
0.005	0.075
0.006	0.090
0.007	0.105
0.008	0.120
0.009	0.135
0.010	0.150

m.	° ′	s.	′ ″
1	0 15	1	0 15
2	0 30	2	0 30
3	0 45	3	0 45
4	1 0	4	1 0
5	1 15	5	1 15
6	1 30	6	1 30
7	1 45	7	1 45
8	2 0	8	2 0
9	2 15	9	2 15
10	2 30	10	2 30
11	2 45	11	2 45
12	3 0	12	3 0
13	3 15	13	3 15
14	3 30	14	3 30
15	3 45	15	3 45
16	4 0	16	4 0
17	4 15	17	4 15
18	4 30	18	4 30
19	4 45	19	4 45
20	5 0	20	5 0
21	5 15	21	5 15
22	5 30	22	5 30
23	5 45	23	5 45
24	6 0	24	6 0
25	6 15	25	6 15
26	6 30	26	6 30
27	6 45	27	6 45
28	7 0	28	7 0
29	7 15	29	7 15
30	7 30	30	7 30
31	7 45	31	7 45
32	8 0	32	8 0
33	8 15	33	8 15
34	8 30	34	8 30
35	8 45	35	8 45
36	9 0	36	9 0
37	9 15	37	9 15
38	9 30	38	9 30
39	9 45	39	9 45
40	10 0	40	10 0
41	10 15	41	10 15
42	10 30	42	10 30
43	10 45	43	10 45
44	11 0	44	11 0
45	11 15	45	11 15
46	11 30	46	11 30
47	11 45	47	11 45
48	12 0	48	12 0
49	12 15	49	12 15
50	12 30	50	12 30
51	12 45	51	12 45
52	13 0	52	13 0
53	13 15	53	13 15
54	13 30	54	13 30
55	13 45	55	13 45
56	14 0	56	14 0
57	14 15	57	14 15
58	14 30	58	14 30
59	14 45	59	14 45
60	15 0	60	15 0

Or to convert Time into Degrees and Parts of Terrestrial Longitude.

Table XXX. Diurnal Variations.

erv. hrs.	10′ m.	20′ m.	30′ m.	1′ m.	2′ m.	3′ m.	4′ m.	5′ m.	6′ m.	7′ m.	8′ m.	9′ m.	Interv. 12 hrs.
m.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	h. m.
0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0
30	0 12.5	0 25.0	0 37.5	0 1.2	0 2.5	0 3.7	0 5.0	0 6.2	0 7.5	0 8.7	0 10.0	0 11.2	0 15
0	0 25.0	0 50.0	1 15.0	0 2.5	0 5.0	0 7.5	0 10.0	0 12.5	0 15.0	0 17.5	0 20.0	0 22.5	0 30
30	0 37.5	1 15.0	1 32.5	0 3.7	0 7.5	0 11.2	0 15.0	0 18.7	0 22.5	0 26.2	0 30.0	0 33.7	0 45
0	0 50.0	1 40.0	2 30.0	0 5.0	0 10.0	0 15.0	0 20.0	0 25.0	0 30.0	0 35.0	0 40.0	0 45.0	1 0
30	1 2.5	2 5.0	3 7.5	0 6.2	0 12.5	0 18.7	0 25.0	0 31.2	0 37.5	0 43.7	0 50.0	0 56.2	1 15
0	1 15.0	2 30.0	3 45.0	0 7.5	0 15.0	0 22.5	0 30.0	0 37.5	0 45.0	0 52.5	1 0.0	1 7.5	1 30
30	1 27.5	2 55.0	4 22.5	0 8.7	0 17.5	0 26.2	0 35.0	0 43.7	0 52.5	1 1.2	1 10.0	1 18.7	1 45
0	1 40.0	3 20.0	5 0.0	0 10.0	0 20.0	0 30.0	0 40.0	0 50.0	1 0.0	1 10.0	1 20.0	1 30.0	2 0
30	1 52.5	3 45.0	5 37.5	0 11.2	0 22.5	0 33.7	0 45.0	0 56.2	1 7.5	1 18.7	1 30.0	1 41.2	2 15
0	2 5.0	4 10.0	6 15.0	0 12.5	0 25.0	0 37.5	0 50.0	1 2.5	1 15.0	1 27.5	1 40.0	1 52.5	2 30
30	2 17.5	4 35.0	6 52.5	0 13.7	0 27.5	0 41.2	0 55.0	1 8.7	1 22.5	1 36.2	1 50.0	2 3.7	2 45
0	2 30.0	5 0.0	7 30.0	0 15.0	0 30.0	0 45.0	1 0.0	1 15.0	1 30.0	1 45.0	2 0.0	2 15.0	3 0
30	2 42.5	5 25.0	8 7.5	0 16.2	0 32.5	0 48.7	1 5.0	1 21.2	1 37.5	1 53.7	2 10.0	2 26.2	3 15
0	2 55.0	5 50.0	8 45.0	0 17.5	0 35.0	0 52.5	1 10.0	1 27.5	1 45.0	2 2.5	2 20.0	2 37.5	3 30
30	3 7.5	6 15.0	9 22.5	0 18.7	0 37.5	0 56.2	1 15.0	1 33.7	1 52.5	2 11.2	2 30.0	2 48.7	3 45
0	3 20.0	6 40.0	10 0.0	0 20.0	0 40.0	1 0.0	1 20.0	1 40.0	2 0.0	2 20.0	2 40.0	3 0.0	4 0
30	3 32.5	7 5.0	10 37.5	0 21.2	0 42.5	1 3.7	1 25.0	1 46.2	2 7.5	2 28.7	2 50.0	3 11.2	4 15
0	3 45.0	7 30.0	11 15.0	0 22.5	0 45.0	1 7.5	1 30.0	1 52.5	2 15.0	2 37.5	3 0.0	3 22.5	4 30
30	3 57.5	7 55.0	11 52.5	0 23.7	0 47.5	1 11.2	1 35.0	1 58.7	2 22.5	2 46.2	3 10.0	3 33.7	4 45
0	4 10.0	8 20.0	12 30.0	0 25.0	0 50.0	1 15.0	1 40.0	2 5.0	2 30.0	2 55.0	3 20.0	3 45.0	5 0
30	4 22.5	8 45.0	13 7.5	0 26.2	0 52.5	1 18.7	1 45.0	2 11.2	2 37.5	3 3.7	3 30.0	3 56.2	5 15
0	4 35.0	9 10.0	13 45.0	0 27.5	0 55.0	1 22.5	1 50.0	2 17.5	2 45.0	3 12.5	3 40.0	4 7.5	5 30
30	4 47.5	9 35.0	14 22.5	0 28.7	0 57.5	1 26.2	1 55.0	2 23.7	2 52.5	3 21.2	3 50.0	4 18.7	5 45
0	5 0.0	10 0.0	15 0.0	0 30.0	1 0.0	1 30.0	2 0.0	2 30.0	3 0.0	3 30.0	4 0.0	4 30.0	6 0
30	5 12.5	10 25.0	15 37.5	0 31.2	1 2.5	1 33.7	2 5.0	2 36.2	3 7.5	3 38.7	4 10.0	4 41.2	6 15
0	5 25.0	10 50.0	16 15.0	0 32.5	1 5.0	1 37.5	2 10.0	2 42.5	3 15.0	3 47.5	4 20.0	4 52.5	6 30
30	5 37.5	11 15.0	16 52.5	0 33.7	1 7.5	1 41.2	2 15.0	2 48.7	3 22.5	3 56.2	4 30.0	5 3.7	6 45
0	5 50.0	11 40.0	17 30.0	0 35.0	1 10.0	1 45.0	2 20.0	2 55.0	3 30.0	4 5.0	4 40.0	5 15.0	7 0
30	6 2.5	12 5.0	18 7.5	0 36.2	1 12.5	1 48.7	2 25.0	3 1.2	3 37.5	4 13.7	4 50.0	5 26.2	7 15
0	6 15.0	12 30.0	18 45.0	0 37.5	1 15.0	1 52.5	2 30.0	3 7.5	3 45.0	4 22.5	5 0.0	5 37.5	7 30
30	6 27.5	12 55.0	19 22.5	0 38.7	1 17.5	1 56.2	2 35.0	3 13.7	3 52.5	4 31.2	5 10.0	5 48.7	7 45
0	6 40.0	13 20.0	20 0.0	0 40.0	1 20.0	2 0.0	2 40.0	3 20.0	4 0.0	4 40.0	5 20.0	6 0.0	8 0
30	6 52.5	13 45.0	20 37.5	0 41.2	1 22.5	2 3.7	2 45.0	3 26.2	4 7.5	4 48.7	5 30.0	6 11.2	8 15
0	7 5.0	14 10.0	21 15.0	0 42.5	1 25.0	2 7.5	2 50.0	3 32.5	4 15.0	4 57.5	5 40.0	6 22.5	8 30
30	7 17.5	14 35.0	21 52.5	0 43.7	1 27.5	2 11.2	2 55.0	3 38.7	4 22.5	5 6.2	5 50.0	6 33.7	8 45
0	7 30.0	15 0.0	22 30.0	0 45.0	1 30.0	2 15.0	3 0.0	3 45.0	4 30.0	5 15.0	6 0.0	6 45.0	9 0
30	7 42.5	15 25.0	23 7.5	0 46.2	1 32.5	2 18.7	3 5.0	3 51.2	4 37.5	5 23.7	6 10.0	6 56.2	9 15
0	7 55.0	15 50.0	23 45.0	0 47.5	1 35.0	2 22.5	3 10.0	3 57.5	4 45.0	5 32.5	6 20.0	7 7.5	9 30
30	8 7.5	16 15.0	24 22.5	0 48.7	1 37.5	2 26.2	3 15.0	4 3.7	4 52.5	5 41.2	6 30.0	7 18.7	9 45
0	8 20.0	16 40.0	25 0.0	0 50.0	1 40.0	2 30.0	3 20.0	4 10.0	5 0.0	5 50.0	6 40.0	7 30.0	10 0
30	8 32.5	17 5.0	25 37.5	0 51.2	1 42.5	2 33.7	3 25.0	4 16.2	5 7.5	5 58.7	6 50.0	7 41.2	10 15
0	8 45.0	17 30.0	26 15.0	0 52.5	1 45.0	2 37.5	3 30.0	4 22.5	5 15.0	6 7.5	7 0.0	7 52.5	10 30
30	8 57.5	17 55.0	26 52.5	0 53.7	1 47.5	2 41.2	3 35.0	4 28.7	5 22.5	6 16.2	7 10.0	8 3.7	10 45
0	9 10.0	18 20.0	27 30.0	0 55.0	1 50.0	2 45.0	3 40.0	4 35.0	5 30.0	6 25.0	7 20.0	8 15.0	11 0
30	9 22.5	18 45.0	28 7.5	0 56.2	1 52.5	2 48.7	3 45.0	4 41.2	5 37.5	6 33.7	7 30.0	8 26.2	11 15
0	9 35.0	19 10.0	28 45.0	0 57.5	1 55.0	2 52.5	3 50.0	4 47.5	5 45.0	6 42.5	7 40.0	8 37.5	11 30
30	9 47.5	19 35.0	29 22.5	0 58.7	1 57.5	2 56.2	3 55.0	4 53.7	5 52.5	6 51.2	7 50.0	8 48.7	11 45
0	10 0.0	20 0.0	30 0.0	1 0.0	2 0.0	3 0.0	4 0.0	5 0.0	6 0.0	7 0.0	8 0.0	9 0.0	12 0
n.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	m.
2	0.8	1.7	2.5	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.7	1
4	1.7	3.3	5.0	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.3	1.5	2
6	2.5	5.0	7.5	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	3
8	3.3	6.7	10.0	0.3	0.7	1.0	1.3	1.7	2.0	2.3	2.7	3.0	4
0	4.2	8.3	12.5	0.4	0.8	1.2	1.7	2.1	2.5	2.9	3.3	3.7	5
2	5.0	10.0	15.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	6
4	5.8	11.7	17.5	0.6	1.2	1.7	2.3	2.9	3.5	4.1	4.7	5.2	7
6	6.7	13.3	20.0	0.7	1.3	2.0	2.7	3.3	4.0	4.7	5.3	6.0	8
8	7.5	15.0	22.5	0.7	1.5	2.2	3.0	3.7	4.5	5.2	6.0	6.7	9
0	8.3	16.7	25.0	0.8	1.7	2.5	3.3	4.2	5.0	5.8	6.7	7.5	10
2	9.2	18.3	27.5	0.9	1.8	2.7	3.7	4.6	5.5	6.4	7.3	8.2	11
4	10.0	20.0	30.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	12
6	10.8	21.7	32.5	1.1	2.2	3.2	4.3	5.4	6.5	7.6	8.7	9.7	13
8	11.7	23.3	35.0	1.2	2.3	3.5	4.7	5.8	7.0	8.2	9.3	10.5	14

Table XXXI. Shewing the lengths of *horizontal lines* equivalent to the several ascending and descending planes, the length of the plane being unity; in reference to the different classes of Engines, including the gross load, with engine and tender.

Gradients.	First Class Engines, Load 100 tons. Equivalent Horizontal Planes.			First Class Engines, Load 50 tons. Equivalent Horizontal Planes.			Second Class Engines, Load 80 tons. Equivalent Horizontal Planes.			Second Class Engines, Load 40 tons. Equivalent Horizontal Planes.		
	As.	Dec.	Mean.	As.	Dec.	Mean.	As.	Dec.	Mean.	As.	Dec.	Mean.
1 in 90	2.50	1.00	1.75	1.99	1.00	1.49	2.60	1.00	1.80	2.07	1.00	1.53
95	2.42	1.00	1.71	1.94	1.00	1.47	2.51	1.00	1.75	2.02	1.00	1.51
100	2.39	1.00	1.69	1.89	1.00	1.44	2.44	1.00	1.72	1.97	1.00	1.48
110	2.23	1.00	1.61	1.81	1.00	1.40	2.38	1.00	1.69	1.88	1.00	1.44
120	2.12	1.00	1.56	1.74	1.00	1.37	2.20	1.00	1.60	1.80	1.00	1.40
130	2.04	1.00	1.52	1.68	1.00	1.34	2.10	1.00	1.55	1.74	1.00	1.37
140	1.96	1.00	1.46	1.64	1.00	1.32	2.03	1.00	1.51	1.69	1.00	1.34
160	1.84	0.83	1.33	1.56	0.83	1.20	1.90	0.83	1.36	1.60	0.83	1.21
180	1.79	0.83	1.31	1.49	0.83	1.16	1.80	0.83	1.31	1.53	0.83	1.18
200	1.67	0.83	1.25	1.44	0.83	1.13	1.72	0.83	1.27	1.48	0.83	1.15
250	1.53	0.83	1.18	1.36	0.83	1.09	1.58	0.83	1.20	1.42	0.83	1.12
300	1.45	0.83	1.14	1.30	0.83	1.06	1.48	0.83	1.15	1.32	0.83	1.07
350	1.38	0.83	1.10	1.25	0.83	1.04	1.41	0.83	1.12	1.27	0.83	1.05
400	1.33	0.83	1.08	1.22	0.83	1.02	1.36	0.83	1.09	1.24	0.83	1.03
500	1.27	0.83	1.05	1.18	0.83	1.01	1.28	0.83	1.05	1.19	0.83	1.01
750	1.18	0.83	1.01	1.12	0.88	1.00	1.19	0.83	1.01	1.13	0.88	1.00
1000	1.13	0.85	1.00	1.09	0.91	1.00	1.14	0.86	1.00	1.09	0.91	1.00
1500	1.09	0.90	1.00	1.06	0.94	1.00	1.09	0.91	1.00	1.07	0.94	1.00

Gradients.	Third Class Engines, Load 80 tons. Equivalent Horizontal Planes.			Third Class Engines, Load 40 tons. Equivalent Horizontal Planes.			Fourth Class Engines Load 60 tons. Equivalent Horizontal Planes.			Fourth Class Engines, Load 30 tons. Equivalent Horizontal Planes.		
	As.	Dec.	Mean.	As.	Dec.	Mean.	As.	Dec.	Mean.	As.	Dec.	Mean.
1 in 90	2.66	1.00	1.83	2.14	1.00	1.57	2.51	1.00	1.75	2.00	1.00	1.50
95	2.58	1.00	1.79	2.08	1.00	1.54	2.44	1.00	1.72	1.95	1.00	1.47
100	2.50	1.00	1.75	2.02	1.00	1.51	2.36	1.00	1.68	1.90	1.00	1.45
110	2.36	1.00	1.68	1.93	1.00	1.46	2.33	1.00	1.66	1.82	1.00	1.41
120	2.25	1.00	1.62	1.85	1.00	1.42	2.14	1.00	1.57	1.75	1.00	1.37
130	2.15	1.00	1.57	1.78	1.00	1.39	2.05	1.00	1.52	1.69	1.00	1.34
140	2.07	1.00	1.53	1.73	1.00	1.36	1.97	1.00	1.48	1.64	1.00	1.32
160	1.94	0.83	1.43	1.64	0.83	1.23	1.85	0.83	1.34	1.56	0.83	1.20
180	1.83	0.83	1.33	1.57	0.83	1.20	1.75	0.83	1.29	1.50	0.83	1.16
200	1.75	0.83	1.29	1.52	0.83	1.17	1.68	0.83	1.25	1.45	0.83	1.14
250	1.60	0.83	1.21	1.41	0.83	1.12	1.54	0.83	1.18	1.35	0.83	1.09
300	1.50	0.83	1.16	1.34	0.83	1.08	1.46	0.83	1.14	1.30	0.83	1.06
350	1.43	0.83	1.13	1.29	0.83	1.06	1.39	0.83	1.10	1.26	0.83	1.04
400	1.37	0.83	1.10	1.25	0.83	1.04	1.34	0.83	1.08	1.22	0.83	1.02
500	1.30	0.83	1.06	1.20	0.83	1.01	1.23	0.83	1.03	1.18	0.83	1.01
750	1.20	0.83	1.01	1.13	0.87	1.00	1.18	0.83	1.01	1.12	0.83	1.00
1000	1.15	0.85	1.00	1.10	0.90	1.00	1.13	0.87	1.00	1.09	0.91	1.00
1500	1.10	0.90	1.00	1.07	0.93	1.00	1.09	0.91	1.00	1.06	0.94	1.00

Mean Class Engines, Load 50 tons. By W. G.

Gradients.	Equivalent Horizontal Planes. As.	Dec.	Mean.	Gradients.	Equivalent Horizontal Planes. As.	Dec.	Mean.
1 in 200	1.28	0.72	1.00	1 in 500	1.12	0.88	1.00
250	1.25	0.75	1.00	550	1.10	0.90	1.00
300	1.22	0.78	1.00	600	1.08	0.92	1.00
350	1.20	0.80	1.00	650	1.05	0.95	1.00
400	1.17	0.83	1.00	700	1.03	0.97	1.00
450	1.15	0.85	1.00	Level	1.00	1.00	1.00

Velocities in Miles an Hour. By Dr Lardner.

Gradients.	Ascen. Plane.	Dec. Plane.	Mean or Level.
	Miles.	Miles.	Miles.
1 in 177	22.25	41.32	31.78
265	24.87	39.13	32.00
330	25.26	37.07	31.16
400	26.87	36.75	31.81
63[illegible]	27.35	34.30	30.82
590	27.37	33.16	30.26
650	29.03	32.58	30.80
Level or Mean .			31.23

Table XXXII. Computation of Cuttings and Embankments, the formation-level or base being 30 feet, and length one chain.

Depth of Cutting in Feet.	Slopes, 1 to 1. Half width at top in Feet.	Slopes, 1 to 1. Content in cubic yards per chain.	Slopes, 1 to 1. Content of 1 perpendicular foot in breadth.	Slopes, 1½ to 1. Half width at top in Feet.	Slopes, 1½ to 1. Content in cubic yards per chain.	Slopes, 1½ to 1. Content of 1 perpendicular foot in breadth.	Slopes, 2 to 1. Half width at top in Feet.	Slopes, 2 to 1. Content in cubic yards per chain.	Slopes, 2 to 1. Content of 1 perpendicular foot in breadth.
1	16	75.78	2.44	16.5	77.00	2.44	17	78.22	2.44
2	17	156.42	4.89	18.0	161.33	4.89	19	166.22	4.89
3	18	242.00	7.33	19.5	253.00	7.33	21	264.00	7.33
4	19	332.44	9.78	21.0	352.00	9.78	23	371.55	9.78
5	20	427.78	12.22	22.5	453.33	12.22	25	488.89	12.22
6	21	528.00	14.67	24.0	572.00	14.67	27	616.00	14.67
7	22	633.11	17.11	25.5	693.00	17.11	29	752.89	17.11
8	23	743.11	19.56	27.0	821.33	19.56	31	899.55	19.56
9	24	858.00	22.00	28.5	957.00	22.00	33	1056.00	22.00
10	25	977.78	24.44	30.0	1100.00	24.44	35	1222.22	24.44
11	26	1102.44	26.89	31.5	1250.33	26.89	37	1398.22	26.89
12	27	1232.00	29.33	33.0	1408.00	29.33	39	1584.00	29.33
13	28	1366.44	31.78	34.5	1573.00	31.78	41	1779.55	31.78
14	29	1505.78	34.22	36.0	1745.33	34.22	43	1984.89	34.22
15	30	1650.00	36.66	37.5	1925.00	36.66	45	2200.00	36.66
16	31	1799.11	39.11	39.0	2112.00	39.11	47	2424.89	39.11
17	32	1953.11	41.55	40.5	2306.33	41.55	49	2659.55	41.55
18	33	2112.00	43.99	42.0	2508.00	43.99	51	2904.00	43.99
19	34	2275.78	46.44	43.5	2717.00	46.44	53	3158.22	46.44
20	35	2444.44	48.89	45.0	2933.33	48.89	55	3422.22	48.89
21	36	2618.00	51.33	46.5	3157.00	51.33	57	3696.00	51.33
22	37	2796.44	53.77	48.0	3388.00	53.77	59	3979.55	53.77
23	38	2979.78	56.21	49.5	3626.33	56.21	61	4272.89	56.21
24	39	3168.00	58.66	51.0	3872.00	58.66	63	4576.00	58.66
25	40	3361.11	61.10	52.5	4125.00	61.10	65	4888.89	61.10
26	41	3599.11	63.55	54.0	4385.33	63.55	67	5211.55	63.55
27	42	3762.00	65.99	55.5	4653.00	65.99	69	5544.00	65.99
28	43	3969.78	68.43	57.0	4928.00	68.43	71	5886.22	68.43
29	44	4182.44	70.88	58.5	5210.33	70.88	73	6238.22	70.88
30	45	4400.00	73.32	60.0	5500.00	73.32	75	6600.00	73.32
31	46	4622.44	75.77	61.5	5797.00	75.77	77	6971.55	75.77
32	47	4849.78	78.22	63.0	6101.33	78.22	79	7352.89	78.22
33	48	5082.00	80.67	64.5	6413.00	80.67	81	7744.00	80.67
34	49	5319.11	83.11	66.0	6732.00	83.11	83	8144.89	83.11
35	50	5561.11	85.55	67.5	7058.33	85.55	85	8555.55	85.55
36	51	5808.00	88.00	69.0	7392.00	88.00	87	8976.00	88.00
37	52	6059.78	90.44	70.5	7733.00	90.44	89	9406.22	90.44
38	53	6316.44	92.89	72.0	8081.33	92.89	91	9846.22	92.89
39	54	6578.00	95.33	73.5	8437.00	95.33	93	10296.00	95.33
40	55	6844.44	97.77	75.0	8800.00	97.77	95	10755.55	97.77
41	56	7115.78	100.22	76.5	9170.33	100.22	97	11224.89	100.22
42	57	7392.00	102.66	78.0	9548.00	102.66	99	11704.00	102.66
43	58	7673.11	105.11	79.5	9933.00	105.11	101	12192.89	105.11
44	59	7959.11	107.55	81.0	10325.33	107.55	103	12691.55	107.55
45	60	8250.00	109.99	82.5	10725.00	109.99	105	13200.00	109.99
46	61	8545.78	112.44	84.0	11132.00	112.44	107	13718.22	112.44
47	62	8846.44	114.88	85.5	11546.33	114.88	109	14246.22	114.88
48	63	9152.00	117.33	87.0	11968.00	117.33	111	14784.00	117.33
49	64	9462.44	119.77	88.5	12397.00	119.77	113	15331.55	119.77
50	65	9777.78	122.21	90.0	12833.33	122.21	115	15888.89	122.21

Example of the use of this Table. If a cutting of one Imperial Chain of 100 links in length and 20 feet in depth, were executed on a base or formation-level of 30 feet, how many cubic yards of earth would be thrown out, the slopes being 2 to 1?

To depth 20 feet in the left hand column, and under slopes 2 to 1 at top, there will be found 3422.22 cubic yards. Mr Macneill's Tables give 51.85 x 66=3422.10 cubic yards.